Inverse Geodetic Problem: Computing ϕ, λ, h given X, Y, Z
Solution From Torge

Some useful Angle Functions

$$\mathrm{dd(ang)} := \begin{vmatrix} \mathrm{degree} \leftarrow \mathrm{floor(ang)} \\ \mathrm{mins} \leftarrow (\mathrm{ang} - \mathrm{degree})\cdot 100.0 \\ \mathrm{minutes} \leftarrow \mathrm{floor(mins)} \\ \mathrm{seconds} \leftarrow (\mathrm{mins} - \mathrm{minutes})\cdot 100.0 \\ \mathrm{degree} + \dfrac{\mathrm{minutes}}{60.0} + \dfrac{\mathrm{seconds}}{3600.0} \end{vmatrix}$$

$$\mathrm{r2d} := \frac{180}{\pi}$$

$$\mathrm{radians(ang)} := \begin{vmatrix} d \leftarrow \mathrm{dd(ang)} \\ d\cdot\dfrac{\pi}{180.0} \end{vmatrix}$$

$$\mathrm{dms(ang)} := \begin{vmatrix} \mathrm{degree} \leftarrow \mathrm{floor(ang)} \\ \mathrm{rem} \leftarrow (\mathrm{ang} - \mathrm{degree})\cdot 60 \\ \mathrm{mins} \leftarrow \mathrm{floor(rem)} \\ \mathrm{rem1} \leftarrow (\mathrm{rem} - \mathrm{mins}) \\ \mathrm{secs} \leftarrow \mathrm{rem1}\cdot 60.0 \\ \mathrm{degree} + \dfrac{\mathrm{mins}}{100} + \dfrac{\mathrm{secs}}{10000} \end{vmatrix}$$

From the Geodetic Reference System 1980:

$a := 6378137\mathrm{m}$

$e_2 := 0.00669438002290$

Given the following Cartesian Coordinates:

$X := 354327.587\mathrm{m}$

$Y := -4606955.685\mathrm{m}$

$Z := 4382483.757\mathrm{m}$

Solution

$$\lambda := \left(\mathrm{atan}\left(\frac{Y}{X}\right)\right) \qquad l := \mathrm{if}(\lambda < 0, -1, 1)$$

The longitude of the point is:

$$l\,\mathrm{dms}(|\lambda|\cdot\mathrm{r2d}) = -85.360704728$$

iterate ϕ

$$\phi_1 := \left(\mathrm{atan}\left(\frac{Z}{\sqrt{X^2 + Y^2}}\right)\right)$$

The value ϕ_1 represents the current estimate of the latitude. Here it is the initial estimate.

$$N := \frac{a}{\sqrt{1 - e_2\cdot\left(\sin\left(\phi_1\right)\right)^2}}$$

N is the radius of curvature in the prime vertical

$$N = 6388271.36419801\ \mathrm{m}$$

$h := 0\text{m}$ The initial estimate of the height is taken as zero.

$$\phi := \left[\text{atan}\left[\left(\frac{Z}{\sqrt{X^2 + Y^2}} \right) \cdot \left(1 - e_2 \cdot \frac{N}{N + h} \right)^{-1} \right] \right]$$

$\text{dms}(\phi \cdot \text{r2d}) = 43.403865442$

$\phi - \phi_1 = 0.00335432$

$\Delta\phi := (\phi - \phi_1) \cdot \text{r2d}$

$\text{dms}(|\Delta\phi|) = 0.1131878238$

Test to see how close the current estimate of the latitude is to the estimate used to determine the current estimate

$\Delta\phi = 0.1921883995$

$\phi_1 := \phi$

Updating the current estimate to the value just computed.

$$h := \frac{\sqrt{X^2 + Y^2}}{\cos(\phi)} - N$$

h is updated with a new value based on the value for the latitude.

$h = 429.968746328\ \text{m}$

Iteration No. 2

$$N := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi_1))^2}}$$

$N = 6388343.22979735\ \text{m}$

$$\phi := \left[\text{atan}\left[\left(\frac{Z}{\sqrt{X^2 + Y^2}} \right) \cdot \left(1 - e_2 \cdot \frac{N}{N + h} \right)^{-1} \right] \right]$$

$\text{dms}(\phi \cdot \text{r2d}) = 43.40386077$

$\phi - \phi_1 = -0.000000227$ $\Delta\phi := (\phi - \phi_1) \cdot \text{r2d}$

$\text{dms}(|\Delta\phi|) = 0.000004673$

$\phi_1 := \phi$

$$h := \frac{\sqrt{X^2 + Y^2}}{\cos(\phi)} - N$$

$h = 356.72114\ \text{m}$

Iteration No. 3

$$N := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi_1))^2}}$$

$N = 6388343.22494281\ \text{m}$

$$\phi := \left[\operatorname{atan}\left[\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\cdot\left(1-e_2\cdot\frac{N}{N+h}\right)^{-1}\right]\right]$$

$\operatorname{dms}(\phi\cdot r2d) = 43.403861566$

$\phi - \phi_1 = 0.000000039$

$\Delta\phi := (\phi - \phi_1)\cdot r2d$

$\operatorname{dms}(|\Delta\phi|) = 0.000000796$

$\phi_1 := \phi$

$$h := \frac{\sqrt{X^2+Y^2}}{\cos(\phi)} - N$$

$h = 356.96142\ m$

Iteration No.4

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_1))^2}}$$

$N = 6388343.22576977\ m$

$$\phi := \left[\operatorname{atan}\left[\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\cdot\left(1-e_2\cdot\frac{N}{N+h}\right)^{-1}\right]\right]$$

The latitude of the point is:

$\operatorname{dms}(\phi\cdot r2d) = 43.403861563$

$\phi - \phi_1 = -0$

$\Delta\phi := (\phi - \phi_1)\cdot r2d$

$\operatorname{dms}(|\Delta\phi|) = 0.000000003$

$$h := \frac{\sqrt{X^2+Y^2}}{\cos(\phi)} - N$$

The height of the point is:

$h = 356.95982\ m$