Inverse Geodetic Problem: Computing ϕ, λ, h given X, Y, Z
Solution by Hirvonen and Moritz

Some useful Angle Functions

$$dd(ang) := \begin{array}{|l} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \frac{minutes}{60.0} + \frac{seconds}{3600.0} \end{array}$$

$$radians(ang) := \begin{array}{|l} d \leftarrow dd(ang) \\ d\cdot\frac{\pi}{180.0} \end{array}$$

$$dms(ang) := \begin{array}{|l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \frac{mins}{100} + \frac{secs}{10000} \end{array}$$

$$r2d := \frac{180}{\pi}$$

From the Geodetic Reference System 1980:

$a := 6378137m$

$e_2 := 0.00669438002290$

Given the following Cartesian Coordinates:

$X := 354327.587m$

$Y := -4606955.685m$

$Z := 4382483.757m$

Solution

$$\lambda := \left(atan\left(\frac{Y}{X}\right)\right)$$

$$l := if(\lambda < 0, -1, 1)$$

The longitude of the point is:

$$l\,dms(|\lambda|\cdot r2d) = -85.360704728$$

iterate ϕ

$$\phi_1 := \left[atan\left[\left(\frac{1}{1-e_2}\right)\cdot\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\right]\right]$$

The value ϕ_1 represents the current estimate of the latitude. Here it is the initial estimate.

$$dms(\phi_1\cdot r2d) = 43.403865442$$

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_1))^2}}$$

N is the radius of curvature in the prime vertical

$$N = 6388343.22979735\ m$$

$$\phi := \left[\operatorname{atan}\left[\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\cdot\left(1+\frac{e_2\cdot N\cdot\sin(\phi_1)}{Z}\right)\right]\right]$$

Updating a new value for the latitude

$$\operatorname{dms}(\phi\cdot r2d) = 43.403861577$$

$$\Delta\phi := (\phi - \phi_1)\cdot r2d$$

$$\operatorname{dms}(|\Delta\phi|) = 0.000003866$$

The difference in the new estimate and the value used to compute this new estimate re compared for convergence

$$\Delta\phi = -0.000010738$$

$$\phi_1 := \phi$$

Updating the new current estimate of the latitude

Iteration No. 2

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_1))^2}}$$

$$N = 6388343.22578123 \text{ m}$$

$$\phi := \left[\operatorname{atan}\left[\left(\frac{Z}{\sqrt{X^2+Y^2}}\right)\cdot\left(1+\frac{e_2\cdot N\cdot\sin(\phi_1)}{Z}\right)\right]\right]$$

The latitude of the point is:

$$\operatorname{dms}(\phi\cdot r2d) = 43.403861563$$

$$\phi - \phi_1 = -0.000000001$$

$$\operatorname{dms}(|\Delta\phi|) = 0.000000014$$

Convergence criteria used here was 0.0002". If the criteria was smaller, another iteration would be required.

$$h := \frac{\sqrt{X^2+Y^2}}{\cos(\phi)} - N$$

The height of the point is:

$$h = 356.95983 \text{ m}$$