SODANO FORMULAS

- DEVELOPED FOR VERY LONG GEODESICS
- DIRECT SOLUTION

$$\tan\beta = (\tan\phi)(1-f) \qquad \text{when } \phi \leq 45^\circ$$

OR

$$\cot\beta = (\cot\phi)/(1-f) \qquad \text{when } \phi > 45^\circ$$

FORMULAS FOR ϕ_1 & ϕ_2

$$\cos\beta_0 = \cos\beta_1 \sin\alpha_{12}$$

$$g = \cos\beta_1 \cos\alpha_{12}$$

$$m_1 = \left(1 + \frac{e'^2}{2}\sin^2\beta_1\right)\left(1 - \cos^2\beta_0\right)$$

$$\phi_s = s/b$$

$$a_1 = \left(1 + \frac{e'^2}{2}\sin^2\beta_1\right)\left(\sin^2\beta_1 \cos\phi_s + g \sin\beta_1 \sin\phi_s\right)$$

$$\phi_0 = \phi_s + a_1\left(-\frac{e'^2}{2}\sin\phi_s\right) + m_1\left(-\frac{e'^2}{4}\phi_s + \frac{e'^2}{4}\sin\phi_s\cos\phi_s\right)$$

$$+ a_1^2\left(\frac{5e'^4}{8}\sin\phi_s\cos\phi_s\right)$$

$$+ m_1^2\left(\frac{11e'^4}{64}\phi_s + \frac{13e'^4}{64}\sin\phi_s\cos\phi_s - \frac{e'^4}{8}\phi_s\cos^2\phi_s\right.$$

$$\left. + \frac{5e'^4}{32}\sin\phi_s\cos^3\phi_s\right)$$

$$+ a_1 m_1\left(\frac{3e'^4}{8}\sin\phi_s + \frac{e'^4}{4}\phi_s\cos\phi_s - \frac{5e'^4}{8}\sin\phi_s\cos^2\phi_s\right)$$

$\phi_0 \rightarrow$ RADIANS

$$\cot\alpha_{21} = \frac{g\cos\phi_0 - \sin\beta_1\sin\phi_0}{\cos\beta_0}$$

FOR MERIDIONAL ARCS, CONSIDER α_{21} AS HAVING $0°$ REFERENCE & OBTAIN the SIGN of cot BY DISREGARDING DENOMINATOR.
FOR other GEODESICS, REPLACE cot by tan when $|\cot\alpha_{21}| > 1$, BY TAKING RECIPROCAL OF QUOTIENT'S VALUE

IF $(0° \leq \alpha_{12} \leq 180°)$ & cot (tan) of α_{21} is (+) or (-), α_{21} is IN QUADRANT III OR IV, RESPECTIVELY
IF $(180° \leq \alpha_{12} < 360°)$ & cot (tan) of α_{21} is (+) or (-), α_{21} is IN QUADRANT I OR II, RESPECTIVELY

$$\cot\lambda = \frac{\cos\beta_1 \cos\phi_0 - \sin\beta_1 \sin\phi_0 \cos\alpha_{12}}{\sin\phi_0 \sin\alpha_{12}}$$

FOR MERIDIONAL ARCS, CONSIDER λ AS HAVING $0°$ REFERENCE ANGLE, & OBTAIN SIGN of cot by DISREGARDING $\sin\alpha_{12}$
For other GEODESICS, REPLACE cot by tan when $|\cot\lambda| > 1$, BY TAKING the RECIPROCAL OF QUOTIENT'S VALUE

QUADRANT & SIGN OF λ:	WHEN $0° < \phi_0 \leq 180°$ ($\sin\phi_0$ CONSIDERED POSITIVE)	WHEN $180° < \phi_0 \leq 360°$ ($\sin\phi_0$ CONSIDERED NEGATIVE)
AND ($0° < \alpha_{12} \leq 180°$)	then if cot (tan) of λ is (+) or (−), λ is IN QUAD I or II RESPECTIVELY	then if cot (tan) of λ is (+) or (−), λ is IN QUAD III or IV RESPECTIVELY
AND ($180° < \alpha_{12} < 360°$)	then if cot (tan) of λ is (+) or (−), the ASSOCIATED λ IS IN QUAD III or IV RESPECTIVELY, & λ OBTAINED BY SUBTRACTING 360°	then if cot (tan) of λ is (+) or (−), the ASSOCIATED λ IS IN QUAD I or II RESPECTIVELY & λ is OBTAINED BY SUBTRACTING 360°

$$\frac{\Delta\lambda - \lambda}{\cos\beta_0} = -f\phi_0 + a_1\left(\frac{3}{2}f^2 \sin\phi_s\right)$$

$$+ m_1\left(\frac{3}{4}f^2\phi_s - \frac{3}{4}f^2 \sin\phi_s \cos\phi_s\right) \quad \text{RADIANS}$$

$$\lambda_2 = \lambda_1 + \Delta\lambda$$

IF $|\lambda_2| > 180°$, MODIFY λ_2 BY ADDING OR SUBTRACTING 360°, ACCORDING TO WHETHER IT IS INITIALLY POSITIVE OR NEG.

$$\sin\beta_2 = \sin\beta_1 \cos\sigma_0 + g \sin\sigma_0$$

$$\cos\beta_2 = \left[(\cos\beta_0)^2 + (g\cos\sigma_0 - \sin\beta_1 \sin\sigma_0)^2\right]^{1/2}$$

$\tan\beta_2 = \sin\beta_2 / \cos\beta_2$ USE WHICHEVER HAS

or

$\cot\beta_2 = \cos\beta_2 / \sin\beta_2$ SMALLER ABSOLUTE VALUE

then

$$\tan\phi_2 = \frac{\tan\beta_2}{(1-f)}$$

or

$$\cot\phi_2 = (1-f)\tan\beta_2$$

INVERSE SOLUTION

$\Delta\lambda = (\lambda_1 - \lambda_2)$ or $(\lambda_2 - \lambda_1)$ + sign opposite of $(\lambda_2-\lambda_1)(360°)$

USE WHICHEVER $\Delta\lambda$ HAS ABSOLUTE VALUE < OR > 180°,
ACCORDING TO WHETHER THE SHORTER OR BACK-SIDES LONGER
GEODESIC IS INTENDED
FOR MERIDIAN ARCS $(|\Delta\lambda| = 0°$ or $180°$ or $360°)$ USE EITHER $\Delta\lambda$
BUT CONSIDER IT (+) for SHORTER & (−) for LONGER

$\tan\beta = (\tan\phi)(1-f)$ when $|\phi| \le 45°$

or

$\cot\beta = \cot\phi/(1-f)$ when $|\phi| > 45°$

$$a_0 = \sin\beta_1 \sin\beta_2$$

$$b_0 = \cos\beta_1 \cos\beta_2$$

$$\cos\phi = a_0 + b_0 \cos\Delta\lambda$$

$$\sin\phi = \pm\left[(\sin\Delta\lambda \cos\beta_2)^2 + (\sin\beta_2 \cos\beta_1 - \sin\beta_1 \cos\beta_2 \cos\Delta\lambda)^2\right]^{1/2}$$

$\sin\phi$ is (+) for shorter arc & (−) for longer

$$c = \frac{(b_0 \sin\Delta\lambda)}{\sin\phi}$$

$$m = 1 - c^2$$

$$\begin{aligned}
s/b = {} & (1+f+f^2)\phi + a_0\left[(f+f^2)\sin\phi - \left(\frac{f^2}{2}\right)\phi^2 \csc\phi\right] \\
& + m\left[-\left(\frac{f+f^2}{2}\right)\phi - \left(\frac{f+f^2}{2}\right)\sin\phi\cos\phi + \left(\frac{f^2}{2}\right)\phi^2 \cot\phi\right] \\
& + a_0^2\left[-\left(\frac{f^2}{2}\right)\sin\phi\cos\phi\right] \\
& + m^2\left[\left(\frac{f^2}{16}\right)\phi + \left(\frac{f^2}{16}\right)\sin\phi\cos\phi - \left(\frac{f^2}{2}\right)\phi^2\cot\phi \right. \\
& \qquad \left. - \left(\frac{f^2}{8}\right)\sin\phi\cos^3\phi\right] \\
& + a_0 m\left[\left(\frac{f^2}{2}\right)\phi^2\csc\phi + \left(\frac{f^2}{2}\right)\sin\phi\cos^2\phi\right]
\end{aligned}$$

$$\frac{\lambda - \Delta\lambda}{c} = (f+f^2)\phi + a_0\left[-\left(\frac{f^2}{2}\right)\sin\phi - f^2\phi^2\csc\phi\right]$$

$$+m\left[-\left(\frac{5f^2}{4}\right)\phi + \left(\frac{f^2}{4}\right)\sin\phi\cos\phi + (f^2)\phi^2\cot\phi\right]$$

$$\cot\alpha_{12} = \frac{\sin\beta_2\cos\beta_1 - \cos\lambda\sin\beta_1\cos\beta_2}{\sin\lambda\cos\beta_2}$$

$$\cot\alpha_{21} = \frac{\sin\beta_2\cos\beta_1\cos\lambda - \sin\beta_1\cos\beta_2}{\sin\lambda\cos\beta_2}$$

IF $\Delta\lambda$ (+) & cot (tan) of α_{12} IS (+) OR (−), α_{12} IS IN QUAD I OR II, RESPECTIVELY

IF $\Delta\lambda$ (−) & cot (tan) of α_{12} IS (+) OR (−), α_{12} IS IN QUAD III OR IV, RESPECTIVELY

IF $\Delta\lambda$ (+) & cot (tan) of α_{21} IS (+) OR (−), α_{21} IS IN QUAD III OR IV, RESPECTIVELY

IF $\Delta\lambda$ (−) & cot (tan) of α_{21} IS (+) OR (−), α_{21} IS IN QUAD I OR II, RESPECTIVELY