Puissant Method - Direct Problem

Some useful angle functions:

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang + 0.0000000001) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins + 0.0000000001) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \frac{minutes}{60.0} + \frac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d\cdot \frac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \frac{mins}{100} + \frac{secs}{10000} \end{array} \right.$$

$$r2d := \frac{180.0}{\pi}$$

Constants:

$a := 6378160$

$b := 6356774.7193$

$$e2 := \frac{a^2 - b^2}{a^2}$$

$e2 = 0.0066945418$ e2 is the first eccentricity squared

Given data:

$\phi := -37.39155571$

$\lambda := 43.55306630$

$p := if(\phi < 0, -1, 1)$

$l := if(\lambda < 0, -1, 1)$

$\phi_1 := p\cdot radians(|\phi|)$

$\lambda_1 := l\cdot radians(|\lambda|)$

$s := 54972.161$

$\alpha_{12} := radians(127.1027080)$

Solution:

$$M_1 := \frac{a\cdot(1 - e2)}{\sqrt{\left[1 - e2\cdot\left(\sin\left(\phi_1\right)\right)^2\right]^3}}$$

$M_1 = 6359277.924313$

M_1 is the radius of curvature in the meridian for the first latitude while N_1 is the radius of curvature in the prime vertical

$$N_1 := \frac{a}{\sqrt{1 - e2\cdot\left(\sin\left(\phi_1\right)\right)^2}}$$

$N_1 = 6386142.43900121$

$$B := \frac{1}{M_1} \qquad B = 0.000000157$$

$$C := \frac{\tan(\phi_1)}{2 \cdot M_1 \cdot N_1} \qquad C = -9.50002 \times 10^{-15}$$

$$D := \frac{3 \cdot e2 \cdot \sin(\phi_1) \cdot \cos(\phi_1)}{2 \cdot \left[1 - e2 \cdot \left(\sin(\phi_1)^2\right)\right]} \qquad D = -0.004869$$

$$E := \frac{1 + 3 \cdot (\tan(\phi_1))^2}{6 \cdot N_1^2} \qquad E = 1.138621 \times 10^{-14}$$

$$h := \frac{s \cdot \cos(\alpha_{12})}{M_1} \qquad h = -0.005223$$

$$\delta\phi := s \cdot \cos(\alpha_{12}) \cdot B - s^2 \cdot (\sin(\alpha_{12}))^2 \cdot C - h \cdot s^2 \cdot (\sin(\alpha_{12}))^2 \cdot E$$

$$\Delta\phi := s \cdot \cos(\alpha_{12}) \cdot B - s^2 \cdot (\sin(\alpha_{12}))^2 \cdot C - h \cdot s^2 \cdot (\sin(\alpha_{12}))^2 \cdot E - \delta\phi^2 \cdot D$$

$$\phi_2 := \begin{vmatrix} \text{dms}\left[(\phi_1 + \Delta\phi) \cdot \text{r2d}\right] \ \text{ if } \left[(\phi_1 + \Delta\phi) > 0\right] \\ \text{dms}\left[(|\phi_1 + \Delta\phi|) \cdot \text{r2d}\right] \cdot (-1) \ \text{ if } \left[(\phi_1 + \Delta\phi) < 0\right] \end{matrix}$$

The latitude of the second point is:

$$\phi_2 = -37.570912894$$

$$p := \text{if}(\phi_2 < 0, -1, 1)$$

$$\phi_2 := p \cdot \text{radians}(|\phi_2|)$$

$$N_2 := \frac{a}{\sqrt{1 - e2 \cdot (\sin(\phi_2))^2}} \qquad N_2 = 6386250.478147$$

N_2 is the radius of curvature in the prime vertical for the second latitude

$$\Delta\lambda := \frac{s}{N_2} \cdot \frac{\sin(\alpha_{12})}{\cos(\phi_2)} \cdot \left[1 - \frac{s^2}{6 \cdot N_2^2} \cdot \left[1 - \frac{(\sin(\alpha_{12}))^2}{(\cos(\phi_2))^2}\right]\right]$$

$$\lambda_2 := \begin{cases} \text{dms}\left[\left(\lambda_1 + \Delta\lambda\right)\cdot \text{r2d}\right] & \text{if } \left[\left(\lambda_1 + \Delta\lambda\right) > 0\right] \\ \text{dms}\left[\left(\left|\lambda_1 + \Delta\lambda\right|\right)\cdot \text{r2d}\right]\cdot(-1) & \text{if } \left[\left(\lambda_1 + \Delta\lambda\right) < 0\right] \end{cases}$$

The longitude of the second point is:

$\lambda_2 = 44.252481670$

$$\phi_m := \frac{\phi_1 + \phi_2}{2}$$

ϕ_m is the mean latitude

$$\Delta\alpha := \frac{\Delta\lambda\cdot\sin\left(\phi_m\right)}{\cos\left(\frac{\Delta\phi}{2}\right)} + \frac{\Delta\lambda^3}{12}\cdot\left[\frac{\sin\left(\phi_m\right)}{\cos\left(\frac{\Delta\phi}{2}\right)} - \frac{\left(\sin\left(\phi_m\right)\right)^3}{\left(\cos\left(\frac{\Delta\phi}{2}\right)\right)^3}\right]$$

$$\alpha_{21} := \left(\alpha_{12} + \Delta\alpha + \pi\right)\cdot \text{r2d}$$

The back azimuth from point 2 to point 1 is:

$\text{dms}\left(\alpha_{21}\right) = 306.52073377$