Ferris State College
Construction Department

INVERSE PROBLEM FOR SPACE RECTANGULAR COORDINATES

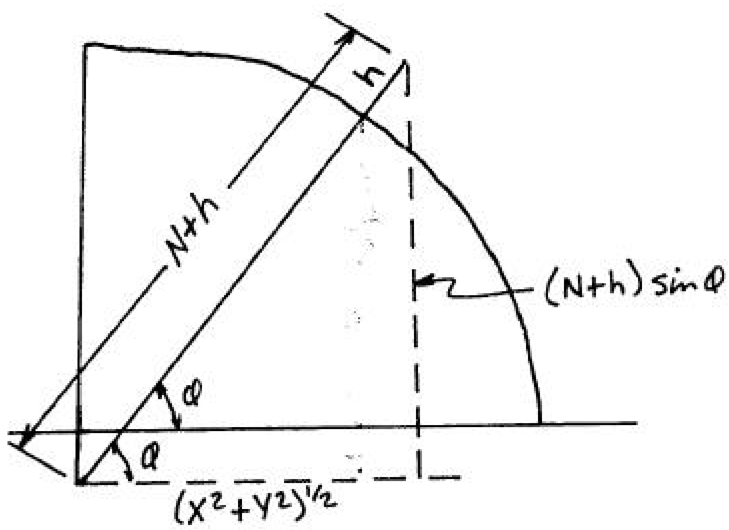
- GIVEN φ, λ & h, COMPUTE SPACE RECTANGULAR COORDINATES AS

$$X = (N+h)\cos\varphi\cos\lambda$$

$$Y = (N+h)\cos\varphi\sin\lambda$$

$$Z = \left[N(1-e^2)+h\right]\sin\varphi$$

- INVERSE PROBLEM – COMPUTE φ, λ, h GIVEN X, Y, Z & ELLIPSOID PARAMETERS
- NOT STRAIGHTFORWARD SINCE N FUNCTION OF φ
- SOLUTION BY HIRVONEN & MORITZ



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- FIND LONGITUDE BY

$$\tan \lambda = \frac{Y}{X}$$

- FROM FIGURE,

$$\tan \varphi = \frac{(N+h)\sin \varphi}{\sqrt{X^2+Y^2}}$$

- BUT, $Z = N \sin \varphi - e^2 N \sin \varphi + h \sin \varphi$

OR, $(N+h)\sin \varphi = Z + e^2 N \sin \varphi$

SO

$$\tan \varphi = \frac{Z + e^2 N \sin \varphi}{\sqrt{X^2+Y^2}}$$

- SOLVE EQUATION BY ITERATION. FIRST WRITE

$$\tan \varphi = \frac{Z}{\sqrt{X^2+Y^2}} \left[1 + \frac{e^2 N \sin \varphi}{Z}\right]$$

- FOR FIRST APPROXIMATION, TAKE $h = \phi$, $Z = N(1-e^2)\sin \varphi$

then

$$\tan \varphi_1 = \frac{Z}{\sqrt{X^2+Y^2}} \left[1 + \frac{e^2}{1-e^2}\right]$$

$$= \left[\frac{1}{1-e^2}\right] \left[\frac{Z}{\sqrt{X^2+Y^2}}\right]$$

- APPROXIMATION EXACT WHEN $h = \phi$ & CAN BE USED TO FIND FIRST APPROXIMATION of φ
- ITERATE TO CONVERGENCE
- FROM EQUATIONS DEFINING X & Y, find h:

$$h = \frac{\sqrt{X^2 + Y^2}}{\cos\varphi} - N$$

- FROM EQUATION DEFINING Z,

$$h = \frac{Z}{\sin\varphi} - N + e^2 N$$

- IN POLAR REGIONS, LATTER FORMULA MORE STABLE WHILE the FORMER WOULD BE BETTER IN EQUATORIAL REGIONS