# The Inverse Geodetic Problem Using the Gauss Mid-Latitude Method

Some useful angle functions:

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang + 0.0000000001) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins + 0.000000001) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d\cdot\dfrac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{array} \right.$$

$$r2d := \frac{180.0}{\pi}$$

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The following data refer to a given reference system

$a := 6378160\cdot m$ $\quad f := \dfrac{1}{298.257222028}$ $\quad b := a - a\cdot f$

$b = 6356775.23702048\ m$

first eccentricity squared: $\quad e_2 := \dfrac{a^2 - b^2}{a^2}$ $\quad e_2 = 0.006694380024537$

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Given data:

$\phi_A := -37.39155571$ $\quad \lambda_A := 43.55306630$

$p := if(\phi_A < 0, -1, 1)$ $\quad l := if(\lambda_A < 0, -1, 1)$

$\phi_1 := p\cdot radians(|\phi_A|)$ $\quad \lambda_1 := l\cdot radians(|\lambda_A|)$

$\phi_B := -37.570912874$ $\quad \lambda_B := 44.252481672$

$p := if(\phi_B < 0, -1, 1)$ $\quad l := if(\lambda_B < 0, -1, 1)$

$\phi_2 := p\cdot radians(|\phi_B|)$ $\quad \lambda_2 := l\cdot radians(|\lambda_B|)$

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Inverse Problem:

The mean latitude:

$$\phi_m := \frac{\phi_1 + \phi_2}{2} \qquad \phi_m\cdot r2d = -37.80342859$$

The difference in longitude and latitude:

$\Delta\lambda := \lambda_2 - \lambda_1$ $\Delta\lambda \cdot r2d = 0.49837603$

$\Delta\phi := \phi_2 - \phi_1$ $\Delta\phi \cdot r2d = -0.29821434$

$W_m := \sqrt{1 - e_2 \cdot (\sin(\phi_m))^2}$ $W_m = 0.99874163$

$N_m := \frac{a}{W_m}$ $N_m = 6386196.22720554\ m$

$A_m := \frac{1}{N_m}$ $A_m = 0.00000016\ \frac{1}{m}$

$M_m := \frac{a \cdot (1 - e_2)}{W_m^{\ 3}}$ $M_m = 6359439.64734370\ m$

$B_m := \frac{1}{M_m}$ $B_m = 0.00000016\ \frac{1}{m}$

$F := \frac{1}{12} \cdot \sin(\phi_m) \cdot (\cos(\phi_m))^2$ $F = -0.03188828$

$\Delta A := \Delta\lambda \cdot \left( \frac{\sin(\phi_m)}{\cos\left(\frac{\Delta\phi}{2}\right)} + F \cdot \Delta\lambda^3 \right)$ $dms(\Delta A \cdot r2d) = -0.58597381$

$\Delta\phi_p := \Delta\phi \cdot \left( \frac{\sin\left(\frac{\Delta\phi}{2}\right)}{\frac{\Delta\phi}{2}} \right)$ $\Delta\phi_p \cdot r2d = -0.29821401$

$\Delta\lambda_p := \Delta\lambda \cdot \left( \frac{\sin\left(\frac{\Delta\lambda}{2}\right)}{\frac{\Delta\lambda}{2}} \right)$ $\Delta\lambda_p \cdot r2d = 0.49837446$

$X_1 := \Delta\lambda_p \cdot \frac{\cos(\phi_m)}{A_m}$ $X_1 = 43890.19860931\ m$

$X_2 := \Delta\phi_p \cdot \frac{\cos\left(\frac{\Delta\lambda}{2}\right)}{B_m}$ $X_2 = -33099.40217128\ m$

$s_i := \sqrt{X_1^{\ 2} + X_2^{\ 2}}$ $s_i = 54971.99248763\ m$

$$s := s_i \cdot \frac{\dfrac{s_i}{2 \cdot N_m}}{\sin\left(\dfrac{s_i}{2 \cdot N_m}\right)}$$

The distance between points 1 and 2 is:

$s = 54972.16220630 \text{ m}$

$$\alpha_{12} := \begin{cases} \text{dms}\left[\left(\text{atan}\left(\dfrac{X_1}{X_2}\right) - \dfrac{\Delta A}{2}\right) \cdot \text{r2d}\right] & \text{if } (X_1 > 0 \wedge X_2 > 0) \\ \text{dms}\left[\left(\text{atan}\left(\dfrac{X_1}{X_2}\right) + \pi - \dfrac{\Delta A}{2}\right) \cdot \text{r2d}\right] & \text{if } (X_1 > 0 \wedge X_2 < 0) \\ \text{dms}\left[\left(\text{atan}\left(\dfrac{X_1}{X_2}\right) + \pi - \dfrac{\Delta A}{2}\right) \cdot \text{r2d}\right] & \text{if } (X_1 < 0 \wedge X_2 < 0) \\ \text{dms}\left[\left(\text{atan}\left(\dfrac{X_1}{X_2}\right) + 2\pi - \dfrac{\Delta A}{2}\right) \cdot \text{r2d}\right] & \text{if } (X_1 < 0 \wedge X_2 > 0) \end{cases}$$

The forward azimuth from point 1 to point 2:

$\alpha_{12} = 127.10270778$

$$\alpha := \text{radians}(\alpha_{12})$$

$$\alpha_{21} := \begin{cases} \text{dms}[(\alpha + \Delta A + \pi) \cdot \text{r2d}] & \text{if } [(\alpha + \Delta A) < \pi] \\ \text{dms}[(\alpha + \Delta A) \cdot \text{r2d}] & \text{if } [(\alpha + \Delta A) > \pi] \end{cases}$$

The back azimuth from point 2 to point 1 is:

$\alpha_{21} = 306.52073397$