The Direct Geodetic Problem Using the Gauss Mid-Latitude Method

Some useful angle functions:

$$dd(ang) := \begin{vmatrix} degree \leftarrow floor(ang + 0.0000000001) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins + 0.000000001) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{vmatrix}$$

$$radians(ang) := \begin{vmatrix} d \leftarrow dd(ang) \\ d\cdot \dfrac{\pi}{180.0} \end{vmatrix}$$

$$dms(ang) := \begin{vmatrix} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{vmatrix}$$

$$r2d := \frac{180.0}{\pi}$$

The following data refer to a given reference system

$a := 6378160\cdot m$ $\quad f := \dfrac{1}{298.25000158005}$ $\quad b := a - a\cdot f$

$b = 6356774.71930860\ m$

first eccentricity squared: $\quad e_2 := \dfrac{a^2 - b^2}{a^2}$ $\quad e_2 = 0.00669454$

Given data:

$\phi := -37.39155571$ $\quad \lambda := 43.55306630$

$p := if(\phi < 0, -1, 1)$ $\quad l := if(\lambda < 0, -1, 1)$

$\phi_1 := p\cdot radians(|\phi|)$ $\quad \lambda_1 := l\cdot radians(|\lambda|)$

$s := 54972.161m$ $\quad az := 127.1027080$

$\alpha_{12} := radians(az)$

Direct Problem:

The first, or initial, iteration

The radius of curvature in the meridian, M, and Prime Vertical, N, for the first station:

$$M := \frac{a \cdot (1 - e_2)}{\left[1 - e_2 \cdot (\sin(\phi_1))^2\right]^{1.5}}$$ $M = 6359277.92432075 \text{ m}$

$$N := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi_1))^2}}$$ $N = 6386142.43899800 \text{ m}$

Compute an initial approximation of the change in latitude, $\Delta\phi$, and change in longitude, $\Delta\lambda$:

$$\Delta\lambda := \frac{s \cdot \sin(\alpha_{12})}{N \cdot \cos(\phi_1)}$$ $l := \text{if}(\Delta\lambda < 0, -1, 1)$

$l \cdot \text{dms}(|\Delta\lambda| \cdot r2d) = 0.29469537$

$$\Delta\phi := \frac{s \cdot \cos(\alpha_{12})}{M \cdot \cos\left(\frac{\Delta\lambda}{2}\right)}$$ $p := \text{if}(\Delta\phi < 0, -1, 1)$

$p \cdot \text{dms}(|\Delta\phi| \cdot r2d) = -0.17573922$

$\Delta\lambda_b := \Delta\lambda$ $\Delta\phi_b := \Delta\phi$ The subscript b indicates that the changes are the base values for the change in longitude and latitude to be used to see when the iteration stops.

Begin the second iteration towards the solution

Compute the latitude and longitude of the second point

$\phi_2 := \phi_1 + \Delta\phi$ $\lambda_2 := \lambda_1 + \Delta\lambda$

$$\phi_m := \frac{\phi_1 + \phi_2}{2}$$ The estimate of the mean latitude

Compute the radius of curvature in the meridian and prime vertical based on the mean latitude

$$M_m := \frac{a \cdot (1 - e_2)}{\left[1 - e_2 \cdot (\sin(\phi_m))^2\right]^{1.5}}$$ $M_m = 6359439.76713680 \text{ m}$

$$N_m := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi_m))^2}}$$ $N_m = 6386196.61404326 \text{ m}$

Compute the change in azimuth and then the corresponding change in longitude and latitude:

$$\Delta\alpha := \Delta\lambda \cdot \frac{\sin(\phi_m)}{\cos\left(\frac{\Delta\phi}{2}\right)} + \frac{\Delta\lambda^3}{12} \cdot \left[\frac{\sin(\phi_m)}{\cos\left(\frac{\Delta\phi}{2}\right)} - \frac{1}{(\cos(\phi_m))^3 \cdot \left(\cos\left(\frac{\Delta\phi}{2}\right)\right)^3} \right]$$

$$\text{dms}(\Delta\alpha \cdot \text{r2d}) = -0.58553673$$

$$\Delta\lambda := \frac{s \cdot \sin\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right)}{N_m \cdot \cos(\phi_m)}$$

$$l := \text{if}(\Delta\lambda < 0, -1, 1)$$

$$l \cdot \text{dms}(|\Delta\lambda| \cdot \text{r2d}) = 0.29541520$$

$$\Delta\phi := \frac{s \cdot \cos\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right)}{M_m \cdot \cos\left(\frac{\Delta\lambda}{2}\right)}$$

$$p := \text{if}(\Delta\phi < 0, -1, 1)$$

$$p \cdot \text{dms}(|\Delta\phi| \cdot \text{r2d}) = -0.17535888$$

$$\text{dms}(|\Delta\lambda - \Delta\lambda_b| \cdot \text{r2d}) = 0.00071983$$

$$\text{dms}(|\Delta\phi - \Delta\phi_b| \cdot \text{r2d}) = 0.00038034$$

This calculation determines the difference in the change in latitude and longitude during this iteration. If the value is sufficiently low, the iteration stops. Here, it will continue.

$$\Delta\lambda_b := \Delta\lambda \qquad \Delta\phi_b := \Delta\phi$$

Begin the third iteration:

$$\phi_2 := \phi_1 + \Delta\phi \qquad \lambda_2 := \lambda_1 + \Delta\lambda$$

$$\phi_m := \frac{\phi_1 + \phi_2}{2}$$

$$M_m := \frac{a \cdot (1 - e_2)}{\left[1 - e_2 \cdot (\sin(\phi_m))^2\right]^{1.5}}$$

$$M_m = 6359439.19539692 \text{ m}$$

$$N_m := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi_m))^2}}$$

$$N_m = 6386196.42266145 \text{ m}$$

$$\Delta\alpha := \Delta\lambda\cdot\frac{\sin(\phi_m)}{\cos\left(\frac{\Delta\phi}{2}\right)} + \frac{\Delta\lambda^3}{12}\cdot\left[\frac{\sin(\phi_m)}{\cos\left(\frac{\Delta\phi}{2}\right)} - \frac{1}{(\cos(\phi_m))^3\cdot\left(\cos\left(\frac{\Delta\phi}{2}\right)\right)^3}\right]$$

$$\text{dms}(\Delta\alpha\cdot r2d) = -0.58597669$$

$$\Delta\lambda := \frac{s\cdot\sin\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right)}{N_m\cdot\cos(\phi_m)} \qquad l := \text{if}(\Delta\lambda < 0, -1, 1)$$

$$l\cdot\text{dms}(|\Delta\lambda|\cdot r2d) = 0.29541536$$

$$\Delta\phi := \frac{s\cdot\cos\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right)}{M_m\cdot\cos\left(\frac{\Delta\lambda}{2}\right)} \qquad p := \text{if}(\Delta\phi < 0, -1, 1)$$

$$p\cdot\text{dms}(|\Delta\phi|\cdot r2d) = -0.17535737$$

$$\text{dms}(|\Delta\lambda - \Delta\lambda_b|\cdot r2d) = 0.00000017$$

$$\text{dms}(|\Delta\phi - \Delta\phi_b|\cdot r2d) = 0.00000151$$

Since the difference is still greater than 0.01", another iteration will be run.

$$\Delta\lambda_b := \Delta\lambda \qquad \Delta\phi_b := \Delta\phi$$

Begin the fourth iteration:

$$\phi_2 := \phi_1 + \Delta\phi \qquad \lambda_2 := \lambda_1 + \Delta\lambda$$

$$\phi_m := \frac{\phi_1 + \phi_2}{2}$$

$$M_m := \frac{a\cdot(1 - e_2)}{\left[1 - e_2\cdot(\sin(\phi_m))^2\right]^{1.5}} \qquad M_m = 6359439.19312923\ \text{m}$$

$$N_m := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_m))^2}} \qquad N_m = 6386196.42190237\ \text{m}$$

$$\Delta\alpha := \Delta\lambda \cdot \frac{\sin(\phi_m)}{\cos\left(\frac{\Delta\phi}{2}\right)} + \frac{\Delta\lambda^3}{12} \cdot \left[\frac{\sin(\phi_m)}{\cos\left(\frac{\Delta\phi}{2}\right)} - \frac{1}{(\cos(\phi_m))^3 \cdot \left(\cos\left(\frac{\Delta\phi}{2}\right)\right)^3} \right]$$

$$\text{dms}(\Delta\alpha \cdot \text{r2d}) = -0.58597679$$

$$\Delta\lambda := \frac{s \cdot \sin\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right)}{N_m \cdot \cos(\phi_m)}$$

$$l := \text{if}(\Delta\lambda < 0, -1, 1)$$

$$l \cdot \text{dms}(|\Delta\lambda| \cdot \text{r2d}) = 0.29541536$$

$$\Delta\phi := \frac{s \cdot \cos\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right)}{M_m \cdot \cos\left(\frac{\Delta\lambda}{2}\right)}$$

$$p := \text{if}(\Delta\phi < 0, -1, 1)$$

$$p \cdot \text{dms}(|\Delta\phi| \cdot \text{r2d}) = -0.17535737$$

$$\text{dms}(|\Delta\lambda - \Delta\lambda_b| \cdot \text{r2d}) = 0.00000000$$

Stop the calculations.

$$\text{dms}(|\Delta\phi - \Delta\phi_b| \cdot \text{r2d}) = 0.00000000$$

The latitude and longitude of the second point are:

$$\phi_2 := \phi_1 + \Delta\phi$$

$$p := \text{if}(\phi_2 < 0, -1, 1)$$

$$\phi_2 := p \cdot \text{dms}(|\phi_2| \cdot \text{r2d})$$

The latitude of point 2 is:

$$\phi_2 = -37.570913081$$

$$\lambda_2 := \lambda_1 + \Delta\lambda$$

$$l := \text{if}(\lambda_2 < 0, -1, 1)$$

$$\lambda_2 := l \cdot \text{dms}(|\lambda_2| \cdot \text{r2d})$$

The longitude of point 2 is:

$$\lambda_2 = 44.252481660$$

The back azimuth from 2 to 1 is:

$$i := \text{if}[(\alpha_{12} + \Delta\alpha) < \pi, \pi, -\pi]$$

$$\alpha_{21} := \text{dms}[(\alpha_{12} + \Delta\alpha + i) \cdot \text{r2d}]$$

$$\alpha_{21} = 306.52073121$$