Transformation Between Cartesian to Geodetic Coordinates - Borkowski Method

Some useful angle functions

$$\mathrm{dd(ang)} := \begin{array}{|l} \mathrm{degree} \leftarrow \mathrm{floor(ang)} \\ \mathrm{mins} \leftarrow (\mathrm{ang} - \mathrm{degree})\cdot 100.0 \\ \mathrm{minutes} \leftarrow \mathrm{floor(mins)} \\ \mathrm{seconds} \leftarrow (\mathrm{mins} - \mathrm{minutes})\cdot 100.0 \\ \mathrm{degree} + \dfrac{\mathrm{minutes}}{60.0} + \dfrac{\mathrm{seconds}}{3600.0} \end{array}$$

$$\mathrm{radians(ang)} := \begin{array}{|l} d \leftarrow \mathrm{dd(ang)} \\ d\cdot\dfrac{\pi}{180.0} \end{array}$$

$$\mathrm{dms(ang)} := \begin{array}{|l} \mathrm{degree} \leftarrow \mathrm{floor(ang)} \\ \mathrm{rem} \leftarrow (\mathrm{ang} - \mathrm{degree})\cdot 60 \\ \mathrm{mins} \leftarrow \mathrm{floor(rem)} \\ \mathrm{rem1} \leftarrow (\mathrm{rem} - \mathrm{mins}) \\ \mathrm{secs} \leftarrow \mathrm{rem1}\cdot 60.0 \\ \mathrm{degree} + \dfrac{\mathrm{mins}}{100} + \dfrac{\mathrm{secs}}{10000} \end{array}$$

$$\mathrm{r2d} := \frac{180}{\pi}$$

Constants for GRS 80:

$a := 6378137$ $\quad b := 6356752.3141$ $\quad e_2 := 0.00669438002290$

$ep_2 := 0.00673949677548$ $\quad f := 0.00335281068118$

a and b are the semi-major and semi-minor axes of the ellipsoid respectively, e_2 and ep_2 are the first and second eccentricities squared respectively, f is the flattening

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Given Quantities:

$X := 472239.0061$ $\quad Y := -4493054.0133$ $\quad Z := 4487560.5408$

Solution:

$\lambda := \mathrm{atan2}(X, Y)$ $\qquad \mathrm{dms}(\lambda\cdot\mathrm{r2d}) = -84.00000000$

Designating x' by r and z' by z

$$r := \frac{X}{\cos(\lambda)} \qquad r = 4517803.010902$$

$$z := Z \qquad z = 4487560.5408$$

$$E := \frac{b\cdot z - \left(a^2 - b^2\right)}{a\cdot r} \qquad E = 0.980525$$

$$F := \frac{b \cdot z + \left(a^2 - b^2\right)}{a \cdot r}$$ $F = 0.999427$

$$P := \frac{4}{3} \cdot (E \cdot F + 1)$$ $P = 2.63995$

$$Q := 2 \cdot \left(E^2 - F^2\right)$$ $Q = -0.07485$

$$D := P^3 + Q^2$$ $D = 18.404296$

$$v := \sqrt[3]{\sqrt{D} - Q} - \sqrt[3]{\sqrt{D} + Q}$$ $v = 0.018901$

$$G := \frac{\sqrt{\left(E^2 + v\right)} + E}{2}$$ $G = 0.98532$

$$t := \sqrt{G^2 + \frac{F - v \cdot G}{2 \cdot G - E}} - G$$ $t = 0.415198$

The latitude of the point is:

$$\phi := \operatorname{atan}\left[a \cdot \frac{\left(1 - t^2\right)}{2 \cdot b \cdot t}\right]$$ $\operatorname{dms}(\phi \cdot r2d) = 45.00000000$

The height above the ellipsoid is:

$$H := (r - a \cdot t) \cdot \cos(\phi) + (z - b) \cdot \sin(\phi)$$ $H = 300.000015$