The Inverse Geodetic Problem using the method by Bowring

Some useful angle functions:

$$dd(ang) := \left|\begin{array}{l} degree \leftarrow floor(ang + 0.0000000001) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins + 0.0000000001) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{array}\right.$$

$$radians(ang) := \left|\begin{array}{l} d \leftarrow dd(ang) \\ d\cdot\dfrac{\pi}{180.0} \end{array}\right.$$

$$dms(ang) := \left|\begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{array}\right.$$

The following data refer to a given reference system

$a := 6378137\cdot m$ $\qquad f := \dfrac{1}{298.257222101}$ $\qquad b := a - a\cdot f$

$b = 6356752.31414036\ m$

second eccentricity squared: $\quad e_{p2} := \dfrac{a^2 - b^2}{b^2}$ $\qquad e_{p2} = 0.006739496775479$

Given data:

$\phi_1 := radians(30)$ $\qquad \lambda_1 := radians(10)$

$\phi_2 := radians(30.444814320)$ $\qquad \lambda_2 := radians(10.451308964)$

Common equations:

$$A := \sqrt{1 + e_{p2}\cdot\left(\cos\left(\phi_1\right)\right)^4} \qquad A = 1.00189369$$

$$B := \sqrt{1 + e_{p2}\cdot\left(\cos\left(\phi_1\right)\right)^2} \qquad B = 1.002524126$$

$$C := \sqrt{1 + e_{p2}} \qquad C = 1.00336409$$

$$w := A\cdot\frac{\left(\lambda_2 - \lambda_1\right)}{2} \qquad w = 0.006589169$$

$$\Delta\phi := \phi_2 - \phi_1 \qquad \Delta\phi = 0.013032486$$

$$\Delta\lambda := \lambda_2 - \lambda_1 \qquad \Delta\lambda = 0.01315343$$

Inverse Problem:

$$D := \frac{\Delta\phi}{2\cdot B}\cdot\left(1 + \frac{3\cdot e_{p2}}{4\cdot B^2}\cdot\Delta\phi\cdot\sin\left(2\cdot\phi_1 + \frac{2}{3}\cdot\Delta\phi\right)\right)$$

$D = 0.006500207$

$$E := \sin(D)\cdot\cos(w)$$

$E = 0.00650002$

$$F := \frac{1}{A}\cdot\sin(w)\cdot\left(B\cdot\cos\left(\phi_1\right)\cdot\cos(D) - \sin\left(\phi_1\right)\cdot\sin(D)\right)$$

$F = 0.005688442$

$$G := \operatorname{atan2}(E, F)$$

$G = 0.718910505$

$$\sigma := 2\cdot\operatorname{asin}\left(\sqrt{E^2 + F^2}\right)$$

$\sigma = 0.017275473$

$$H := \operatorname{atan}\left[\frac{1}{A}\cdot\left(\sin\left(\phi_1\right) + B\cdot\cos\left(\phi_1\right)\cdot\tan(D)\right)\cdot\tan(w)\right]$$

$H = 0.00332551$

$$\alpha_{12} := G - H$$

$\operatorname{dms}\left(\alpha_{12}\cdot\frac{180}{\pi}\right) = 41.00000004$

$$\alpha_{21} := G + H + \pi$$

$\operatorname{dms}\left(\alpha_{21}\cdot\frac{180}{\pi}\right) = 221.22518717$

$$s := \frac{a\cdot C\cdot\sigma}{B^2}$$

$s = 109999.999633107\ \text{m}$