GEOCENTRIC TO GEODETIC COORDINATE TRANSFORMATION

BORKOWSKI, K.M., 1989. "ACCURATE ALGORITHM TO TRANSFORM GEOCENTRIC TO GEODETIC COORDINATES", BULLETIN GÉODÉSIQUE, 63(1): 50-56

PROBLEM: GIVEN X, Y, Z, FIND ϕ, λ, H

RECOGNIZE:

$$X = (N+H)\cos\phi\cos\lambda$$
$$Y = (N+H)\cos\phi\sin\lambda$$

$$\therefore \frac{Y}{X} = \frac{(N+H)\cos\phi\sin\lambda}{(N+H)\cos\phi\cos\lambda} = \tan\lambda$$

$$\underline{\underline{\lambda = \tan^{-1}(Y/X)}}$$

BORKOWSKI'S DIRECT ALGORITHM:

$$t = \pm\left[G^2 + \frac{F - vG}{2G - E}\right]^{1/2} - G$$

where:
$$E = \frac{bz' - (a^2 - b^2)}{ax'}$$

$$F = \frac{bz' + (a^2 - b^2)}{ax'}$$

$$G = \frac{\pm (E^2 + v)^{1/2} + E}{2}$$

$$v = (\sqrt{D} - Q)^{1/3} - (\sqrt{D} + Q)^{1/3}$$

$$D = P^3 + Q^2$$

$$P = (4/3)(EF + 1)$$

$$Q = 2(E^2 - F^2)$$

FROM OUR DISCUSSION ON MERIDIAN ELLIPSE & CARTESIAN COORDINATES,

$$X = x' \cos\lambda$$
$$Y = x' \sin\lambda$$
$$Z = z'$$

where: $x' = x + H\cos\varphi$
$z' = z + H\sin\varphi$

THE VALUE x IS THE x-COORDINATE IN THE MERIDIAN ELLIPSE & z IS THE CORRESPONDING z-COORDINATE. THE x' & z' COORDINATES REPRESENT THE x & y COORDINATES IN THE MERIDIAN ELLIPSE ACCOUNTING FOR A POINT ABOVE (OR BELOW) THE SURFACE OF THE ELLIPSOID. THESE COORDINATES ARE FOUND BY

$$x' = X/\cos\lambda = Y/\sin\lambda$$
$$z' = Z$$

CONTINUING WITH BORKOWSKI'S ALGORITHM, THE LATITUDE AND HEIGHT ARE COMPUTED AS

$$\phi = \tan^{-1}\left(\frac{a\,(1-t^2)}{2bt}\right)$$

$$H = (x' - at)\cos\phi$$

NOTES: IN THE DEVELOPMENT, BORKOWSKI DEFINES t AS

$$t \equiv \tan\left(\pi/4 - \beta/2\right)$$

WHERE β IS THE REDUCED (PARAMETRIC) LATITUDE. EVALUATION LEADS TO A FOUR-DEGREE POLYNOMIAL.

$$t^4 + 2Et^3 + 3Ft - 1 = 0$$

THE SOLUTION IS FOUND USING FERRARI'S SOLUTION TO THE QUARTIC EQUATION. THUS,

$$t = \pm\left[\frac{G^2 + (F - vG)}{2G - E}\right]^{1/2} - G$$

BECAUSE BOTH t & G HAVE $\pm$ TERMS, THERE ARE 4 SOLUTIONS FOR ϕ & H. PROVIDED $a > b$ AND $\phi > 0$, ONE NEED ONLY USE THE POSITIVE SQUARE ROOT FOR BOTH VARIABLES.

FOR VALUES OF $D < \phi$, BORKOWSKI RECOMMENDS USING the following EQUATION for v to ELIMINATE WORKING WITH COMPLEX NUMBERS.

$$v = 2(-P)^{1/2} \cos\left\{\frac{1}{3}\cos^{-1}\left[\frac{Q}{P}(-P)^{-1/2}\right]\right\}$$

FINALLY, the ALGORITHM WILL WORK FOR SOUTHERN LATITUDES PROVIDED that the SEMI-MINOR AXIS, b, IS PRE-ASSIGNED the SAME SIGN AS the z'-COORDINATE.