$$G = \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2 + \left(\frac{\partial z}{\partial \lambda}\right)^2$$

$$= \frac{C^2}{V^2}\left(\cos^2\varphi \sin^2\lambda + \cos^2\varphi\cos^2\lambda\right) = \frac{C^2}{V^2}\cos^2\varphi = (N\cos\varphi)^2 = x^2$$

E & G DEPEND ON φ ONLY

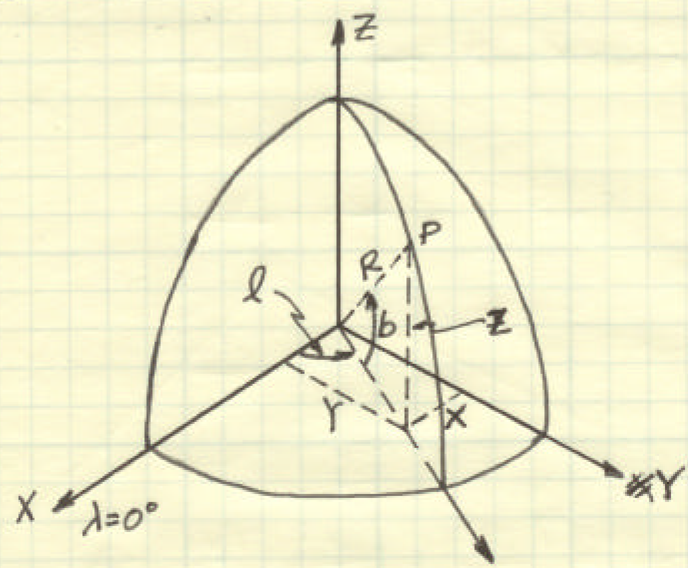
SINCE $\omega = 90°$ ON THE ELLIPSOID, $\cos\omega = 0$, THEREFORE

$$\tan\alpha = \sqrt{\frac{G}{E}}\,\frac{d\lambda}{d\varphi} = \frac{N\cos\varphi}{M}\,\frac{d\lambda}{d\varphi}$$

$$ds = \sqrt{E\,d\varphi^2 + G\,d\lambda^2}$$

$$= \sqrt{M^2 d\varphi^2 + N^2\cos^2\varphi\, d\lambda^2}$$

GAUSSIAN FUNDAMENTAL QUANTITIES ON THE SPHERE



$$X = R\cos b\cos l$$

$$Y = R\cos b\sin l$$

$$Z = R\sin b$$

PARTIAL DERIVATIVES HAVE FORM (R BEING A CONSTANT)

$$\frac{\partial X}{\partial b} = -R\sin b\cos l \qquad \frac{\partial X}{\partial l} = -R\cos b\sin l$$

$$\frac{\partial Y}{\partial b} = -R\sin b\sin l \qquad \frac{\partial Y}{\partial l} = R\cos b\cos l$$

$$\frac{\partial Z}{\partial b} = R\cos b \qquad \frac{\partial Z}{\partial l} = 0$$

GAUSSIAN FUNDAMENTAL QUANTITIES HAVE FORM:

$$E = R^2$$

$$F = 0$$

$$G = R^2 \cos^2 b$$

SINCE WE ARE ON the SPHERE, $M = N = R$
SIMILARLY, PARAMETRIC LINES ARE PERPENDICULAR, thus

$$\cos \omega = 0$$

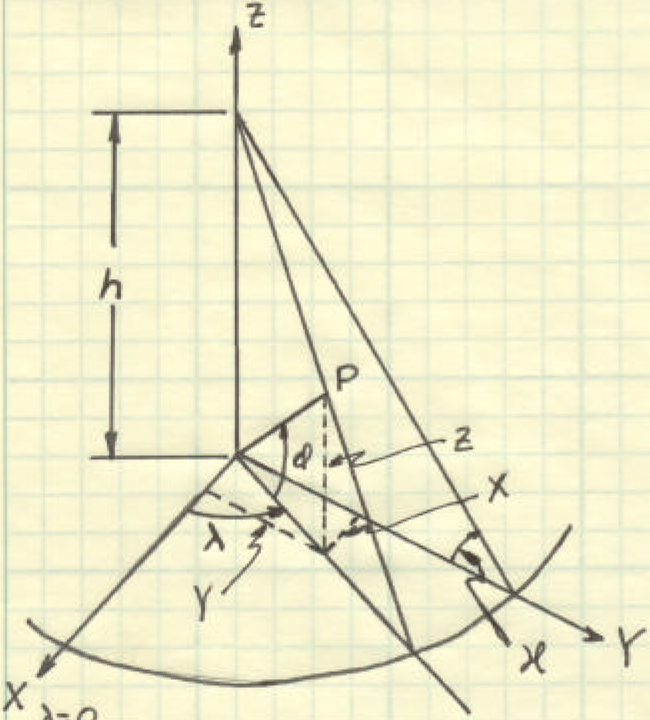
$$\tan \alpha = \frac{R \cos b}{R} \frac{d\ell}{db} = \frac{d\ell}{db} \cos b$$

$$ds = \left[R^2 db^2 + R^2 \cos^2 b \, d\ell^2\right]^{1/2}$$

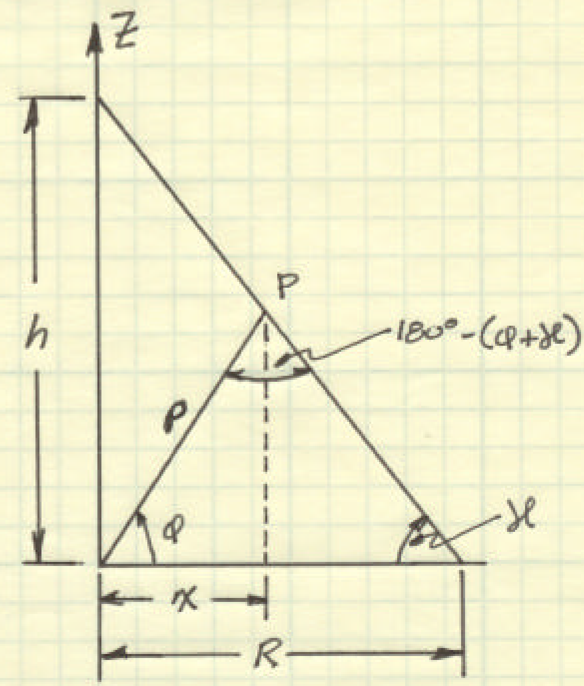
$$= R^2 \left[db^2 + d\ell^2 \cos^2 b\right]^{1/2}$$

HERE, VALUES OF FUNDAMENTAL QUANTITIES E, F, & G
DEPEND ON COORDINATE SYSTEM
- HERE, ORIGIN AT CENTER OF the SPHERE

GAUSSIAN FUNDAMENTAL ON the CONE & CYLINDER



$Z = \rho \sin \varphi$



ON THE RIGHT HAND FIGURE, WE HAVE

$$\tan\chi = \frac{h}{R}$$

$$\rho = R\,\frac{\sin\chi}{\sin[180^\circ - (\varphi+\chi)]}$$ FROM SINE LAW

THEREFORE

$$x = \rho\cos\varphi = R\,\frac{\sin\chi\cos\varphi}{\sin[(180^\circ-\chi)-\varphi]}$$

$$= R\,\frac{\sin(180^\circ-\chi)\cos\varphi}{\sin(180^\circ-\chi)\cos\varphi - \cos(180^\circ-\chi)\sin\varphi}$$

$$= R\,\frac{\cos\varphi}{\cos\varphi - \cot(180^\circ-\chi)\sin\varphi}$$

THE COORDINATES:

$$X = x\cos\lambda = R\,\frac{\cos\lambda}{1+\cot\chi\tan\varphi}$$

$$Y = x\sin\lambda = R\,\frac{\sin\lambda}{1+\cot\chi\tan\varphi}$$

$$Z = z = x\tan\varphi = R\,\frac{\tan\varphi}{1+\cot\chi\tan\varphi}$$

THE PARTIAL DERIVATIVES OF COORDINATES WRT THE VARIABLES

$$\frac{\partial X}{\partial\varphi} = -R\,\frac{\cot\chi\cos\lambda}{(\cos\varphi+\cot\chi\sin\varphi)^2} \qquad \frac{\partial X}{\partial\lambda} = -R\,\frac{\sin\lambda}{1+\cot\chi\tan\varphi}$$

$$\frac{\partial Y}{\partial\varphi} = -R\,\frac{\cot\chi\sin\lambda}{(\cos\varphi+\cot\chi\sin\varphi)^2} \qquad \frac{\partial Y}{\partial\lambda} = R\,\frac{\cos\lambda}{1+\cot\chi\tan\varphi}$$

$$\frac{\partial Z}{\partial\varphi} = R\,\frac{1}{(\cos\varphi+\cot\chi\sin\varphi)^2} \qquad \frac{\partial Z}{\partial\lambda} = 0$$

THE FUNDAMENTIAL QUANTITIES:

$$E = \frac{R^2(1+\cot^2\chi)}{(\cos\varphi+\cot\chi\sin\varphi)^4} \qquad F = 0$$

$$G = \frac{R^2}{(1+\cot\chi \tan\varphi)^2}$$

$$ds = \left[\frac{R^2(1+\cot^2\chi)}{(\cos\varphi + \cot\chi \sin\varphi)^4} d\varphi^2 + \frac{R^2}{(1+\cot\chi \tan\varphi)} d\lambda^2\right]^{1/2}$$

$\cos\omega = 0$ (MEANS PARAMETRIC LINES PERPENDICULAR)

$$\tan\alpha = \frac{\cos^2\varphi}{\sqrt{1+\cot^2\chi}} \frac{d\lambda}{d\varphi}$$

IN CASE OF CYLINDER $\rightarrow$ $\chi = 90°$ & $\cot\chi = 0$

-SO THE FUNDAMENTAL QUANTITIES ARE:

$$E = \frac{R^2}{\cos^4\varphi} \qquad F = 0$$

$$G = R^2$$

AND

$$ds = \sqrt{\frac{d\varphi^2}{\cos^4\varphi} + d\lambda^2}$$

$$\cos\omega = 0$$

$$\tan\alpha = \cos^2\varphi \frac{d\lambda}{d\varphi}$$