The Center
for
Photogrammetric Training

CORRECTIONS TO PHOTO COORDINATES

Surveying Engineering Department
Ferris State University

ANALYTICAL INSTRUMENTATION

- Design characteristics
  - High accuracy
  - High reliability
  - High measuring efficiency
  - Low first cost
  - Low cost of maintenance
  - Operational efficiency
    - Does operator need specialized training
    - Comfort of individual when operating instrument

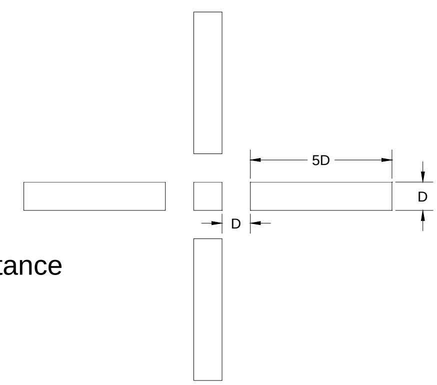
ANALYTICAL INSTRUMENTATION

- Systematic errors associated with comparators
  - Instrument system errors
    - Scaling and periodic errors (spindles, coordinate counter)
    - Affinity errors (different scales)
    - Rectilinearity (bending) errors
    - Lack of orthogonality
  - Backlash and tracking errors
  - Dynamic errors (microscope velocity does not drop to zero at points to be approached during operation)
  - System automation errors
    - Digital resolution
    - Errors due to deviation of direction

ARTIFICIAL TARGETS

- If scale of photo is 1”=500’ and the desired target width is ≈ 0.004” (about 100 μm), D ≈ 2’

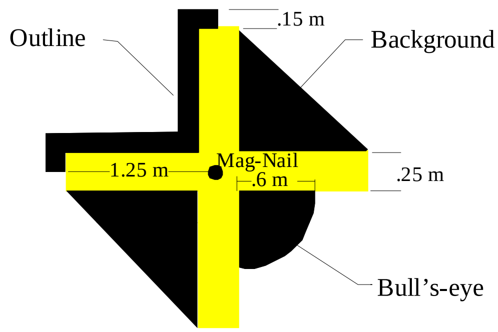
Artificial Target white on dark background



D = Ground Distance

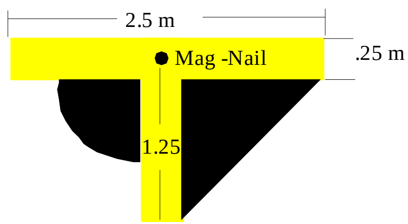
GROUND TARGETS

- Three types
  - Signalized
  - Detail points
  - PUG points
- Suggested patterns from MDOT for standard mapping
- Target highlighted by
  - Background
  - Bull's eye
  - outline

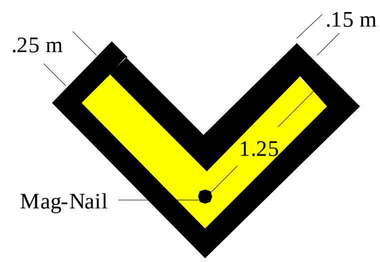


Standard Target

GROUND TARGETS



"T" Target



Chevron

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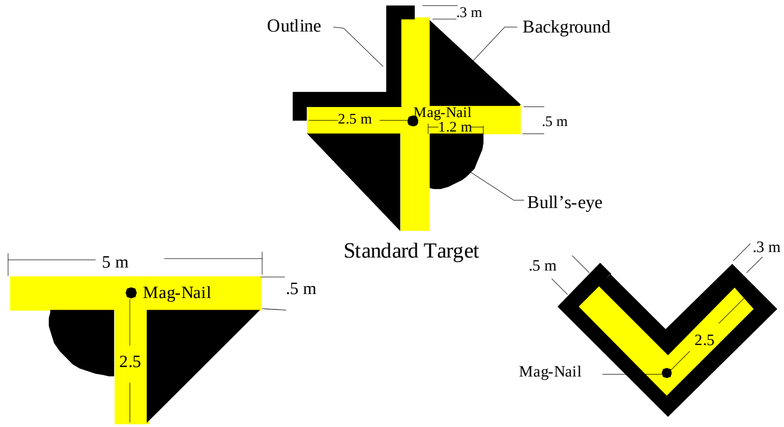
GROUND TARGETS

- MDOT high level target design criteria



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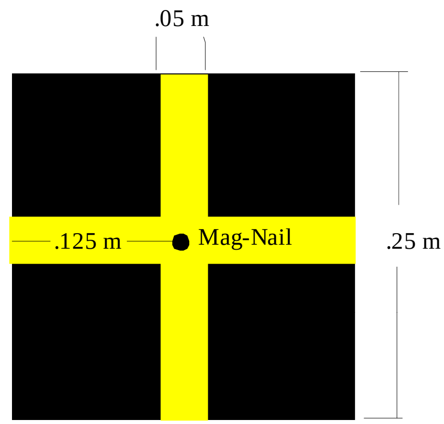
GROUND TARGETS

- MDOT low level target design – required to be square



Low Level Target

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ABBE'S COMPARATOR PRINCIPLES

- To exclusively base the measurement in all cases on a longitudinal graduation with which the distance to be measured is directly compared; and
- To always design the measuring apparatus in such a way that the distance to be measured will be the rectilinear extension of the graduation used as a scale.

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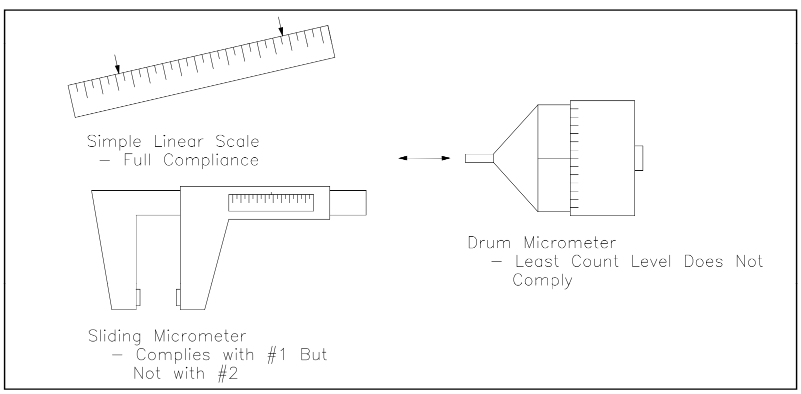
ABBE'S COMPARATOR PRINCIPLES



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BASIC ANALYTICAL PHOTOGRAMMETRY THEORY

- **First Order Theory –** basic collinearity concept
  - – Straight line from object space to inner space
- **Second Order Theory –** corrects for most significant errors unaccounted in First Order Theory
- **Third Order Theory –** other sources of error generally not accounted for

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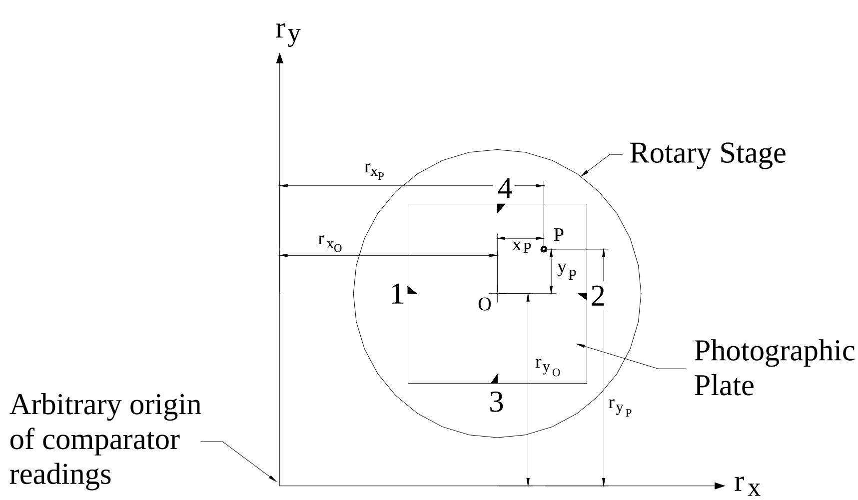
COMPARATOR READINGS



COMPARATOR READINGS

- Measure coordinates of fiducial marks
- Arbitrary coordinates of principal point (assuming photo oriented to stage):

$$r_{x_O} = \frac{r_{x_3} + r_{x_4}}{2} \qquad r_{y_O} = r_{y_1} = r_{y_2}$$

- Photo coordinates:

$$x_P = r_{x_P} - r_{x_O} \qquad y_P = r_{y_P} - r_{y_O}$$

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COMPARATOR READINGS

- If photos placed on stage with no orientation – compute rotation angle

$$\tan\theta = \frac{r_{y_2} - r_{y_1}}{r_{x_2} - r_{x_1}}$$

- 2-D rotation

$$x' = r_x \cos\theta + r_y \sin\theta$$

$$y' = -r_x \sin\theta + r_y \cos\theta$$

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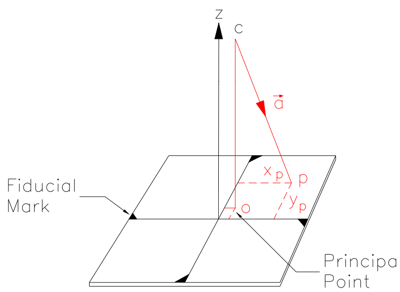
INTERIOR ORIENTATION



- Photogrammetric coordinate system
- Vector from perspective center to point:

$$\vec{a} = \begin{bmatrix} x_p - x_o \\ y_p - y_o \\ 0 - f \end{bmatrix}$$

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FILM DEFORMATION

- Use and processing makes film susceptible to dimensional change
- Isogonal affine transformation

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} - \begin{bmatrix} x_o \\ y_o \end{bmatrix}$$

  – Linear model

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} - \begin{bmatrix} x_o \\ y_o \end{bmatrix}$$

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FILM DEFORMATION

- 8-parameter projective transformation

$$x = \frac{a_1 x' + a_2 y' + a_3}{c_1 x' + c_2 y' + 1} - x_o$$

$$y = \frac{b_1 x' + b_2 y' + b_3}{c_1 x' + c_2 y' + 1} - y_o$$

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FILM DEFORMATION

- Polynomial correction
  - 4-fiducial model

$$\Delta x = x - x' = x + a_0 + a_1 x + a_2 y + a_3 xy$$
$$\Delta y = y - y' = y + b_0 + b_1 x + b_2 y + b_3 xy$$

  - 8-fiducial model

$$\Delta x = x - x' = x + a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 + a_6 x^2 y + a_7 xy$$
$$\Delta y = y - y' = y + b_0 + b_1 x + b_2 y + b_3 xy + b_4 x^2 + b_5 y^2 + b_6 x^2 y + b_7 xy$$

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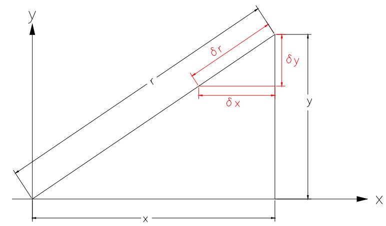
SEIDEL ABERRATION DISTORTION

- Conrady's intuitive development



$$\delta r = k_0 r + k_1 r^3 + k_2 r^5 + \cdots$$

- By similar triangles

$$\frac{\delta r}{r} = \frac{\delta x}{x} = \frac{\delta y}{y}$$

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SEIDEL ABERRATION DISTORTION

- Corrections to x and y coordinates:

$$\delta x = \frac{\delta r}{r} x \qquad \delta y = \frac{\delta r}{r} y$$

- Corrected coordinates:

$$x_c = x - \delta x = x\left(1 - \frac{\delta r}{r}\right)$$
$$= x\left(1 - k_0 - k_1 r^2 - k_2 r^4 - \cdots\right)$$
$$y_c = y - \delta y = y\left(1 - \frac{\delta r}{r}\right)$$
$$= y\left(1 - k_0 - k_1 r^2 - k_2 r^4 - \cdots\right)$$

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Example

A camera calibration report displays the following information:

Field Angle	7.5^0	15^0	22.7^0	30^0	35^0	40^0
Symmetric radial distortion, μm	4	6	4	-1	-6	-3
Decentering distortion, μm	0	0	0	1	1	2

If the photo coordinates of a point are x = +33.148 mm and y = -14.921 mm, what are the coordinates corrected for radial lens distortion? The calibrated focal length of the camera is 152.560 mm.

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Correcting Photographic Coordinates for Radial Lens Distortion

Given the following values:

$x := 33.148$ $\quad y := -14.921$ $\quad f := 152.560$

Factor to convert degrees into radians: $torad := \frac{\pi}{180}$

$$ang := \begin{pmatrix} 7.5 \\ 15 \\ 22.7 \\ 30 \\ 35 \\ 40 \end{pmatrix} \qquad distort := \begin{pmatrix} 4 \\ 6 \\ 5 \\ -1 \\ -6 \\ -3 \end{pmatrix}$$

To compute the distortion at the point, we first need to compute the radial distance from the principal point

$$dist := f \cdot \tan(ang \cdot torad) \qquad dist = \begin{pmatrix} 20.085 \\ 40.878 \\ 63.817 \\ 88.081 \\ 106.824 \\ 128.013 \end{pmatrix}$$

The radial distance from the principal point to the point is:

$$r := \sqrt{x^2 + y^2} \qquad r = 36.351$$

Thus, the point lies between the 7.5   and 15^0 field angles. Perform a linear interpolation to find the radial distortion at that point.

$$\&r := \frac{\left[4 + \left(\frac{\text{distort}_1 - \text{distort}_0}{\text{dist}_1 - \text{dist}_0}\right)\cdot(\text{dist}_1 - r)\right]}{1000} \qquad \&r = 0.0044$$

The corrected photographic coordinates become:

$$x_c := \left(1 - \frac{\&r}{r}\right)\cdot x \qquad x_c = 33.144$$

$$y_c := \left(1 - \frac{\&r}{r}\right)\cdot y \qquad y_c = -14.919$$

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Correcting Photographic Coordinates for Radial Lens Distortion

Given the following values:

$$x := 33.148 \qquad y := -14.921 \qquad f := 152.560 \qquad r := \sqrt{x^2 + y^2}$$

$$k_0 := -0.2231\times 10^{-3} \qquad k_1 := 0.450110^{-7} \qquad k_2 := -0.181710^{-11}$$

Using the polynomial to compute the photographic coordinates corrected for radial lens distortion:

$$x_c := \left(1 + k_0 + k_1\cdot r^2 + k_2\cdot r^4\right)\cdot x \qquad x_c = 33.142$$

$$y_c := \left(1 + k_0 + k_1\cdot r^2 + k_2\cdot r^4\right)\cdot y \qquad y_c = -14.919$$

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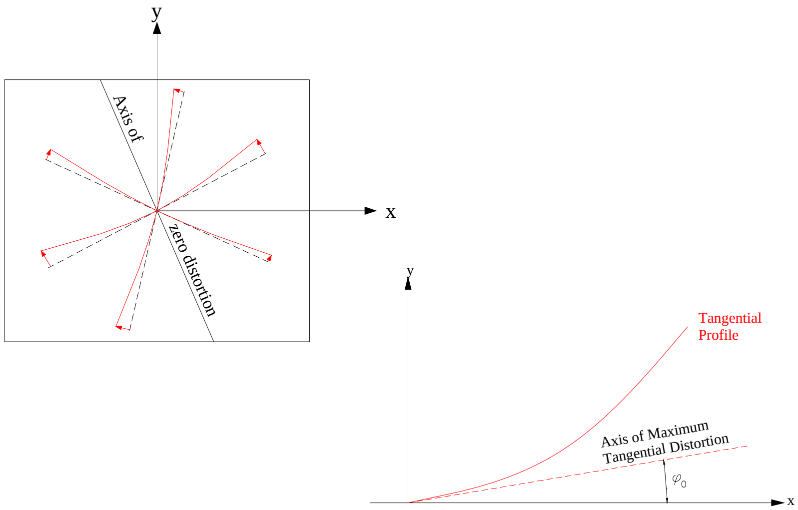
DECENTERING DISTORTION

- Always one radial line (axis of zero tangential distortion) which remains straight



DECENTERING DISTORTION

- D. Brown – Thin Prism Model

$$\delta x = -\left(J_1 r^2 + J_2 r^4\right)\sin\varphi_0 = -J\sin\varphi_o$$

$$\delta y = \left(J_1 r^2 + J_2 r^4\right)\cos\varphi_0 = J\cos\varphi_0$$

- Conrady-Brown Model

$$\delta x = \left(J_1 r^2 + J_2 r^4\right)\left[\left(1 + \frac{2x^2}{r^2}\right)\sin\varphi_o - \frac{2xy}{r^2}\cos\varphi_o\right]$$

$$\delta y = \left(J_1 r^2 + J_2 r^4\right)\left[\frac{2x^2}{r^2}\sin\varphi_o - \left(1 + \frac{2xy}{r^2}\right)\cos\varphi_o\right]$$

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DECENTERING DISTORTION

- Revised Conrady-Brown Model

$$\delta x = \left[P_1\left(r^2 + 2x^2\right) + 2P_2 xy\right]\left[1 + P_3 r^2 + P_4 r^4 + \cdots\right]$$
$$\delta y = \left[2P_1 xy + P_2\left(r^2 + 2y^2\right)\right]\left[1 + P_3 r^2 + P_4 r^4 + \cdots\right]$$

- Corrected photo coordinates:

$$x_c = x - \delta x$$
$$y_c = y - \delta y$$

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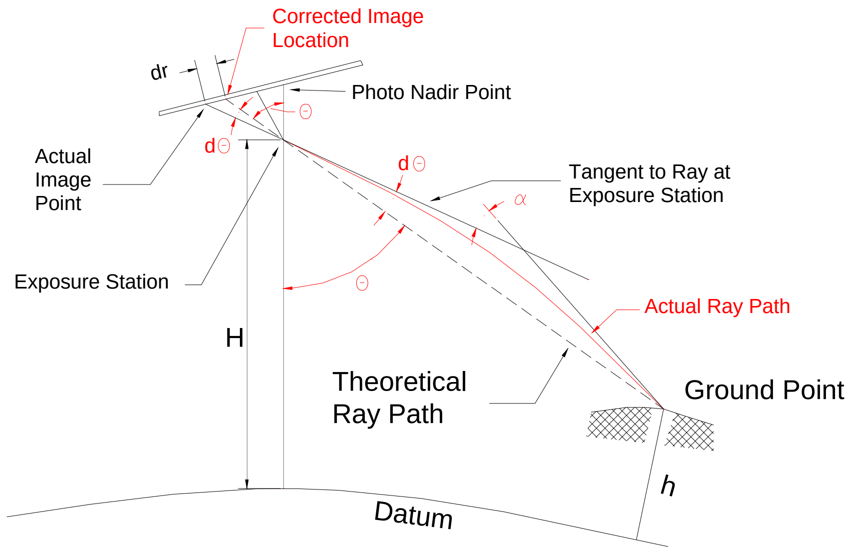
ATMOSPHERIC REFRACTION



ATMOSPHERIC REFRACTION

- Snell's Law

$$(n_i + dn)\sin\theta_i = n_i \sin(\theta_i d\alpha)$$

- Generalizing and simplification

$$d\alpha = \frac{dn}{n}\tan\theta$$

$$\alpha = \int_{\alpha_P}^{\alpha_o} d\alpha = \tan\theta \int_{n_P}^{n_o} \frac{dn}{n} = \tan\theta \ln(n)\Big|_{n_P}^{n_o}$$

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ATMOSPHERIC REFRACTION

- Generalizing

$$d\theta = \alpha/2 = K\tan\theta$$

$$d\theta'' = 0.206K\tan\theta$$

- For vertical photography, $d\theta$ can be shown with respect to r

$$r = f\tan\theta$$

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ATMOSPHERIC REFRACTION

- Differentiating:

$$\frac{dr}{d\theta} = f \sec^2\theta$$

$$dr = f \sec^2\theta \, d\theta$$

$$= f\left(1 + \tan^2\theta\right)d\theta$$

$$= f\left(1 + r^2/f^2\right)d\theta$$

$$dr = \frac{f^2 + r^2}{f}\,d\theta$$

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ATMOSPHERIC REFRACTION

- Expressing dr as a function of K

$$d\theta = K\tan\theta$$

$$dr = \frac{f^2 + r^2}{f}K\tan\theta$$

$$= \frac{f^2 + r^2}{f}K\left(\frac{r}{f}\right)$$

$$dr = K\left(r + \frac{r^3}{f^2}\right)$$

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ATMOSPHERIC REFRACTION

- Cartesian components of radial displacement:

$$\delta x = x\left(\frac{\delta r}{r}\right) = K\left(1+\frac{r^2}{f^2}\right)x$$

$$\delta y = y\left(\frac{\delta r}{r}\right) = K\left(1+\frac{r^2}{f^2}\right)y$$

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ATMOSPHERIC REFRACTION

- 1959 ARDC Model

$$K = \left[\frac{2410\,H}{H^2 - 6H + 250} - \frac{2410\,h}{h^2 - 6h + 250}\left(\frac{h}{H}\right)\right]\cdot 10^{-6}$$

- Saastamoinen Model

$$K = \left\{\frac{2335}{H}\left[(1-0.02257h)^{5.256} - (1-0.02257H)^{5.256}\right] - 277.0(1-0.02257H)^{4.256}\right\}\cdot 10^{-6}$$

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Flying Height in m	For Radial Distance r of the Image Point from the Photo Center, in mm									Coefficients	
	12	24	50	63	78	94	111	131	153	$k_1 \cdot 10^{-2}$	$k_2 \cdot 10^{-6}$
For Ground Elevation – 0 m above sea level											
3000	0.4	0.9	1.9	2.6	3.4	4.5	5.9	7.9	10.7	3.4	1.53
6000	0.7	1.5	3.3	4.4	5.9	7.7	10.1	13.5	18.3	6.1	2.50
9000	0.9	1.9	4.2	5.7	7.5	9.9	13.0	17.3	23.4	7.7	2.23
For Ground Elevation – 500 m above sea level											
3000	0.3	0.7	1.6	2.1	2.8	3.7	4.9	6.4	8.8	2.8	1.25
6000	0.7	1.3	3.0	4.0	5.3	6.9	9.1	12.2	15.4	5.4	2.3
9000	0.9	1.8	3.9	5.3	7.0	9.2	12.0	16.0	21.7	7.2	2.99
For Ground Elevation – 1000 m above sea level											
3000	0.3	0.6	1.3	1.7	2.2	2.9	3.9	5.1	6.9	2.2	0.99
6000	0.6	1.2	2.7	3.6	4.8	6.3	8.2	10.9	14.5	4.8	2.08
9000	0.8	1.6	3.6	4.9	6.5	8.5	11.2	14.9	20.1	6.7	2.76
For Ground Elevation – 1500 m above sea level											
3000	0.2	0.4	0.8	1.2	1.6	2.2	2.8	3.8	5.1	1.6	0.74
6000	0.5	1.1	2.4	3.2	4.2	5.5	7.3	9.7	13.1	4.2	1.87
9000	0.7	1.5	3.4	4.5	6.0	7.8	10.3	13.8	18.6	6.1	2.59

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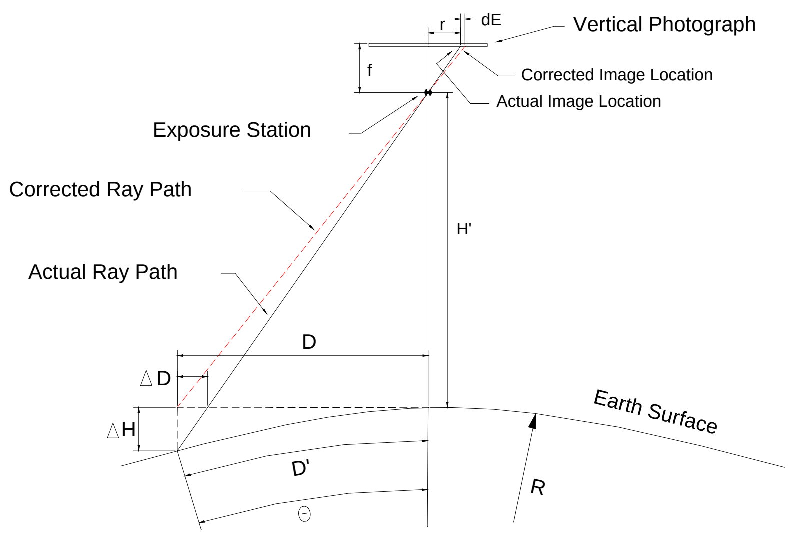
EARTH CURVATURE



EARTH CURVATURE

- From the diagram

$$\theta = \frac{D'}{R} \approx \frac{D}{R} \qquad \therefore \cos\theta \approx \cos\left(\frac{D}{R}\right) = 1 - \frac{D^2}{2R^2} + \cdots$$

$$\Delta H = R - R\cos\theta = R\left(1 - \cos\theta\right) = R\left(1 - 1 + \frac{D^2}{2R^2} - \cdots\right)$$

$$\therefore \Delta H \approx \frac{D^2}{2R}$$

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EARTH CURVATURE

- From the diagram we can write

$$dE = \frac{f}{H'}\Delta D$$

- But

$$dE = \frac{fD^3}{2H^2R}$$

- Therefore

$$dE = \frac{H'r^3}{2Rf^2}$$

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