Analytical Aerotriangulation

Center for Photogrammetric Training

Ferris State University

RCB

Two-Photo Intersection

- Extend Single Photo Resection to “m” photos
- Additional subscripts
  - j = specific point designation, 1 to n
  - i = specific photo designation, 1 to m
    - (m = 2 here)

Observation Equations

- Equations related to photo coordinate observations:

$$\mathbf{V}_{ij} + \overset{e}{\mathbf{B}}_{ij} \overset{e}{\Delta}_i + \overset{s}{\mathbf{B}}_{ij} \overset{s}{\Delta}_j + \mathbf{f}_{ij} = \mathbf{0}$$

- Represents observation (x_J, y_J) on i^{th} photo
- If all points imaged on all photos, have:

2 · m · n observations

Observation Equations

- All observation equations:

$$\mathbf{V}_{1j} + \overset{e}{\mathbf{B}}_{1j} \overset{e}{\Delta}_1 + \overset{s}{\mathbf{B}}_{1j} \overset{s}{\Delta}_j + \mathbf{f}_{1j} = \mathbf{0}$$

$$\mathbf{V}_{2j} + \overset{e}{\mathbf{B}}_{2j} \overset{e}{\Delta}_2 + \overset{s}{\mathbf{B}}_{2j} \overset{s}{\Delta}_j + \mathbf{f}_{2j} = \mathbf{0}$$

$$\overset{e}{\mathbf{V}}_1 - \overset{e}{\Delta}_1 + \overset{e}{\mathbf{f}}_1 = \mathbf{0}$$

$$\overset{e}{\mathbf{V}}_2 - \overset{e}{\Delta}_2 + \overset{e}{\mathbf{f}}_2 = \mathbf{0}$$

$$\overset{s}{\mathbf{V}}_j - \overset{s}{\Delta}_j + \overset{s}{\mathbf{f}}_j = \mathbf{0}$$

Observation Equations

- Collect first 2 equations, we have

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} + \begin{bmatrix} \overset{e}{\mathbf{B}}_1 & \mathbf{0} \\ \mathbf{0} & \overset{e}{\mathbf{B}}_2 \end{bmatrix} \begin{bmatrix} \overset{e}{\Delta}_1 \\ \overset{e}{\Delta}_2 \end{bmatrix} + \begin{bmatrix} \overset{s}{\mathbf{B}}_1 \\ \overset{s}{\mathbf{B}}_2 \end{bmatrix} \Delta_s + \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} = \mathbf{0}$$

- or

$$\mathbf{V} + \overset{e}{\mathbf{B}}\overset{e}{\Delta} + \overset{s}{\mathbf{B}}\overset{s}{\Delta} + \mathbf{f} = \mathbf{0}$$

Observation Equations

- Take next 2 equations

$$\begin{bmatrix} \overset{e}{\mathbf{V}}_1 \\ \overset{e}{\mathbf{V}}_2 \end{bmatrix} + \begin{bmatrix} -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \overset{e}{\Delta}_1 \\ \overset{e}{\Delta}_2 \end{bmatrix} + \begin{bmatrix} \overset{e}{\mathbf{f}}_1 \\ \overset{e}{\mathbf{f}}_2 \end{bmatrix} = \mathbf{0}$$

- or

$$\overset{e}{\mathbf{V}} - \overset{e}{\Delta} + \overset{e}{\mathbf{f}} = \mathbf{0}$$

Observation Equations

- Equations on direct observations of survey points

$$\begin{bmatrix} \overset{s}{\mathbf{V}}_1 \\ \vdots \\ \overset{s}{\mathbf{V}}_j \\ \vdots \\ \overset{s}{\mathbf{V}}_n \end{bmatrix} - \begin{bmatrix} \overset{s}{\boldsymbol{\Delta}}_1 \\ \vdots \\ \overset{s}{\boldsymbol{\Delta}}_j \\ \vdots \\ \overset{s}{\boldsymbol{\Delta}}_n \end{bmatrix} + \begin{bmatrix} \overset{s}{\mathbf{f}}_1 \\ \vdots \\ \overset{s}{\mathbf{f}}_j \\ \vdots \\ \overset{s}{\mathbf{f}}_n \end{bmatrix} = \mathbf{0}$$

- or

$$\overset{s}{\mathbf{V}} - \overset{s}{\boldsymbol{\Delta}} + \overset{s}{\mathbf{f}} = \mathbf{0}$$

Observation Equations

- Collecting

$$\begin{bmatrix} \mathbf{V} \\ \overset{e}{\mathbf{V}} \\ \overset{s}{\mathbf{V}} \end{bmatrix} + \begin{bmatrix} \overset{e}{\mathbf{B}} & \overset{s}{\mathbf{B}} \\ -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \overset{e}{\boldsymbol{\Delta}} \\ \overset{s}{\boldsymbol{\Delta}} \end{bmatrix} + \begin{bmatrix} \mathbf{f} \\ \overset{e}{\mathbf{f}} \\ \overset{s}{\mathbf{f}} \end{bmatrix} = \mathbf{0}$$

- Redefining

$$\overline{\mathbf{V}} + \overline{\mathbf{B}}\overline{\boldsymbol{\Delta}} + \overline{\mathbf{f}} = \mathbf{0}$$

Normal Equations

- Developed as before:

$$\left(\bar{B}^T \bar{W} \bar{B}\right)\bar{\Delta} + \bar{B}^T \bar{W} \bar{f} = 0$$

- Expanded form:

$$\begin{bmatrix} \overset{e}{B}^T W \overset{e}{B} + \overset{e}{W} & \overset{e}{B}^T W \overset{s}{B} \\ \overset{s}{B}^T W \overset{e}{B} & \overset{s}{B}^T W \overset{s}{B} + \overset{s}{W} \end{bmatrix} \begin{bmatrix} \overset{e}{\Delta} \\ \overset{s}{\Delta} \end{bmatrix} + \begin{bmatrix} \overset{e}{B}^T Wf - \overset{e}{W}\overset{e}{f} \\ \overset{s}{B}^T Wf - \overset{s}{W}\overset{s}{f} \end{bmatrix} = 0$$

Expanding Sub-Matrices

$$\left(\overset{e}{B}^T W \overset{e}{B} + \overset{e}{W}\right)_{12,12} = \begin{bmatrix} \overset{e}{B}_1^T W_1 \overset{e}{B}_1 + \overset{e}{W}_1 & 0 \\ 0 & \overset{e}{B}_2^T W_2 \overset{e}{B}_2 + \overset{e}{W}_2 \end{bmatrix}$$

is a block diagonal form of 6 x 6 sub-matrices

Expanding Sub-Matrices

$$\left(\overset{e}{B}^T W \overset{s}{B}\right)_{12,3n} = \begin{bmatrix} \overset{e}{B}_1^T W_1 \overset{s}{B}_1 \\ \overset{e}{B}_2^T W_2 \overset{s}{B}_2 \end{bmatrix}$$

Sub-matrix full provided all points (n) imaged and observed on both photos

Expanding Sub-Matrices

$$\left(\overset{s}{B}^T W \overset{s}{B} + \overset{s}{W}\right)_{3n,3n} = \overset{s}{B}_1^T W_1 \overset{s}{B}_1 + \overset{s}{B}_2^T W_2 \overset{s}{B}_2 + \overset{s}{W}$$

Sub-matrix – block diagonal <u>provided</u> $\left(\overset{s}{W}\right)$ block diagonal with sub-matrices of same form

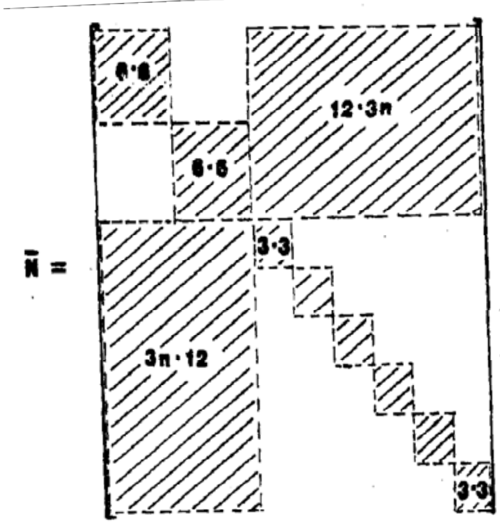
Expanding Sub-Matrices

$$\left(\overset{e}{B}{}^T W f - \overset{e}{W}\,\overset{e}{f}\right)_{12,1} = \begin{bmatrix} \overset{e}{B}{}_1^T W_1 f_1 - \overset{e}{W}_1\,\overset{e}{f}_1 \\ \overset{e}{B}{}_2^T W_2 f_2 - \overset{e}{W}_2\,\overset{e}{f}_2 \end{bmatrix}$$

$$\left(\overset{s}{B}{}^T W f - \overset{s}{W}\,\overset{s}{f}\right)_{3n,1} = \overset{s}{B}{}_1^T W_1 f_1 + \overset{s}{B}{}_2^T W_2 f_2 - \overset{s}{W}\,\overset{s}{f}$$

Normal Coefficient Matrix

- Assume 2 photos and 6 points. All points appear on both photos



Normal Coefficient Matrix

- Assume: pt #1 on photo #1 only, pt 2 on both photos, pt #3 only on photo #2