PROBABILITY

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COUNTING

- Difficult to determine number of elements in finite sample space directly
- THEOREM: If sets $A_1, A_2, \ldots, A_k$ contain $n_1, n_2, \ldots, n_k$ elements respectively, then there are $n_1 \times n_2 \times \ldots \times n_k$ ways of choosing the first element of A_1, then element A_2, ..., finally an element of A_k

PERMUTATION

- When r objects are chosen from a set of n distinct objects, in any particular order or arrangement
- ${}_nP_r$ = n(n-1)(n-2)...(n-r+1)
- In terms of factorials

$${}_nP_r = \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)!}{(n-r)!} = \frac{n!}{(n-r)!}$$

COMBINATION

- The number of ways in which r objects can be selected from a set of n objects is

$$\binom{n}{r} = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$$

- or, in factorial notation

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

PROBABILITY

- Classical Probability Concept
  - If there are n equally likely possibilities, of which one must occur and S are regarded as favorable, or as a "success", then the probability of a "success" is

$$P(A) = S/n$$

AXIOMS OF PROBABILITY

- Real numbers from 0 to 1

$$0 \le P(A) \le 1$$

- Probability of an event from sample space occurring is 1

P(S) = 1

- Probability functions additive

$$P(A \cup B) = P(A) + P(B)$$

GENERALIZATION OF THIRD AXIOM

- If $A_1, A_2, \ldots, A_n$ are mutually exclusive events in a sample space S, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n = P(A_1) + P(A_2) + \cdots P(A_n)$$

RULE OF CALCULATING PROBABILITY OF EVENT

- If A is an event in finite sample space S, then P(A) equals the sum of the probabilities of the individual outcomes comprising A

GENERAL ADDITION RULE

- If A and B are any events in S

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If A and B are mutually exclusive so that

$$P(A \cap B) = 0$$

The additional rule theorem reduces to 3rd axiom -- called special addition rule

PROBABILITY OF COMPLEMENT

- If A and A′ are complementary events $P(A') = 1 - P(A)$

- What is probability of at least 1 head is a coin is tossed 6 times in succession
  - 2^6 sample points since each toss has 2 outcomes. Then P(E) = 1 - P(E′)

$$P(E') = 1/64 \quad P(E) = 1 - 1/64 = 63/64$$

CONDITIONAL PROBABILITY

- The conditional probability of B relative to A is given as

$$P(B|A)=\frac{P(A\cap B)}{P(A)}$$

- B is independent of A iff P(B|A) = P(B)

GENERAL MULTIPLICATION RULE

- If A and B are any events in S, then

$$\begin{aligned} P(A\cap B) &= P(A)\cdot P(B\mid A) \\ &= P(B)\cdot P(A\mid B) \end{aligned}$$

- Special Multiplication Rule - given A and B being independent events

$$P(A\cap B)=P(A)\,P(B)$$

RULE OF ELIMINATION

- Also called Rule of Total Probability
- If $B_1, B_2, \ldots, B_n$ are mutually exclusive events of which one must occur, then

$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A | B_i)$$

BAYES' THEOREM

- If $B_1, B_2, \ldots, B_n$ are mutually exclusive events of which one must occur, then

$$P(B_r | A) = \frac{P(B_r) \cdot P(A | B_r)}{\sum_{i=1}^{n} P(B_i) \cdot P(A | B_i)}$$

- Probabilities $P(B_i)$ called "a priori" probabilities