EXAMPLE OF CHOLESKY DECOMPOSITION

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If A is a matrix defined as $A = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 10 & 2 \\ -2 & 2 & 5 \end{bmatrix}$ then the LU factorization can be shown as:

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & -2 \\ 0 & 9 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

Factoring out the diagonal elements results in:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1/2 & -1/2 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1 \end{bmatrix} = LDL^T$$

Performing the Cholesky decomposition yields:

$$L^* = LD^{1/2} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -1/2 & 1/3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 1 & \sqrt{3} \end{bmatrix}$$

Then, verifying the results, $L^*L^{*T} = A$:

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 1 & \sqrt{3} \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & \sqrt{3} \end{bmatrix} = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 10 & 2 \\ -2 & 2 & 5 \end{bmatrix} = A$$