Wetherell Land Surveying LLC
Patrick N. Johnson, P.S.
510 Michigan Ave., P.O. Box 219 Baldwin, MI 49304
(231) 745-3441 • Fax (231) 745-8494

PROJECT: GAUSSIAN ELIMINATION

REFERENCE: STEVEN J. LEON, "LINEAR ALGEBRA WITH APPLICATIONS"

DATE:
CUSTOMER:

MATRIX A - NONSINGULAR MATRIX. HAS THE FOLLOWING FORM INITIALLY

$$A^{(1)} = \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & \cdots & a_{1n}^{(1)} \\ \boxed{\begin{matrix} a_{21}^{(1)} \\ \vdots \\ a_{n1}^{(1)} \end{matrix}} & \begin{matrix} a_{22}^{(1)} \\ \\ a_{n2}^{(1)} \end{matrix} & \begin{matrix} a_{23}^{(1)} \\ \\ a_{n3}^{(1)} \end{matrix} & \begin{matrix} \cdots \\ \\ \cdots \end{matrix} & \begin{matrix} a_{2n}^{(1)} \\ \\ a_{nn}^{(1)} \end{matrix} \end{bmatrix}$$

ASSUME $a_{11}^{(1)} \neq \phi$

BY ELEMENTARY ROW OPERATIONS, WANT TO MAKE ALL ELEMENTS BELOW THE DIAGONAL IN COLUMN 1 $= \phi$

- ROW OPERATION: ADDING A MULTIPLE OF ROW 1 TO EACH ROW BELOW THE DIAGONAL

$a_{11}^{(1)}$ IS CALLED THE PIVOT

CREATE A NEW MATRIX, $A^{(2)}$:

$$A^{(2)} = \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & \cdots & a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & \cdots & a_{2n}^{(2)} \\ \vdots & \boxed{\begin{matrix} \vdots \\ a_{n2}^{(2)} \end{matrix}} & & & \\ 0 & & a_{n3}^{(2)} & \cdots & a_{nn}^{(2)} \end{bmatrix}$$

DEFINE: $m_{k1} = \dfrac{a_{k1}^{(1)}}{a_{11}^{(1)}}$ for $k = 2, 3, \ldots n$

then $a_{kj}^{(2)} = a_{kj}^{(1)} - m_{k1} a_{1j}^{(1)} \quad (2 \leq k \leq n,\ 2 \leq j \leq n)$

STEP REQUIRES $(n-1)$ DIVISIONS, $(n-1)^2$ MULTIPLICATIONS, & $(n-1)^2$ ADDITIONS/SUBTRACTIONS

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NEXT STEP, $a_{22}^{(2)}$ BECOMES PIVOT & THROUGH ROW OPERATIONS, ELEMENTS IN COLUMN 2 BELOW DIAGONAL BECOME ϕ

ASSUME $a_{22}^{(2)} \neq \phi$, then

$$m_{k2} = \frac{a_{k2}^{(2)}}{a_{22}^{(2)}} \quad \text{for } k = 3, 4, \cdots, n$$

FORMS NEW MATRIX

$$A^{(3)} = \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & \cdots & a_{1n}^{(1)} \\ 0 & a_{22}^{(2)} & a_{23}^{(2)} & \cdots & a_{2n}^{(2)} \\ 0 & 0 & a_{33}^{(3)} & \cdots & a_{3n}^{(3)} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & a_{n3}^{(3)} & \cdots & a_{nn}^{(3)} \end{bmatrix}$$

WHERE $a_{kj}^{(3)} = a_{kj}^{(2)} - m_{k2}\, a_{2j}^{(2)} \quad (3 \le k \le n,\ 3 \le j \le n)$

STEP REQUIRES $(n-2)$ DIVISIONS, $(n-2)^2$ MULTIPLICATIONS & $(n-2)^2$ ADDITIONS/SUBTRACTIONS

CONTINUE PROCESS UNTIL ALL COLUMNS ARE PROCESSED

TOTAL PROCESS REQUIRES

$$(n-1) + (n-2) + \cdots + 1 = \frac{n(n-1)}{2} \quad \text{DIVISIONS}$$

$$(n-1)^2 + (n-2)^2 + \cdots + 1^2 = \frac{n(2n-1)(n-1)}{6} \quad \text{MULTIPLICATIONS}$$

$$(n-1)^2 + \cdots + 1^2 = \frac{n(2n-1)(n-1)}{6} \quad \text{ADDITIONS/SUBTRACTIONS}$$

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ALGORITHM:

For $i = 1, 2, \ldots, n-1$

  For $k = i+1, \ldots, n$

    SET $m_{ki} = \dfrac{a_{ki}^{(i)}}{a_{ii}^{(i)}}$ PROVIDED $a_{ii}^{(i)} \neq 0$

    For $j = i+1, \ldots, n$

      SET $a_{kj}^{(i+1)} = a_{kj}^{(i)} - m_{ki} a_{ij}^{(i)}$

    END FOR LOOP

  END FOR LOOP

END FOR LOOP

SEE ATTACHED SPREADSHEET FOR EXAMPLE

Solution of Linear Equation by Gaussian Elimination

A(1) =	1	3	-2	1	5	=	Augmented Coefficient Matrix
	2	-1	6	2	26		
	3	2	-5	1	-4		
	2	6	1	-4	1		

		A(2) =	1	3	-2	1	5	Line same as A(1)
m(2,1)	2		0	-7	10	0	16	
m(3,1)	3		0	-7	1	-2	-19	
m(4,1)	2		0	0	5	-6	-9	

		A(3) =	1	3	-2	1	5	Line same as A(2)
			0	-7	10	0	16	Line same as A(2)
m(3,2)	1		0	0	-9	-2	-35	
m(3,4)	0		0	0	5	-6	-9	

		A(4) =	1	3	-2	1	5	Line same as A(3)
			0	-7	10	0	16	Line same as A(3)
			0	0	-9	-2	-35	Line same as A(3)
m(4,4)	-0.556		0	0	0	-7.11	-28.44444	

Solution:

x(1) =	1
x(2) =	2
x(3) =	3
x(4) =	4

1	3	-2	1	5									
2	-1	6	2	26									
3	2	-5	1	-4									
2	6	1	-4	1									
						1	3	-2	1	5			
		m(2,1)	=A4/A3			=A4-D9*A3	=B4-D9*B3	=C4-D9*C3	=D4-D9*D3	=E4-D9*E3			
		m(3,1)	=A5/A3		A(2) =	=A5-D10*A3	=B5-D10*B3	=C5-D10*C3	=D5-D10*D3	=E5-D10*E3			
		m(4,1)	=A6/A3			=A6-D11*A3	=B6-D11*B3	=C6-D11*C3	=D6-D11*D3	=E6-D11*E3			
						1	3	-2	1	5			
						0	-7	10	0	16			
		m(3,2)	=H10/H9		A(3) =	=G10-D16*G9	=H10-D16*H9	=I10-D16*I9	=J10-D16*J9	=K10-D16*K9			
		m(3,4)	=H11/H9			=G11-D17*G10	=H11-D17*H10	=I11-D17*I10	=J11-D17*J10	=K11-D17*K10			
						1	3	-2	1	5		x(1) =	=(K20-(J20*N23+I20*N22+H20*N21))/G20
						0	-7	10	0	16		x(2) =	=(K21-J21*N23-I21*N22)/H21
					A(4) =	0	0	-9	-2	-35		x(3) =	=(K22-J22*N23)/I22
		m(4,4)	=I17/I16			=G17-D23*G16	=H17-D23*H16	=I17-D23*I16	=J17-D23*J16	=K17-D23*K16		x(4) =	=K23/J23