VERTICAL PHOTOGRAPHS

Center for Photogrammetric Training
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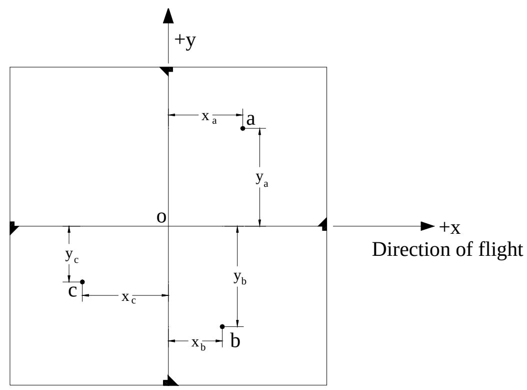
RCB

DEFINITIONS

- Vertical photograph – photo taken with the optical axis coinciding with the direction of gravity
- Tilted photograph – photo taken with the optical axis unintentionally tilted from the vertical by a small amount, usually less that 3^o
  - Near vertical photo
- Exposure station – space position of the front nodal point at the instant of exposure
- Flying height – elevation of exposure station above sea level or datum

DEFINITIONS

- x-axis of the photograph – line on photo between opposite fiducials which most nearly parallels the direction of flight
- y-axis – line on photo normal to x-axis with positive being 90° counter-clockwise from +x



COORDINATE SYSTEM USING CORNER FIDUCIALS

- x', y' coordinates – arbitrary

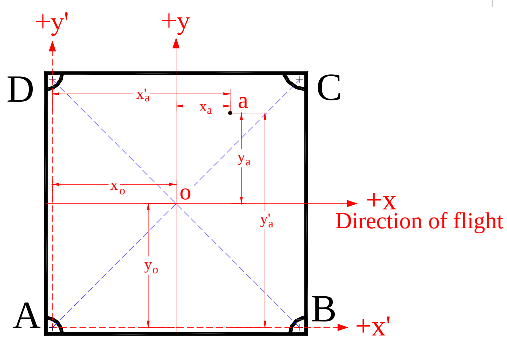
$$x_a = x'_a - x_o$$

$$y_a = y'_a - y_o$$

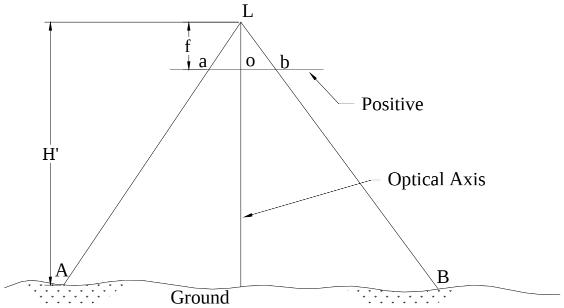
- where

$$x_o = \frac{x'_B + x'_C}{4}$$

$$y_o = \frac{y'_D + y'_C}{4}$$



SCALE OVER FLAT TERRAIN



- Utilizing similar triangles

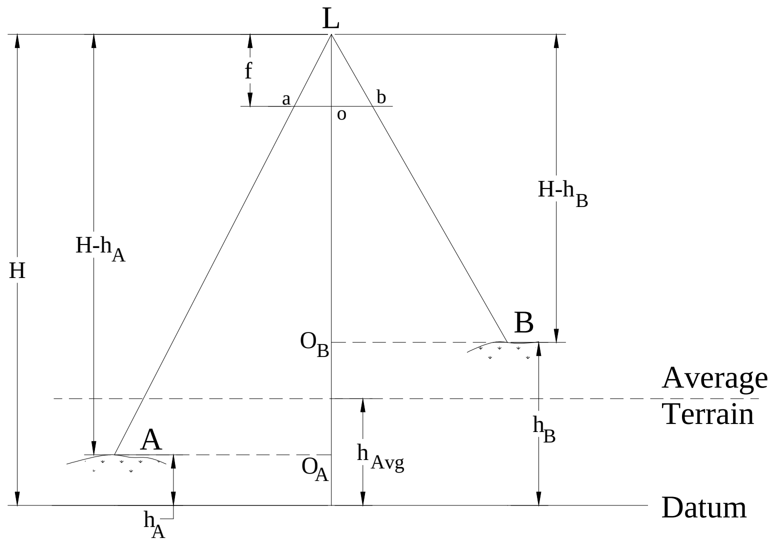
$$S = \frac{ab}{AB} = \frac{f}{H'}$$

SCALE OVER FLAT TERRAIN

- Ex (6-1): Vertical photo taken over flat terrain with 152.4 mm focal length camera at a height of 1830 m above ground. What is the photo scale?
- Solution:

$$S = \frac{f}{H'} = \frac{152.4\ \text{mm}}{1830\ \text{m}} = \frac{1\ \text{mm}}{12\ \text{m}} \quad \Rightarrow \quad 1:12,\bar{0}00$$

SCALE OVER VARIABLE TERRAIN



SCALE OVER VARIABLE TERRAIN

- By similar triangles, Loa and LAO_A:

$$S_A = \frac{ao}{AO_A} = \frac{La}{LA}$$

- By similar triangles LO_AA and Loa

$$\frac{La}{LA} = \frac{f}{H - h_A}$$

- Scale is:

$$S_A = \frac{ao}{AO_A} = \frac{La}{LA} = \frac{f}{H - h_A}$$

AVERAGE SCALE

- Defines overall mean scale of vertical photo over variable terrain

$$S_{Avg} = \frac{f}{H - h_{Avg}}$$

- where:

$$h_{Avg} = \frac{h_A + h_B + \cdots + h_i}{i}$$

SCALE OVER VARIABLE TERRAIN

- Ex: (6-2): The highest, average, and lowest terrain points are 610, 460, and 310 m above sea level respectively. Calculate the maximum scale, minimum scale, and average scale if the flying height above mean sea level is 3000 m and the camera focal length is 152.4 mm.

SCALE OVER VARIABLE TERRAIN

- Maximum scale

$$S_{Max} = \frac{152.4\text{ mm}}{3000\text{ m} - 610\text{ m}} = \frac{1\text{ mm}}{16\text{ m}} \Rightarrow 1:15{,}700$$

- Minimum scale

$$S_{Min} = \frac{152.4\text{ mm}}{3000\text{ m} - 310\text{ m}} = \frac{1\text{ mm}}{17.7\text{ m}} \Rightarrow 1:17{,}700$$

- Average scale

$$S_{Avg} = \frac{152.4\text{ mm}}{3000\text{ m} - 460\text{ m}} = \frac{1\text{mm}}{16.7\text{ m}} \Rightarrow 1:16{,}700$$

SCALE OVER VARIABLE TERRAIN

- How to determine scale if the area on the photography is inaccessible?
- If map of area is available, it can be used to help determine scale
- Use relationship

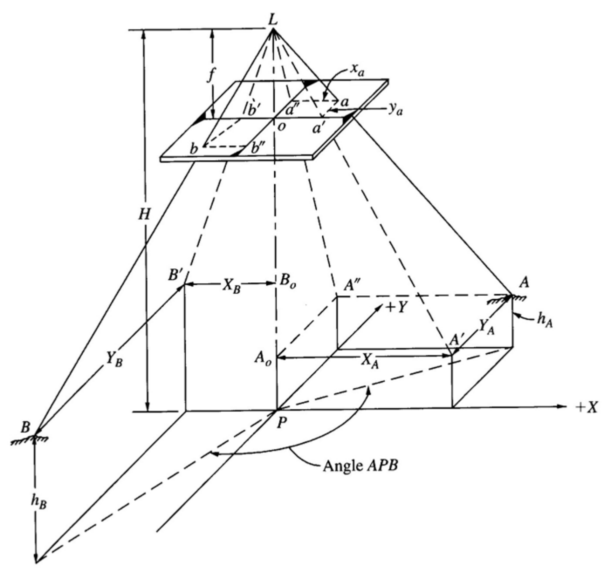
$$S_{Photo} = \left(\frac{d_{Photo}}{d_{Map}}\right) S_{Map}$$

SCALE OVER VARIABLE TERRAIN

- Ex (6-4): The length of an airport runway measures 160 mm on a vertical photograph. On a map, whose scale is 1:24,000, the runway scales 103 mm. What is the scale of the photograph at the runway elevation
- Solution:

$$S = \left(\frac{160\ \text{mm}}{103\ \text{mm}}\right)\left(\frac{1}{24{,}000}\right) = 1{:}15{,}400$$

GROUND COORDINATES FROM VERTICAL PHOTO



GROUND COORDINATES FROM VERTICAL PHOTO

- From similar triangles La'o and LA'A_O

$$\frac{oa'}{A_O A'} = \frac{f}{H - h_A} = \frac{x_a}{X_A} \Rightarrow X_A = \left(\frac{H - h_A}{f}\right) x_a$$

- From similar triangles La"o and LA"A_O

$$\frac{oa''}{A_O A''} = \frac{f}{H - h_A} = \frac{y_a}{Y_A} \Rightarrow Y_A = \left(\frac{H - h_A}{f}\right) y_a$$

GROUND COORDINATES FROM VERTICAL PHOTO

- Ex (6-6): A vertical photo was taken with a 152.4 mm focal length camera at a flying height of 1385 m above the datum. The measured photo coordinates and ground elevations are given in the following table, Determine the horizontal length of line AB

Point	x	y	Elevation
a	-52.35 mm	-48.27 mm	204 m
b	40.64 mm	43.88 mm	148 m

GROUND COORDINATES FROM VERTICAL PHOTO

- Solution:

$$X_A = \left(\frac{1385\,\text{m} - 204\,\text{m}}{152.4\,\text{mm}}\right)(-52.35\,\text{mm}) = -405.7\ \text{m}$$

$$Y_A = \left(\frac{1385\,\text{m} - 204\,\text{m}}{152.4\,\text{mm}}\right)(-48.27\,\text{mm}) = -374.1\,\text{m}$$

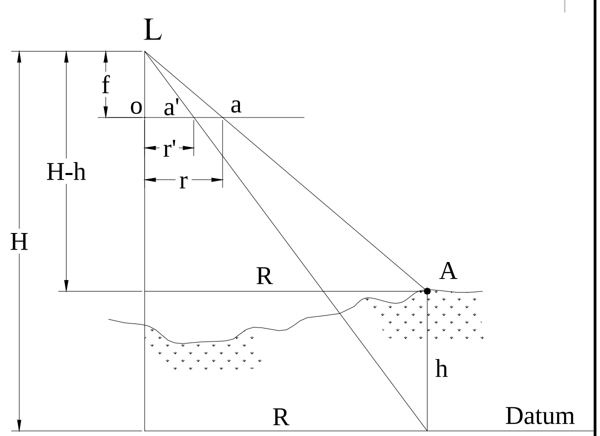
$$X_B = \left(\frac{1385\,\text{m} - 148\,\text{m}}{152.4\,\text{mm}}\right)(40.64\text{mm}) = 329.9\ \text{m}$$

$$Y_B = \left(\frac{1385\,\text{m} - 148\,\text{m}}{152.4\,\text{mm}}\right)(43.88\,\text{mm}) = 356.2\ \text{m}$$

$$AB = \sqrt{[329.9 - (-405.7)]^2 + [356.2 - (374.1)]^2} = 1036\ \text{m}$$

RELIEF DISPLACEMENT

- Shift in photographic position of image caused by elevation above/below the datum



RELIEF DISPLACEMENT

- Relief displacement relationship

$$d = r - r'$$

- From diagram, scale relationships written as

$$\frac{f}{H-h} = \frac{r}{R} \quad \text{and} \quad \frac{f}{H} = \frac{r'}{R}$$

- Rewriting

$$r = \frac{Rf}{H-h} \quad \text{and} \quad r' = \frac{Rf}{H}$$

RELIEF DISPLACEMENT

- Substitute values for r and r'

$$d = \frac{Rf}{H-h} - \frac{Rf}{H}$$

$$= \frac{Rf - Rf + \frac{Rfh}{H}}{H\left(1 - h/H\right)}$$

$$= \frac{Rfh}{H(H-h)}$$

RELIEF DISPLACEMENT

- Rearrange

$$R = \frac{r(H-h)}{f} \qquad \text{and} \qquad R = \frac{r'H}{f}$$

- Substitute first value for R into our formula

$$d = \frac{Rfh}{H(H-h)} = \frac{\left[\frac{r(H-h)}{f}\right]fh}{H(H-h))} \Rightarrow \underline{\underline{d = \frac{rh}{H}}}$$

RELIEF DISPLACEMENT

- Substitute second value for R into same formula

$$d = \frac{Rfh}{H(H-h)} = \frac{\left(\frac{r'H}{f}\right)fh}{H(H-h)} \Rightarrow \underline{\underline{d = \frac{r'h}{H-h}}}$$

  - Formula used when radial distance to datum position can be measured

RELIEF DISPLACEMENT

- Observations
  - Amount of relief increases the farther the point is with respect to the principal point
  - Greater the elevation the greater the amount of relief displacement
  - Increasing flying height decreases the amount of relief displacement
- For elevations above datum, displacement outward while elevations below are displaced inward

RELIEF DISPLACEMENT

- Ex: The datum scale of a photo taken with a 6” focal length lens is 1:12,000. A hilltop lies at an elevation of 1,600’ above the datum, and the image of the hilltop is 2.822” from the principal point. Compute the relief displacement of the hilltop.

RELIEF DISPLACEMENT

- Solution: first solve for flying height

$$H = \frac{f}{S} = \frac{6''}{1:12{,}000} = 6000'$$

- Relief displacement is found as

$$d = \frac{rh}{H} = \frac{(2.822'')(1{,}600')}{6{,}000'} = \underline{\underline{0.753''}}$$

RELIEF DISPLACEMENT

- Relief displacement can be used to determine height of object using:

$$h = \frac{dH}{r}$$

- where d is the difference between the top and bottom of the object. Flying height (H) is height of camera above the base of the object

RELIEF DISPLACEMENT

- Ex: A vertical photo taken from 535 m above the datum. The elevation of the base of a tower is 259 m and the relief displacement d is measured as 54.1 mm. The radial distance to the top of tower is 121.7 mm. What is tower height?

- Solution:
  - Flying height above base of tower

$$H = 535\text{m} - 259\text{m} = 276\,\text{m}$$

  - Height of tower

$$h = \frac{54.1\text{mm}\,(276\text{m})}{121.7\text{mm}} = 123\,\text{m}$$

RELIEF DISPLACEMENT

FLYING HEIGHT OF A VERTICAL PHOTOGRAPH

- Accurate flying height can be determined if elevations of end of line known. Use Pythagorean theorem

$$AB^2 = (X_B - X_A)^2 + (Y_B - Y_A)^2$$

- Substitute in scale relationships for X and Y

$$AB^2 = \left[\left(\frac{H - h_B}{f}\right)x_b - \left(\frac{H - h_A}{f}\right)x_a\right]^2 + \left[\left(\frac{H - h_B}{f}\right)y_b - \left(\frac{H - h_A}{f}\right)y_a\right]^2$$

FLYING HEIGHT OF A VERTICAL PHOTOGRAPH

- Quadratic form:

$$aH^2 + bH + c = 0 \quad \Rightarrow \quad H = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- where:

$$a = \left[(x_a - x_b)^2 + (y_a - y_b)^2\right] / f^2$$

$$b = \{[2x_a(x_b - x_a) + 2y_a(y_b - y_a)]h_A + [2x_b(x_a - x_b) + 2y_b(y_a - y_b)]h_B\} / f^2$$

$$c = \left[(x_a^2 + y_a^2)h_A^2 - (x_a x_b + y_a y_b)2h_A h_B + (x_b^2 + y_b^2)h_B^2\right] / f$$

FLYING HEIGHT OF A VERTICAL PHOTOGRAPH

- Iterative approach: begin by estimating flying height

$$\frac{f}{H-h} = \frac{ab}{AB} \quad \Rightarrow \quad H' = \frac{AB}{ab} f + h_{AVG}$$

- Using H', compute X_A, Y_A, X_B, and Y_B.
- Compute AB' using Pythagorean theorem
- Compare AB' with known value of AB
  - If they agree within prescribed criteria, H' is the flying height – otherwise ...

FLYING HEIGHT OF A VERTICAL PHOTOGRAPH

- Compute new estimate of flying height

$$\frac{H-h_{Avg}}{H'-h_{Avg}} = \frac{AB}{AB'} \quad \Rightarrow \quad H = \frac{AB}{AB'}\left(H'-h_{Avg}\right) + h_{Avg}$$

- Again, compute X_A, Y_A, X_B, and Y_B and a new value for AB'.
- Compare new value for AB' with known value. If not within criteria, iterate

ERROR EVALUATION

- Errors due to random nature of measured quantities and failure of assumptions to be met
- More significant sources of error are:
  1. Errors in photographic measurements
  2. Errors in ground control
  3. Shrinkage/expansion of film and paper
  4. Tilt in the photography

ERROR EVALUATION

- Simple approach – statistical error propagation
  - Calculate rate of change with respect to each variable containing error
- Example: error in computing flying height. If flying height is calculated as

$$H' = f\left(\frac{AB}{ab}\right) = 152.4\,mm\left(\frac{1,524\,m}{127.0\,mm}\right) = 1,829\,m$$

ERROR EVALUATION

- To calculate expected error, dH', caused by error in AB and ab, take derivative of equations with respect to each error source
  - Assume error in ground distance σ_{AB} = 0.50 m and error in photo distance σ_{ab} = 0.20 mm
  - then

$$\frac{\partial H'}{\partial AB} = \frac{f}{ab} = \frac{152.4\,mm}{127.0\,mm} = 1.200$$

  - and

$$\frac{\partial H'}{\partial ab} = \frac{-f\,(AB)}{ab^2} = \frac{-152.4\,mm\,(1524\,m)}{(127.0\,mm)^2} = -14.40\,{}^{m}\!/_{mm}$$

ERROR EVALUATION

- Error propagation formula

$$\sigma_{H'} = \sqrt{\left(\frac{\partial H'}{\partial AB}\right)^2 \sigma_{AB}^2 + \left(\frac{\partial H'}{\partial ab}\right)^2 \sigma_{ab}^2}$$

- From example

$$\sigma_{H'} = \sqrt{(1.200)^2 (0.50\,m)^2 + \left(-14.4\,{}^{m}\!/_{mm}\right)^2 (0.20\,mm)^2}$$

$$= \sqrt{0.36\,m^2 + 8.29\,m^2}$$

$$= \pm 2.9\,m$$