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RELATIVE ORIENTATION

The Center for Photogrammetric Training
Ferris State University

Introduction

The process of relative orientation involves the recreation of the relative angular relationships that existed at the time of photography. This is done by systematically eliminating the y-parallax (p_y) that exist within the model.

The Coplanarity Concept

For relative orientation to exist, the rays emerging from the emergent nodal points of the projectors and the vector containing the air base must form a plane (figure 1). The base vector (B) can be shown as:

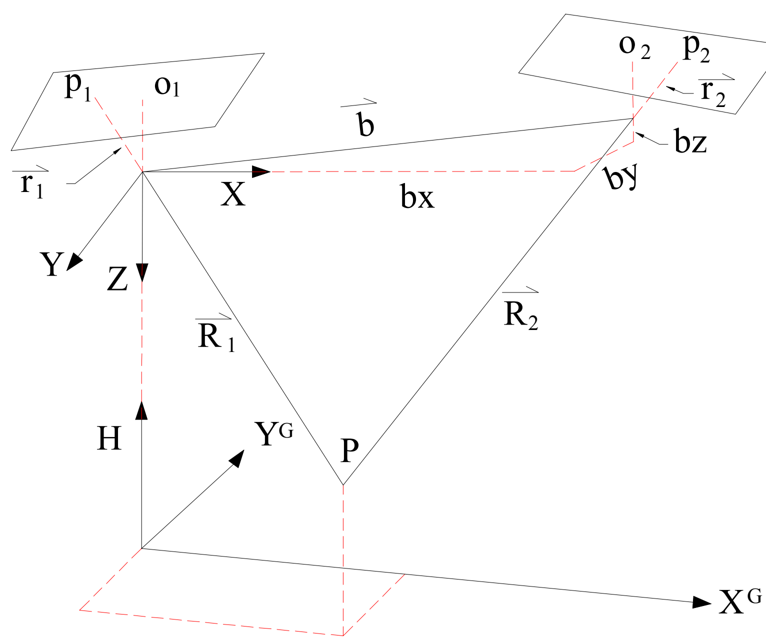

Figure 1. The coplanarity concept.

$$\vec{b} = \begin{bmatrix} bx \\ by \\ bz \end{bmatrix} = \begin{bmatrix} X'' - X' \\ Y'' - Y' \\ Z'' - Z' \end{bmatrix}$$

In relative orientation, all of the intersecting rays are brought into intersection. Thus, the model coordinates derived from either projector must be the same. In other words,

$$X_{p'} = X_{p''} \text{ and } Y_{p'} = Y_{p''}$$

Any difference is parallax. Y-parallax can be expressed as an error shown as

$$p_y = Y_{p'} - Y_{p''}$$

Initially, y-parallax will exist therefore one corrects for it by adding corrections to both projectors. This can be represented mathematically as

$$Y_{p'} + dY' = Y_{p''} + dY''$$

For x-parallax, there is no correction performed because the operator alters either the base (bx) or the Z-wheel. Rearranging the last equation and expressing it as a correction to the coordinates we have

$$dY' - dY'' = Y_{p''} - Y_{p'} = -p_y$$

The parallax at the different points in the model can be represented in terms of the orientation elements. Thus,

$$\begin{aligned} -p_y &= d'Y - dy'' \\ &= d'by - \frac{Y}{Z} \bullet d'bz + X \bullet d\kappa' - Z\left(1 + \frac{Y^2}{Z^2}\right) d\omega' + \frac{XY}{Z} d\phi' - dby'' \\ &\quad + \frac{Y}{Z} dbz'' - (X - b)\, d\kappa'' + Z\left(1 + \frac{Y^2}{Z^2}\right) d\omega'' - \frac{(X-b)\,Y}{Z} d\phi'' \end{aligned}$$

$$-p_y = \frac{XY}{Z}\left(d\phi' - d\phi''\right) - \left(Z + \frac{Y^2}{Z}\right)\left(d\omega' - d\omega''\right) - \frac{Y}{Z}\left(d'bz - dbz'' - b \bullet d\phi''\right)$$

$$+ X\left(d\kappa' - d\kappa''\right) + \left(d'by - dby + d \bullet d\kappa''\right)$$

Rearranging the terms, the correction for parallax can be shown as:
This is sometimes expressed in a simpler form as follows:

$$-p_y = \frac{XY}{Z} \bullet \Delta A - \left(Z + \frac{Y^2}{Z}\right) \bullet B - \frac{Y}{Z} \bullet \Delta C + X \bullet \Delta D + \Delta E$$

where: ΔA, ΔB, ΔC, ΔD, ΔE are referred to as the pseudo-elements.

From the preceding formulas, one can see that the model contains five types of parallax that are proportional to:

$$\frac{XY}{Z} \; ; \; \left(Z + \frac{Y^2}{Z} \right) \; ; \; \frac{Y}{Z} \; ; \; X \; ; \; \text{a constant}$$

Thus, the operator needs to visit five different positions in the model to clear it of parallax. Another significant part of the formulas is that moving one (or both) of the projectors along the base will not affect the orientation of the model. In other words, a movement in the bx direction cannot be used to create intersection of the rays when that intersection does not exist, nor will it destroy the intersection if the model is free of parallax. The bx movement can be used to eliminate the x-parallax existing in the model.

PERFORMING RELATIVE ORIENTATION ON THE STEREOPLOTTER

Relative orientation can be performed in many different fashions. Each of the instrument manufacturers have designed specific methods for accomplishing the relative orientation process. There are two general methods employed in photogrammetry:

1. Independent approach (also called the swing/swing method or two projector method) is when both projectors are moved in the orientation.

2. Dependent approach (also called the one projector method) involves the movement of one projector while the other is fixed in space.

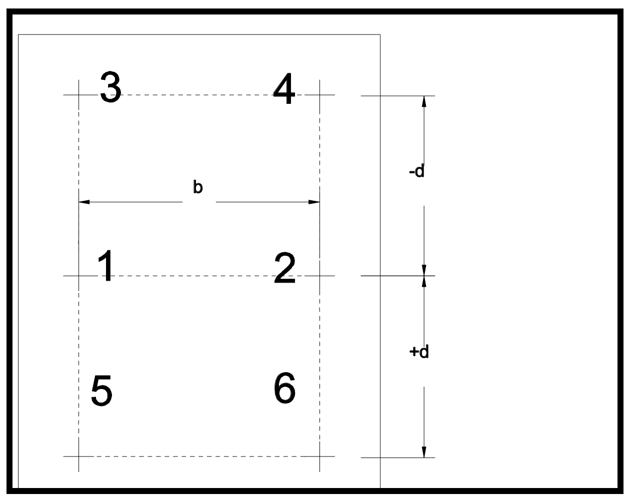

Figure 2. Location of the model points for relative orientation.

The general method for clearing parallax in a stereomodel involving the independent method is:

1. Clear p_y at point 1 with κ".
2. Clear p_y at point 2 with κ'.
3. Clear p_y at point 3 with ϕ". ϕ introduces p_x into the model. This must be eliminated by the Z-wheel.

4. Clear p_y at point 4 using φ'.
5. Overcorrect the p_y by 150% at point 5 (or 6) using the ω" (or ω').
6. The use of ω will introduce parallax in the whole model. Therefore, it is necessary to revisit all of the points again (steps 1-5) and eliminate any residual parallax. This iterative process is continued until no corrections need to be entered. Then go to point 6 (or 5 if point 6 was used in step 5) and check for any parallax. If it exists, there is still parallax in the model that has to be eliminated.

The overcorrection value used in step 5 can also be computed on some of the stereoplotters. Bertil Hallert showed that the overcorrection coefficient using the empirical-unsymmetrical method can be computed as:

$$k = \frac{1}{2}\left(1 + \frac{f^2}{d^2}\right)$$

This coefficient is multiplied by the y-parallax measured at points 5 or 6. This overcorrection is added to the parallax measured at the point.

Hallert also presents an empirical-symmetrical method, which is outlines as follows:

1. Clear the p_y at points 1-4 as outlined above.
2. At point 5, half of the parallax is corrected with ω" and the rest is overcorrected with ω. This overcorrection value is designated as k' where:

   $$k' = 1 + \frac{f^2}{d^2}$$

   The new parallax at points 5 and 3 after the overcorrection are

   $$p_{y_5} = k'p' - p' = p_{y_3}$$

   where p' is the y-parallax (p_y) at point 5 that must be overcorrected.
3. Continue the relative orientation as before.

The procedure for dependent orientation must still involve the correction to the parallax using five elements. Thus the translational elements (by and bz) must be used. The procedure is as follows (using projector II in the change):

1. At point 2 clear p_y with by".
2. At point 1 clear p_y with κ".
3. At point 4 clear p_y with bz".

4. At point 3 clear p_y with φ".
5. At point 6 overcorrect p_y with ω" by 150%.
6. Repeat steps 1-5 until there is no y-parallax at each point.
7. Check point 5 for any y-parallax.

These approaches to relative orientation are valid for flat terrain. Thus, the process may take longer for hilly terrain.

STEREOSCOPIC PLOTTING INSTRUMENTS

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INTRODUCTION

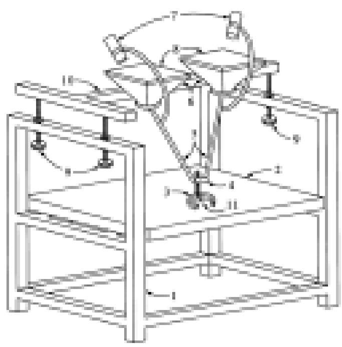
- Stereoplotter
  - Provides rigorous, accurate solution for object point location (X, Y, Z) from images appearing in overlapping pair of photos
- Accuracy depends on 3 orientation processes
  - Interior orientation
  - Relative orientation
  - Absolute orientation

CLAFFICATION OF PLOTTERS

- Four general categories
  1. Direct optical projection instruments
  2. Mechanical or optical-mechanical projection instruments
  3. Analytical stereoplotters
  4. Softcopy or digital stereoplotters
- Other classification methods
  1. By accuracy capability (first-, second-, or third order)
  2. According to whether an “approximate” solution or “theoretically correct” solution is obtained

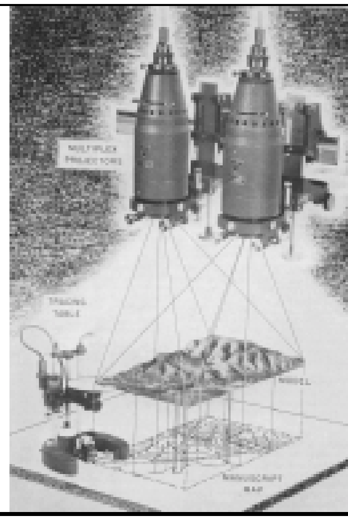
DIRECT OPTICAL PROJECTION STEREOPLOTTERS

1. Main frame
2. Reference table
3. Tracing table
4. Platen
5. Guide rods
6. Projectors
7. Illumination lamps
8. Diapositives
9. Leveling screws
10. Projector bar
11. Tracing pencil



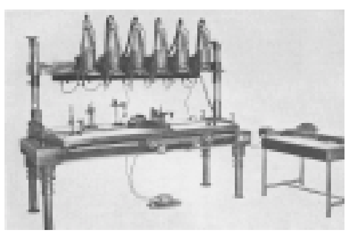
DIRECT
OPTICAL
PROJECTION
PLOTTER

MULTIPLEX
MODEL
SKETCH



DIRECT OPTICAL PROJECTION STEREOPLOTTERS

- Nistri Photomultiplex Model D III



- Kelsh Stereoplotter

PROJECTION SYSTEM

- Light rays projected through projector objective lenses and intercepted below on platen
- Requires operation in dark room
- Lens formula must be satisfied

$$\frac{1}{f'} = \frac{1}{p} + \frac{1}{h}$$

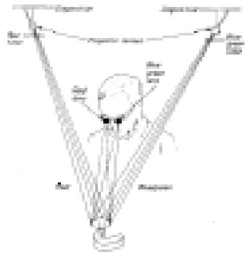
- Intersection must occur within depth of field of projector lens

PROJECTION SYSTEMS

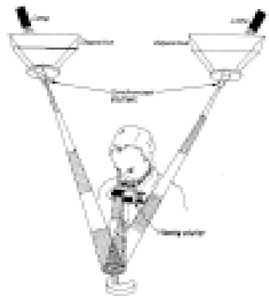
- To recreate relative angular relationships
  - Projectors must have rotational and translational movement capabilities
  - 6 possible for each projector
    - ω (omega) – x rotation also called tilt
    - φ (phi) – y rotation also called tip
    - κ (kappa) – z rotation also called swing
    - X translation
    - Y translation
    - Z translation

VIEWING SYSTEMS

- Anaglyphic system



- Polarized platen viewing similar system
- Stereo-image alternator (SIA)



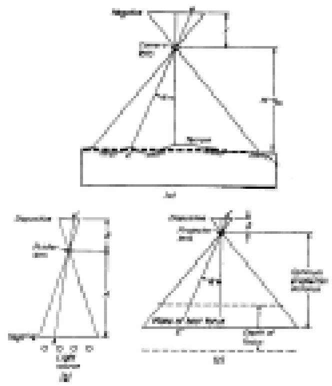
INTERIOR ORIENTATION

- Recreates geometry of the taking camera
- Four steps
  1. Centering diapositives on the projectors
  2. Setting off the proper principal distance
  3. Preparation of the diapositive
  4. Compensation for image distortions

INTERIOR ORIENTATION

- Preparation of diapositive
  - Direct contact printing
    - Principal distance will equal focal length of taking camera
  - Projection printing
    - Seldom used today
    - Necessary for reduced-sized diapositives
    - Must meet following:

$$\frac{A}{B} = \frac{f}{p}$$



INTERIOR ORIENTATION

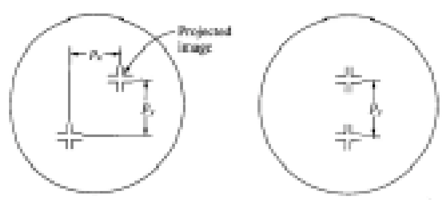
- Compensation for lens distortion
  - Use a "correction plate" in projection printing of diapositive, followed by use of distortion-free lens
  - Vary the projector principal distance by means of a cam
    - Reconstructing true geometry
  - Use projector lens whose distortion characteristics negate camera's distortion

RELATIVE ORIENTATION

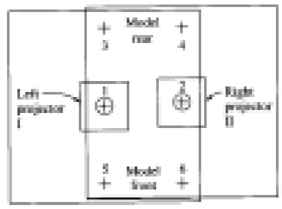
- Recreate the same relative relationship between diapositives that existed at the time of the photography
- Condition: each model point and the two projection centers form a plane in miniature
  - Just like that which existed for the corresponding ground point and the two exposure stations

RELATIVE ORIENTATION

- Since p_x is a function of elevation, it can be removed by raising or lowering platen (Z-wheel)
- What remains is p_y – removed using a rotational or translational element to a projector on the stereoplotter

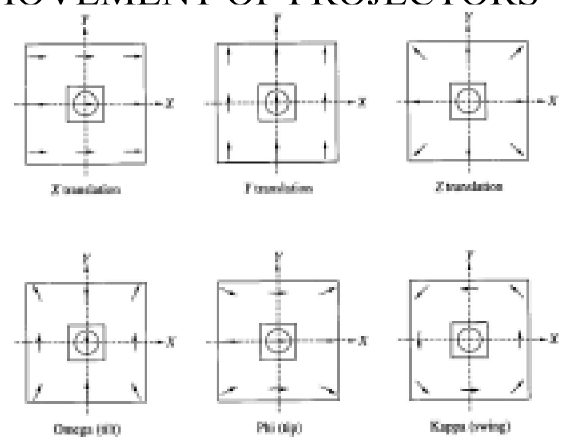


RELATIVE ORIENTATION



- ♦ 6 von Gruber points used to “clear” y-parallax
  - 5 points used to clear the model
  - 6^{th} point used to check the model

MOVEMENT OF PROJECTORS



INDEPENDENT METHOD OF RELATIVE ORIENTATION

1. Clear p_y at 1 with κ”
2. Clear p_y at 2 with κ’
3. Clear p_y at 3 with φ”
4. Clear p_y at 4 with φ’
5. Observe p_y at 5 and overcorrect by introducing 150% correction to the measured parallax with ω”
6. Repeat steps 1-5 until no p_y exists at #5
7. Check for p_y at point 6.

DEPENDENT METHOD OF RELATIVE ORIENTATION

Right projector only:

1. Clear p_y at 2 with by-translation
2. Clear p_y at 1 with κ”
3. Clear p_y at 4 with bz-translation
4. Clear p_y at 3 with φ”
5. Observe p_y at 6 and overcorrect by introducing 150% correction to the measured parallax with ω”
6. Repeat steps 1-5 until no p_y exists at #5
7. Check for p_y at point 5.

ABSOLUTE ORIENTATION

- After relative orientation, a true 3-D model is formed
- Next step:
  - Level model with respect to datum
  - Unknown scale of model is fixed to desired scale for mapping

SELECTING MODEL SCALE

- Model scale constrained by scale of photography and limitations of stereoplotter
- Model scale represented by

$$S_m = \frac{b}{B} = \frac{h}{H'}$$

- Recalling scale of photography, model scale can be represented as

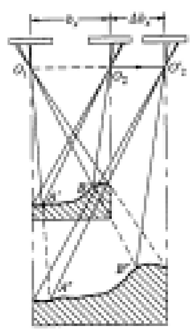
$$S_m = \frac{h}{f} S_p$$

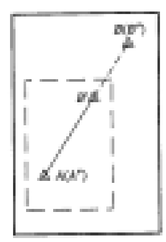
SELECTING MODEL SCALE

- When model scale determined, initial model air base is set off
  - More convenient before relative orientation
    - Scale closer to required model scale
  - Initial model base obtained by multiplying photo base by actual enlargement ratio S_m/S_p

SCALING MODEL

- Model scale changed by varying model base
- If by and bz settings same for each projector, model base consists only of bx motion



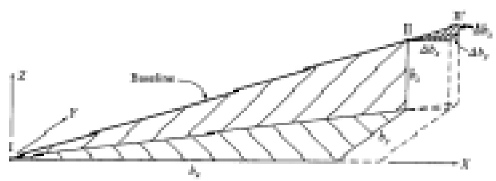


SCALING MODEL

- Minimum of 2 horizontal control points needed
  - Unique solution – no check
- Place floating mark over point A and mark location on plotting sheet
  - Similarly for point B
  - Distance shown as A'B'
- If A'B' does not equal AB, compute change to bx

$$\Delta bx = bx\left(\frac{AB}{A'B'} - 1\right)$$

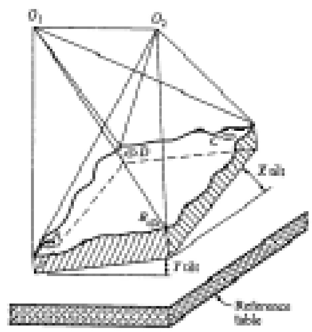
SCALING MODEL



- If by and bz not equal, need to move the right projector from position II to II'

LEVELING THE MODEL

- Requires minimum of 3 vertical control pts.
  - No check
- Proper gears must be placed in instrument for consistent vertical scale
- Two components of tilt
  - X – component (Ω)
  - Y-component (Φ)



LEVELING THE MODEL

- Iterative procedure

1. Set floating mare on model point A and index tracing table dial to read control elevation of point
2. Read model elevation of control point D
3. If difference exists, X-tilt (Ω) applied
   - If model elevation is higher than control elevation, model is tilted up in rear
4. Repeat steps 1-3 until model is level in the direction from A to D

LEVELING THE MODEL

5. Reindex tracing table dial to read control elevation of point A with floating mark set on model point A
6. Read model elevation of control point B
7. If model elevation does not agree with control elevation, introduce Y-tilt (Φ) in similar fashion as set 3
8. Repeat steps 5-7 until model is level in that direction

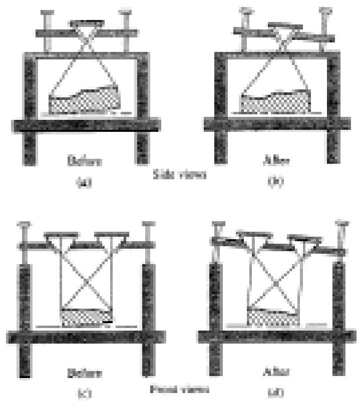
LEVELING THE MODEL

9. Check point D to see if model elevation still conforms to control elevation
   - If line AD is not parallel to Y-axis of stereoplotter then it will be likely not to conform
10. Check point C to see if model elevation conforms to control elevation
    - If elevation does not conform, may indicate an error in relative orientation or blunder in the vertical control

LEVELING THE MODEL

- Methods of introducing corrective tilts – instrument dependent
  - Reference table may be tilted in X and Y directions making them parallel to model datum
  - Using leveling screws to rotate projector bar
  - Introducing corrective tilts to each projector individually

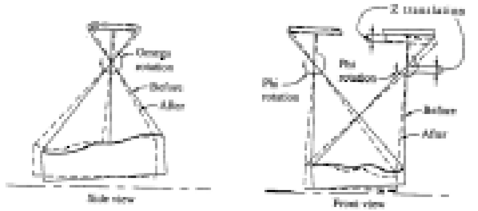
LEVELING THE MODEL BY WITH LEVELING SCREWS TO PROJECTOR BAR



(a) and (b) Correcting X tilt of a model by X tilt of projector bar.
(c) and (d) Correcting Y tilt of a model by Y tilt of projector bar.

LEVELING THE MODEL BY MOVING EACH PROJECTOR

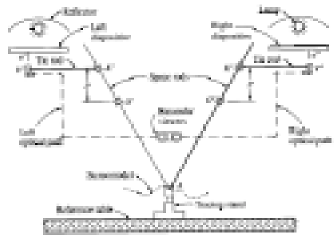
- Trial amount of Ω applied to left projector
  - Remove any p_y with corresponding Ω in right projector at any point in stereomodel
- Trial amount of Φ applied to both projectors
  - Y-parallax removed by removing p_y at one of the corners using a bz motion to either projector



MECHANICAL PROJECTION PLOTTER

- Space rods used to simulate direct optical projection of light rays
- Preferred instrument over direct optical plotters
  - More versatile
  - Higher accuracy
  - Better overall stability
  - Need not be operated in dark room

MECHANICAL PROJECTION PLOTTER



- ♦ Diapositives placed in carriers and illuminated above
- ♦ Space rods are free to rotate about gimbal joints O' and O" and can slide up and down through joints
- ♦ Model air base defined by O'O"
- ♦ Joints fixed in position except their spacing can be changed

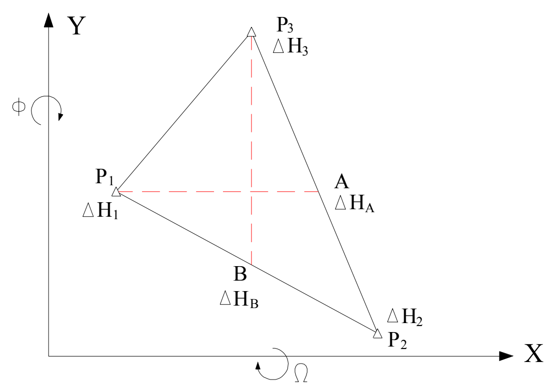
MECHANICAL PROJECTION PLOTTER

- ♦ Viewing system – 2 optical trains of lenses, mirrors, and prisms
- ♦ Reference half mark superimposed on optical axis of each lens V' and V"
- ♦ Movement imparted to lenses from space rods by means of tie rods connected to another set of gimbal joints
  - – These joints fixed in vertical position
  - – Vertical distance O' and O" to corresponding upper gimbal joint K' and K" – principal distance (p)

MECHANICAL PROJECTION PLOTTER

- ♦ Space rods intersect at tracing stand
- ♦ Moving tracing stands moves space rods
- ♦ When properly oriented, moving tracing stand moves lenses V' and V" to a' and a" in corresponding imagery
  - – With half marks merged, floating mark will appear on surface of model point A
- ♦ Orientation very similar to optical plotters

LEVELING MODEL (EMPIRICAL)



LEVELING MODEL (EMPIRICAL)

- Given elevations denoted as H_i', while elevations read at instrument elevation counter called H_i
- Difference is elevation error

$$\Delta H_i = H_i - H_i^{'}$$

- 3 non-collinear control points required for leveling
- As in optical plotters, one height counter, like P_1, is set to correct ground elevation

LEVELING MODEL (EMPIRICAL)

- Elevation errors at A and B determined from simple linear interpolation

$$\Delta H_A = \Delta H_2 + \frac{P_2 A}{P_2 P_3}\left(\Delta H_3 - \Delta H_2\right)$$

$$\Delta H_B = \Delta H_1 + \frac{P_1 B}{P_1 P_2}\left(\Delta H_2 - \Delta H_1\right)$$

LEVELING MODEL (EMPIRICAL)

- Correction formulas depend on
  - Particular instrument
  - Type of material (negative or diapositive)
- For Wild A-7 (diapositive), corrective tilts:

$$\Omega^c = \frac{\Delta H_B - \Delta H_3}{P_3B} \cdot \rho^c$$

$$\Phi^c = \frac{\Delta H_A - \Delta H_1}{P_1A} \cdot \rho^c$$

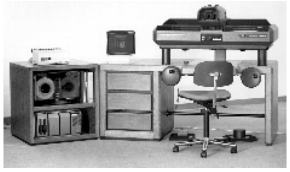
- Where ρ^c is conversion factor from radians to centesimal minutes ($\rho^c = 6366.2$) and distances expressed in ground units

ANALYTICAL PLOTTERS

- Development possible with advances in computers, digital encoders and servosystems
- Use exact mathematical calculations to define stereomodel
- Capabilities
  - Precisely measure x and y photo coordinates on both photos of a stereopair
  - Accurately move to defined x and y photo locations

ANALYTICAL PLOTTERS

- Wild BC-1



- Zeiss P-3

ADVANTAGES OF AP

- No optical or mechanical limitations in forming models
  - Great versatility
  - Can handle vertical, tilted, low oblique, convergent, high oblique, panoramic, terrestrial images, and satellite images
- Accommodate photography from any focal-length camera
- Provide superior results than analog plotters
  - Optical and mechanical errors are not introduced
  - Can correct for any combination of systematic errors
  - Take advantage of redundant observations and incorporate into least squares solution

SOFTCOPY PLOTTERS

- Similar to analytical plotters
  - Rely on digital imagery
  - Less expensive and more versatile than analytical plotters
- Essential component – computer with high-resolution graphics display
  - Must be capable of displaying both photos simultaneously

SOFTCOPY PLOTTERS

- Leica Socet Set

- ZI ImageStation