COMMENT: S&P, V. XXXV, N

The First Method To Adjust a Traverse Based on Statistical Considerations

By HERBERT W. STOUGHTON

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Traverses were used in land surveying in England by the end of the 16th century. Arthur Hopton (1611), Aaron Rathbone (1616), and William Leybourne (1650 and 1653) discussed traverses in their writings. In 1739 Robert A. Gibson published *A Treatise On Land Surveying;* the fourth edition entitled *A Treatise On Practical Surveying* was the first text on surveying printed in the United States. In the American edition of his work Gibson emphasized calculation of latitudes and departures—Northings, Southings, Eastings, and Westings. He stated that the sum of the Northings must equal the sum of the Southings, and that the sum of the Eastings must equal the sum of the Westings. He further stated that for small surveys if the sums as previously stated exceeded one-fifth of a rod per station then the survey must be rerun. If the survey satisfied the aforementioned requirements, Gibson gave a rule to correct the latitudes and departures for each course. This is one of the earliest discussions of traverse closure and traverse adjustment.

Gibson's method to adjust a traverse had no statistical foundation. In 1808 Robert Patterson of Philadelphia, Pennsylvania, wrote Dr. Adrian, the editor of a journal of theoretical and applied mathematics, for a method to adjust the observations based on statistical considerations. The solution to Patterson's question was proposed by Nathaniel Bowditch of Salem, Massachusetts. Bowditch's method is the oldest traverse adjustment technique based on a statistical approach. In the United States surveyors refer to Bowditch's method as the Compass Rule, while in the other English-speaking countries the method is called Bowditch's Rule or Bowditch's method in honor of Nathaniel Bowditch, as is the normal practice to name a theorem or procedure after its developer.

The survey texts printed in the United States today do not mention Bowditch's contribution. In *The Adjustment Of Observations* by Thomas W. Wright and John F. Hayford (pp. 156–158), Bowditch's method was rigorously derived. In *A Treatise On Land Surveying,* William M. Gillespie credits Bowditch for the method (p. 177, eighth edition). *The Theory And Practice Of Surveying* by Robert Gibson revised by James Ryan (1840) is the only text where Bowditch's original paper is reproduced verbatim (footnote, pp. 191–195).

Following is the complete text of Bowditch's paper as it appeared in 1808. The paper was published in *The Analyst or Mathematical Museum.*

The Analyst or Mathematical Museum

[Volume I No. II, 1808, p. 42 (reprinted Volume I No. III, 1808, page 68.]

QUESTION

By Robert Patterson, Philadelphia

For the best satisfactory solution of which, to be adjudged by the Editor, he offers a prize of ten dollars.

In order to find the content of a piece of ground, having a plane level surface, I measured, with a common circumferentor and chain, the bearings and lengths of its several sides, or boundary lines, which I found as follows:

1. N.45°E. 40 perches,
2. S.30°W. 25 ditto,
3. S.5°E. 36 ditto,
4. West 29.6 ditto,
5. N.20°E. 31 ditto, to the place of beginning.

But, upon casting up the difference of latitude and departure, I discovered, what will perhaps always be the case in actual surveys, that some error had been contracted in taking the dimensions. Now it is required to compute the area of this enclosure, on the *most probable supposition* of this error.

The solution to the above question appeared in Volume I No. IV, 1808, pp. 88–93.

ARTICLE XIII.

Solution of Mr. Patterson's Prize Question for correcting a survey, proposed in No. II. p. 42, No. III. p. 68, by Nathaniel Bowditch, to whom the Editor has awarded the prize of ten dollars.

The principles, on which the solution of the Prize Question, p. 42, ought to depend, appear to me to be these.

1. That the error ought to be apportioned among *all* the bearings and distances.

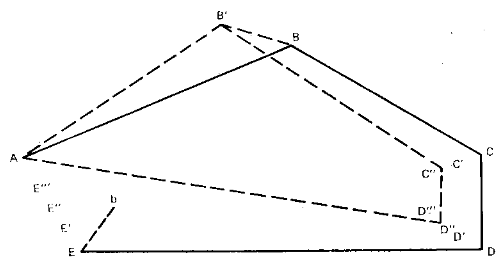
2. That in those lines in which an alteration of the measured distance would tend considerably to correct the error of the survey, a correction ought to be made; but when such alteration would not have that tendency, the length of the line ought to remain unaltered.

3. In the same manner, an alteration ought to be made in the observed bearings, if it would tend considerably to correct the error of the survey, otherwise not.

4. In cases where alterations in the bearings and distance will both tend to correct the error, it will be proper to alter them both, making greater or less alterations according to the greater or less efficacy in correcting the error of the survey.

5. The alterations made in the observed bearing and length of any one of the boundary lines ought to be such that the combined effect of such alterations may tend wholly to correct the error of the survey.

Suppose now that *ABCDE* represent the boundary lines of a field, as plotted from the observed bearings and lengths, and that the last point *E*, instead of falling on the first *A*, is distant from it by the length *AE*. The question will then be, what alterations *BB′*, *CC″*, *DD‴*, &c., must be made in the positions of the points *B,C,D*, &c., so as to obtain the most probable boundaries *AB′C″D‴A?*

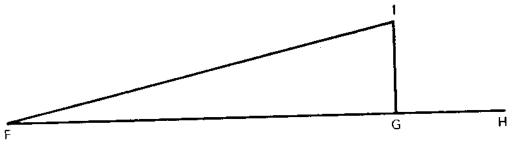


If *AB′* be supposed to be the most probable bearing and length of the first boundary line, the point *B* would be moved through the line *BB′*, and the following points ***C, D, E,*** would in consequence thereof be moved in equal and parallel directions to *C′*, *D′*, *E′*, and the boundary would become *AB′C′D′E′*. Again, if by correcting in the most probable manner the error in the observed bearing and length of *BC*, (or *B′C′*) the point *C′* be moved to *C″*, the points *D′* and *E′* would be moved in equal and parallel directions to *D″* and *E″*, and the boundary line would become *AB′C″D″E″*. In a similar manner, if by correcting the probable error in the bearing and length of *CD*, (or *C″D″*) the point *D″* be moved to *D‴*, the point *E″* would be moved in an equal and parallel direction to *E‴*, and the boundary would become *AB′C″D‴E‴*. Lastly, by correcting the probable error in the bearing and length of the line *DE*, (or *D‴E‴*) the true boundary *AB′C″D‴A* would be obtained.

If we suppose the lines *BB′*, *CC″*, *DD‴*, &c. to be parallel to *AE*, it would satisfy the second, third, fourth, and fifth, of the preceding principles. For the change of position of the points *B*, *C*. &c. being in directions parallel to *AE*, the whole tendency of such change would be to move the point *E* directly towards *A*, conformably to the fifth principle, and by inspecting the figure it will appear that the second, third, and fourth principles would also be satisfied. For, in the first place, it appears that the bearing of the first line *AB* would be altered considerably, but the length but little. This is agreeable to those principles; because an increase of the distance *AB* would move the point *E* in the direction *Eb* parallel to *AB*, and an alteration in the bearing would move it in the direction *Eb′* perpendicular to *AB*. Now the former change would not tend effectually to decrease the distance *AE*, but the latter would be almost wholly exerted in producing that effect. Again, the length of the line *BC* would be considerably changed without altering essentially the bearing; the former alteration would tend greatly to decrease the distance *AE*, but the latter would not produce so sensible an effect. Similar remarks may be made on the changes in the other bearings and distances, but it does not appear to be necessary to enter more largely on this subject.

It remains now to determine the proportion of the lines *BB′*, *CC″*, *DD‴*, &c. To do this we shall observe that in measuring the lengths of any lines the errors would probably be in proportion to their lengths. These supposed errors must however be decreased on those lines where the effect in correcting the error of the survey would be small, by the second and fourth principles.

In observing the bearings of all the boundary lines, equal errors are liable to be committed; however it will be proper, by the third and fourth principles, to suppose the error greater or less, in proportion to the greater or less effect it would produce in correcting the error of the survey.



Now the error of an observed bearing being given, as for example *GFI*, the change of position *GI* of the end of the line *G* would be proportional to the length of the line *FG* (= *FI*), so that the supposed errors both in the length and in the bearing of any boundary line, would produce changes in the position of the end of it proportional to its length. There appears therefore a considerable degree of probability in supposing the lines *BB′*, *C′*, *C″*, *D″*, *DD‴*, &c., to be respectively proportional to the lengths of the boundary lines *AB*, *BC*, *CD*, &c. The main point to be ascertained before adopting this hypothesis is whether a due proportion of the error of the survey is thrown on the bearings and the lengths of the sides. Now it is plain by this hypothesis, that the error in any boundary line is supposed to be wholly in the bearing if the line be perpendicular to *AE*, and wholly in its length when parallel to *AE*; and if the length be the same in both cases, the change of position of the end of the line would in both cases be exactly equal. Thus, if *FGH* be the boundary line, *GI* the change of position of the point *B* in the former case, and *GH* in the latter, we should in the hypothesis have *GI* = *GH*.

To show the probability of this hypothesis it may be observed, that in measuring the length of a line *FGH* of six or eight chains of 50 links each, an error of *one* link might easily be committed by the stretching of the chain, or the unevenness of the surface. This error would be about 1/350 of the whole length. If we therefore suppose *GI* to be 1/350 of *FG*, the angle *GFI* would be about 10′. Now, with such instruments as are generally made use of by surveyors, it is about as probable that an error of 10′ was made in the bearing, as that the above error of 1/350 part was made in measuring the length. We shall therefore adopt it as a principle that the most probable way of apportioning the error of the survey *AE* is to take *BB′*, *C′C″*, *D″D‴*, &c., respectively proportional to the boundary lines *AB*, *BC*, *CD*, &c.

Hence the following practical rule for correcting a survey.

Geometrically. Draw the boundary lines *ABCDE* by means of the observed bearings and lengths, and find the error of the survey *AE*, and let the quotient of *AE* divided by the

Table 1. Traverse Table.						Table 2. Corrections.		Table 3. Correct diff. of latitude and departure.			
Course	Dist.	N.	S.	E.	W.	N.	E.	N.	S.	E.	W.
N.45°E.	40	28.28	–	28.28	–	.02	.02	28.30	–	28.30	–
S.30°W.	25	–	21.65	–	12.50	.02	.01	–	21.63	–	12.49
S.5°E.	36	–	35.86	3.14	–	.02	.02	–	35.84	3.16	–
W.	29.6	–	–	–	29.60	.02	.01	0.02	–	–	29.59
N.20°E.	31	29.13	–	10.60	–	.02	0.2	29.15	–	10.62	–
	161.6	57.41	57.51	42.02	42.10	.10	.08	57.47	57.47	42.08	42.08
			57.41		42.02						
		Errors	0.10		0.08						

sum of all the lines *AB, BC, CD, DE,* be represented by *r*. Through the angular points *B, C, D,* &c., draw the lines *BB′*, *CC″*, &c., parallel to *AE,* and in the same direction that *A* bears from *E*. Take $BB' = r \times AB$, $CC'' = r \times (AB + BC)$, $DD''' = r \times (AB + BC + CD)$, &c. Then through the points, *A, B′, C″, D‴*, &c., draw the corrected boundary lines *ABCDA,* which being determined, the area may be found by dividing the figure into triangles in the usual method.

The proportional parts *BB′*, *CC″*, &c., may be found expeditiously by means of a table of difference of latitude and departure, by finding the page where the sum of the lines $AB + BC + CD + DE$ in the distance column corresponds to *AE* in the departure or difference of latitude column, then find *AB*, $AB + BC$, &c., in the distance column, and the corresponding numbers will be equal to *BB′*, *CC″*, *DD‴*, &c., respectively.

Arithmetically. The area of the field may also be found by means of the tables of difference of latitude and departure, by calculating for each of the observed bearings and lengths, the corresponding difference of latitude and departure; which may be corrected in the following manner. Add up the northings and southings, and find the difference of their sums, which will be the error of the survey in the difference of latitude, which call by the same name as the least sum. Proceed in the same manner with the eastings and westings and find the error of the departure. These errors must be apportioned among the differences of latitude and departure by saying, as the sum of the observed distances $(AB + BC + CD + DE)$ is to any particular distance as (AB), so is the above error in the difference of latitude or departure to the corresponding correction of the difference of latitude or departure depending on that distance.* The corrections being thus calculated and applied to the corresponding differences of latitude and departure, by adding when of the same name, and subtracting when of different names, will give the differences of latitude and departure, from which the area may be calculated by the usual rules.

This last method being made use of for calculating the area in the proposed question, the error in the difference of latitude is found 0.10 N., and the error of the departure 0.08 E., and the sum of all the distances $AB + BC$, &c. = 161.6, as in Table 1.

Hence

161.6:0.10::40:0.02
::25:0.02
&c. &c.
} The corrections of difference of lat. as in Table 2.

And

161.6:0.08::40:0.02
25:0.01
&c. &c.
} The corrections of departure as in Table 2.

The corrections of Table 2 being connected with the corresponding number of Table 1 will give the corrected differences of latitude and departure as in Table 3. By

* These corrections may also be calculated by means of the table of difference of latitude and departure.

these corrected values, the areas were calculated as in Table 4. The method of doing this being taught in books of surveying, it would be useless to repeat it here.

Table 4. Calculation of the Areas.

Dep.	Dep.	North Areas	South Areas
0.00	28.30	800.89	–
12.49	12.49	–	270.16
9.33	21.82	–	782.03
38.92	48.25	0.96	–
28.32	67.22	1959.46	–
		2761.31	1052.19
		1052.19	
		1709.12	
	Half =	854.56	Sq. perches.

So the area is 5 acres, 1 rood, 14 56/100 perches.

N.B. In a subsequent communication Mr. Bowditch informed me that he had used this method of correcting a survey several years ago.

In justice to Mr. Bowditch it is proper to observe that his solution to Question VII. No. II. was true, and would have been published in No. III. (as no other true solution was sent me) had it been received in due time: but the solution published in No. III. was in print before his came to hand.

Through want of room several other learned and ingenious pieces of Mr. Bowditch's are omitted, which will enrich the future Numbers of the *Analyst*. ED. ■

$$(2)\quad C = 2R\left(\sin\frac{\Delta}{2}\right)$$

where R = radius, Δ = central angle in degrees-minutes-seconds, L = arc length, and C = chord length. By combining equations (1) and (2) and simplifying, the following equality results:

$$(3)\quad \frac{(\Delta)(\pi)}{(360^\circ)(L)} = \frac{\sin\frac{\Delta}{2}}{C}$$

The HP–65 program automatically assumes a $\Delta/2$ angle of 181 degrees and reiterates similar to the "Bassham Iteration" until the equality in equation (3) is reached.

Although this program takes about 30 seconds to run, it should be of benefit to HP–65 owners who are faced with solving curve problems given only the arc length and chord.

Ed. Note: As a registered land surveyor of Illinois, it's time to join or rejoin your national professional organization—ACSM.

Program

1. LBL	16. TAN	31. π	46. 2	61. STO 2	76. RCL 2
2. A	17. RCL 6	32. ÷	47. gR↓	62. 3	77. R/S
3. STO 3	18. g	33. RCL 3	48. STO 6	63. 6	78. RCL 6
4. RTN	19. π	34. RCL 5	49. GTO	64. 0	79. f
5. LBL	20. X	35. ÷	50. B	65. RCL 5	80. →D.MS
6. A	21. 1	36. RCL 6	51. LBL	66. X	81. R/S
7. STO 5	22. 8	37. f	52. 2	67. 4	82. RCL 3
8. 1	23. 0	38. COS	53. +	68. ÷	83. R/S
9. 8	24. ÷	39. ÷	54. 2	69. g	84. RCL 1
10. 1	25. −	40. 1	55. ÷	70. π	85. RCL 6
11. STO 6	26. 1	41. −	56. STO 6	71. ÷	86. f
12. RTN	27. 8	42. ÷	57. 2	72. RCL 6	87. TAN
13. LBL	28. 0	43. RCL 6	58. X	73. ÷	88. X
14. B	29. X	44. gx≤y	59. f	74. STO 1	89. R/S
15. f	30. g	45. GTO	60. →D.MS	75. R/S	90. RCL 5
					91. RTN

Operation

Instructions	*Input Data/Units*	*Keys*	*Output Data/Units*
1. Enter program			
2.	C	A	
3.	L	A	
4. Curve Solution		B	R
5.		R/S	Δ (Deg-min-sec)
6.		R/S	Δ/2 (Deg-min-sec)
7.		R/S	C
8.		R/S	T
9.		R/S	L

Norman G. Dennis, G.E. Co., Technical and Support Svs. Dept., Bay St. Louis, Mississippi—Dennis sent copies of pp. 96–97 of *Signals, Systems and Communications,* by B. P. Lathi, assoc. professor of Electrical Engineering, Bradley University, published by John Wiley & Sons, Inc., New York. These pages say that, "This function plays an important role in communication theory and is known as the *sampling function,* abbreviated by *SA(x)* for convenience." The formula then shown:

$$Sa(x) = \frac{\sin x}{x}$$

Dennis goes on to say that, "The $Sa(\;)$ notation was invented some time after 1952," adding that he doesn't know exactly when. He also says that the book mentioned above is "a beautiful clear book."

On 'The First Method To Adjust a Traverse Based on Statistical Considerations' by Herbert W. Stoughton

Published in Surveying and Mapping, Vol. XXXIV, No. 2, June 1974, pp. 145–149

Ira H. Alexander, *Los Angeles, Calif.*—In the June 1974 issue of *Surveying and Mapping,* Herbert W. Stoughton, Assistant Professor of Civil Technology at the State University of New York, Alfred, New

York, presented an interesting article entitled "The First Method To Adjust a Traverse Based on Statistical Considerations." The information contained therein whetted my interest in a variety of facts I have been only partially aware of in years past.

To complete some of the points raised by Mr. Stoughton it should be noted that the man who proposed the original problem, Robert Patterson, was Professor of Mathematics and Natural Philosophy at the University of Pennsylvania. To him belongs the credit of devising the rule so well known to draftsmen and survey calculators of simplifying traverse calculations by rotating bearings of a survey so that a particular course becomes a meridian. This idea is discussed on pages 96 through 98 of *A Treatise on Surveying* by John Gummere, A.M., Fourteenth Edition, Philadelphia, 1846.

In this same work, under discussion of correcting latitudes and departures of a closed traverse, is mentioned the reference to issue No. 4 of the *Analyst* showing that both Nathaniel Bowditch and the editor, Professor Adrain, published solutions to the problem. Gummere gives the results of Bowditch's rule, but does not show the demonstration, stating that it is too long and not of a nature for publication therein.

In my copy of *A Treatise on Surveying* by William M. Gillespie, LL.D., New York, 1892, on pages 131 and 150, reference again is made to the demonstration by Dr. Bowditch of his adjustment rule. In my copy of the *Adjustment of Observations* by Thomas W. Wright and John F. Hayford, Second Edition, New York, 1906, pages 156 to 158, is a discussion of both the solutions of Bowditch and Adrain. Reference therein is made to a solution wherein rigorous analytical methods are employed using mathematical techniques devised by Gauss. The assertion is made that "To Adrain, therefore, is due not only the first derivation of the exponential law of error, but its first application to geodetic work."

In *Elements of Surveying and Leveling* by Charles Davies, LL.D., first published in 1830, and revised in successive editions to the one I have which was copyrighted in 1898, publication being in New York, Bowditch is not mentioned by name, but the essence of the compass rule is.

One point with which I take issue with Mr. Stoughton is his statement that the survey texts printed in the United States today do not mention Bowditch's contribution. In my copy of *Elementary Surveying*, by Breed and Hosmer, Volume I, Seventh Edition, New York, 1938, on page 506 reference is made to the "compass rule," as first given by Dr. Nathaniel Bowditch.

Perhaps this information will be of interest to your readers in your "Comment and Discussion" column.

RESPONSE

HERBERT W. STOUGHTON, RLS, Almond, N.Y.—I would like to reply to the comments of Mr. Ira Alexander concerning my paper which appeared in the June issue of *Surveying and Mapping*. When I prepared my introductory remarks, I did not have a copy of Gummere's *Treatise On Surveying*. As a sidelight to my research, I encountered the name of John Gummere (address: "near Burlington, New Jersey") as a frequent contributor to the *Mathematical Analyst*. Also, I find the name F. R. Hassler of West Point in the 1808 issues of the *Analyst*. I presume that this is Ferdinand Hassler, Director of the U.S. Coast Survey. Further, it is interesting to note that Robert Adrain, editor of the *Analyst*, does not identify Robert Patterson as Professor of Mathematics and Natural Philosophy at the University of Pennsylvania.

Since June, I have acquired the 1892 and the 1900 editions of *A Treatise On Surveying* by William M. Gillespie, revised by Cady Staley, where on pages 131 and 150 Bowditch is credited with the traverse adjustment. Further, my research reveals that in 1900 Charles L. Crandall of Cornell University wrote the paper entitled: "The Adjustment Of A Transit Survey As Compared With That Of A Compass Survey" in the *Proceedings of the American Society of Civil Engineers* [Volume XXVI, No. 10, December 1900, p. 1164 ff.]. Professor Crandall mentions Bowditch's work, and proceeds to present the traverse adjustment technique known as "Crandall's Method" or "Crandall's Rule." Following Crandall's paper George W. Tuttle discusses Crandall's Rule, and then discusses Dr. Adrain's proof of Bowditch's Rule based on least squares in the *Proceedings* [Volume XXVII, No. 2, February 1901, p. 119]. About six years later Bowditch's name is mentioned again by Crandall in his classical book entitled, *A Text-book on Geodesy and Least Squares.* Also, I find only one paper published in *Surveying and Mapping* which refers to Bowditch ["Bowditch's Method Of Adjusting Transit Traverses—A Justification" by B. Goussinsky; October–December 1948, pp. 222–225]. In Bouchard and Moffitt; J. B. Johnson; Tracy; Davis, Foote, and Kelly; Davis and Kelly; and several later editions of Breed and Hosmer (volumes I and II) [besides the 1938 edition mentioned by Mr. Alexander] make no mention of Bowditch when discussing the "Compass Rule," while prominent mention is made of Professor Crandall. There are undoubtedly references to Bowditch's contribution in surveying textbooks published in the United States, but since 1920, these references are few.

Finally, I believe I can elaborate on Mr. Alexander's comments on Mr. Adrain's derivation of the exponential law of error. On page 93 of the *Analyst* (the same page on which the concluding remarks of Bowditch's paper appears) appears the paper entitled: "Research Concerning the Probability of the Errors Which Happen in Making Observations &c." by Robert Adrain. The first six pages are a general theoretical discussion followed by a detailed discussion of four problems. The problems are:

1. Supposing *a*, *b*, *c*, *&c.* to be the observed measures of any quantity x, the most probable value of x is required. (page 98)
2. Given the observed positions of a point in space, to find the most probable position of the point. (page 99)
3. To correct the Dead Reckoning at sea, by an observation of the latitude. (page 103)
4. To correct a survey. (page 106)

On page 109, Adrain concludes his discussion with the following remarks:

"From this investigation it appears, that the rules hitherto given by authors for correcting a survey, are altogether erroneous, and ought to be entirely rejected. The true method here given is exemplified by Mr. Bowditch, in his solution of Mr. Patterson's question of correcting a survey; his practical rule and mine being precisely the same.

"I have applied the principle of this essay to the determination of the most probable value of the earth's ellipticity. &c. but want of room will not permit me to give the investigations at this time."

Author's Note: Also, Adrain's name is misspelled in the article which was also incorrect in the original manuscript. The error appears in line 4, second paragraph, left column, page 145. ■