

A CONVENIENT METHOD OF CUTTING OFF A TRAPEZIUM OF REQUIRED AREA FROM A LARGER FOUR-SIDED FIGURE

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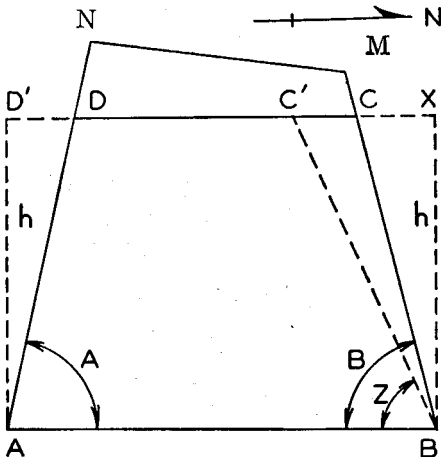
In the surveying of mineral claims, it is frequently required to reduce an oversize mineral claim to a parcel with two parallel sides and of required acreage. The Saskatchewan Regulations for Survey of Mineral Claims require oversize mineral claims to be reduced to 51.65 acres and the parcel remaining surveyed as an excess area of the mineral claim. Figures 1 and 2 indicate typical oversize mineral claims.

Occasionally, on surveys other than that of mineral claims, it is required to cut off a required acreage from a larger parcel, such that the cut-off line is parallel to one side.

From discussion with land surveyors, it is apparent that usual methods require involved solving of a quadratic equation or applying trial distances and then determining how close the acreage of the parcel approaches the required value. After about 6 trials, a very close approximation to the required parcel is obtained.

There are other methods of solving the problem, such as the use of calculus or coördinates, but in each case the solution is involved and not readily adaptable for use of logarithmic tables.

Rather than amend Saskatchewan Regulations, authorizing approximations of the required area of 51.65 acres, Surveys Branch evolved the solution herein given, which may be of general interest to land surveyors. The time involved is a fraction of that required in solving by other known methods.



Theory

Given: A four-sided figure ABMN which exceeds 60 acres in area.

Required: To cut off an area of 51.65 acres by drawing a line parallel to the east side of the figure.

Construction: Suppose CD to be the required line. Draw AD' and BX perpendicular to DC and let h be the length of AD' or BX. Draw a line BC' to meet DC, produced if necessary, at the point C' so that $CC' = DD'$.

Then the area $ABC'D' = \text{area } ABCD$
 $= 51.65 \text{ acres.}$

Let angle ABC' be Z

Then, since $DD' = h \cot A$

$$CX = h \cot B$$

$$C'X = h \cot Z$$

$$\text{and } C'X = DD' + CX$$

Therefore

$$h \cot Z = h \cot A + h \cot B$$

$$\text{and } \cot Z = \cot A + \cot B \quad (1)$$

$$\text{Now } DC = D'C' = AB - (h \cot A + h \cot B)$$

$$\text{Therefore } CD = AB - h \cot Z$$

$$\text{And } h = \frac{AB - DC}{\cot Z} \quad (2)$$

$$\text{The area of } ABCD = \frac{(DC + AB) h}{2} = 1500^2 \text{ (i.e., 51.65 acres or 2,250,000 square feet)}$$

$$\text{But } h = \frac{AB - DC}{\cot Z}$$

$$\text{Therefore } \frac{(DC + AB)}{2} \frac{(AB - DC)}{\cot Z} = 1500^2$$

$$\text{So that } \frac{AB^2 - DC^2}{2 \cot Z} = 1500^2$$

$$\text{and } AB^2 - DC^2 = 1500^2 (2 \cot Z)$$

$$\text{Therefore } CD^2 = AB^2 - 1500^2 (2 \cot Z) \quad (3)$$

In this equation, AB is known, $\cot Z$ can be calculated quickly from formula (1) as shown, so that DC can be found very easily. All that remains is the computation of h from equation (2) and the length of AD.

Following are two examples of the method:

ARCH No. 4 Mineral Claim—Figure 1

Data:

$$\text{Angle } A = 107^\circ 52' 45''$$

$$\text{Angle } B = 104^\circ 56' 00''$$

$$AB = 1423.4 \text{ feet}$$

I. To Calculate Angle Z

$$\cot Z = \cot A + \cot B \quad (1)$$

$$\cot Z = \cot 107^\circ 52' 45'' + \cot 104^\circ 56' 00''$$

$$\cot Z = -0.322590 - 0.266702$$

$$\text{Therefore } \cot Z = -0.589292$$

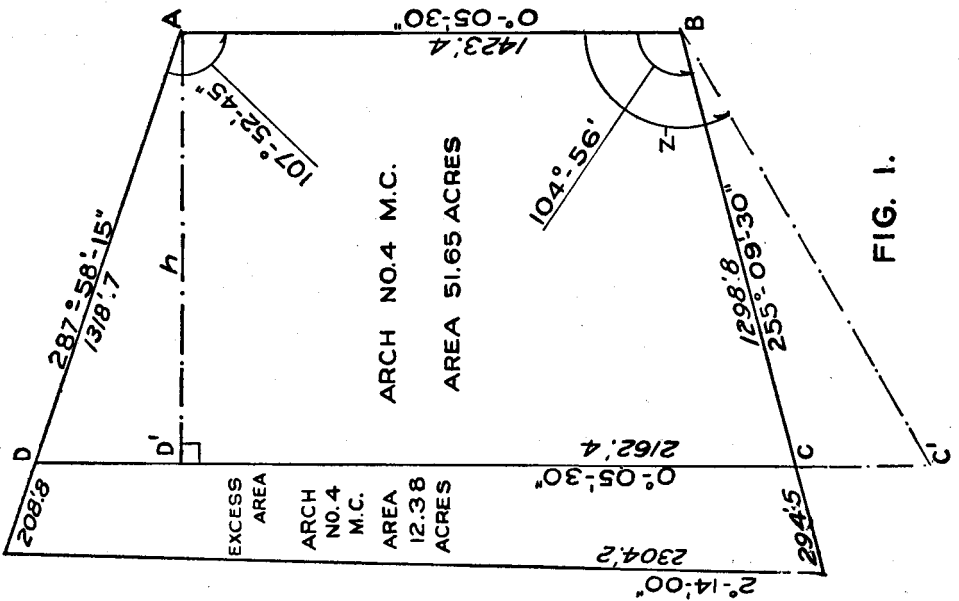


FIG. 1.

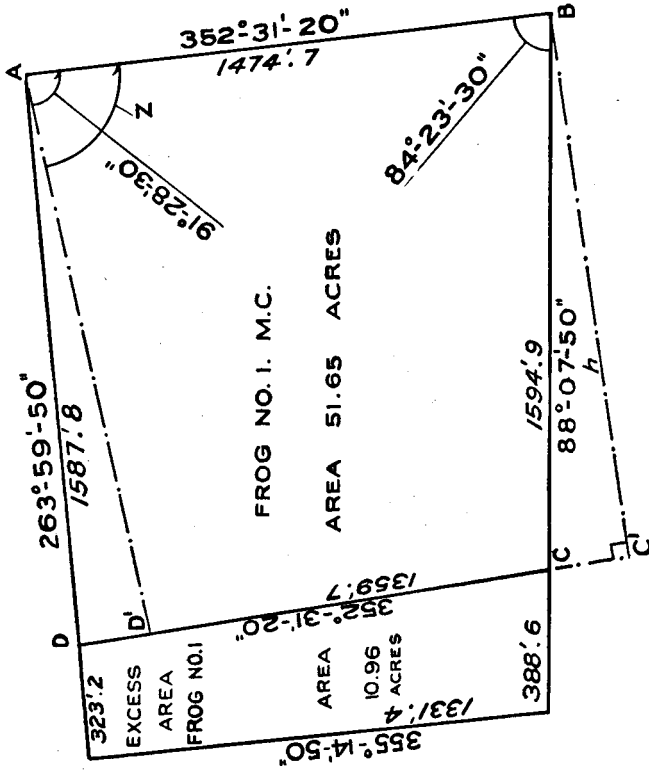


FIG. 2.

II. To Calculate DC

$$DC^2 = AB^2 - 1500^2 (2 \cot Z) \quad (3)$$

$$DC^2 = (1423.4)^2 - 1500^2 (-1.178584)$$

$$DC^2 = 2,026,068 + 2,651,814$$

$$DC^2 = 4,677,882 \text{ feet}$$

$$DC = 2162.4 \text{ feet}$$

III. To calculate perpendicular height h of the figure ABCD

$$h = \frac{AB - DC}{\cot Z} \quad (2)$$

$$= \frac{1423.4 - 2162.4}{-0.589292}$$

$$= \frac{-739.4}{-0.589292}$$

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$$= \underline{\underline{1254.7 \text{ feet}}}$$

IV. To calculate distance AD

$$\frac{h}{AD} = \sin \text{ angle } D'DA$$

$$AD = \frac{h}{\sin D'DA}$$

$$= \frac{1254.7}{\sin 72^\circ 07' 15''}$$

$$= \frac{1254.7}{0.951692}$$

$$= \underline{\underline{1318.7 \text{ feet}}}$$

FROG No. 1 Mineral Claim—Figure 2

Data:

$$\text{Angle A} = 91^\circ 28' 30''$$

$$\text{Angle B} = 84^\circ 23' 30''$$

$$AB = 1474.7 \text{ feet}$$

I. To calculate cot Z:

$$\cot Z = \cot A + \cot B \quad (1)$$

$$\cot Z = \cot 91^\circ 28' 30'' + \cot 84^\circ 23' 30''$$

$$\cot Z = -0.025750 + 0.098198$$

$$\cot Z = 0.072448$$

$$2 \cot Z = 0.144896$$

II. To calculate DC

$$DC^2 = (AB)^2 - 1500^2 (2 \cot Z) \quad (3)$$

$$DC^2 = (1474.7)^2 - 1500^2 (0.144896)$$

$$DC^2 = 2,174,740 - 326,016$$

$$DC^2 = 1,848,724 \text{ feet}$$

$$\text{Therefore } DC = 1359.7 \text{ feet}$$

III. To calculate the perpendicular height h of the figure ABCD

$$h \cot Z = AB - DC$$

$$h = \frac{AB - DC}{\cot Z}$$

$$h = \frac{1474.7 - 1359.7}{0.072448}$$

$$h = \frac{115.0}{0.072448}$$

$$h = 1587.3 \text{ feet}$$

IV. To calculate AD

$$\frac{h}{AD} = \sin 88^\circ 31' 30''$$

$$AD = \frac{h}{\sin 88^\circ 31' 30''}$$

$$AD = \frac{1587.3}{0.999670}$$

$$\text{Therefore } AD = 1587.8 \text{ feet}$$

Sixty thousand man-years—that's the estimated labour force which will be required for construction of the St. Lawrence Seaway and Power Project.

Detailed survey of Longue Sault Rapids was obtained through use of helicopter. First attempts at survey—using gas-filled "kytoons"—failed due to shifting air currents above swift water.

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