Computations for Missing Elements of Closed Traverses

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ABSTRACT: The Surveyor and the Engineer can be faced with incomplete data for a closed traverse. This paper presents a relatively rapid method of determining the numeric value of the missing data when there are not more than two unknowns. The method presented is the simultaneous solution of two equations containing two unknowns. Three cases are developed; namely, two lengths unknown, two azimuths unknown, and a length and an azimuth unknown. It is shown that a knowledge of the field conditions is necessary in some instances.

THE problem of missing elements in a closed traverse is encountered on occasion by an Engineer or a Surveyor. When both elements are missing in one course of the traverse, the solution has nearly always been one of summing the latitudes (or the ordinates) to zero and of summing the departures (or the abscissas) to zero. In this way the missing ordinate and abscissa are obtained. With the ordinate and abscissa of the line known, the length and direction of the course are easily obtained by use of algebra and plane trigonometry.

When the missing elements are not in the same course, the common solution has been to rearrange the traverse so the unknown elements are adjacent and the known elements are adjacent (Figures 1 and 2). Figure 2 is solved by first determining the length and direction of the closing (dashed) line which is constructed common between the known elements and the unknown elements. This closing line forms the third side of a triangle in which at least three and possibly four elements are known; namely, the length of each of the three sides or some combination of length of sides and angles. With this information it is then possible to solve the remaining elements of the triangle. Once the triangle is solved, the traverse is reshuffled into its original shape for the determination of the coordinates of the points.

The foregoing approach, although common, appears to be very time consuming. In solving for the magnitude and sense of an unknown force in Statics, the approach is to sum the forces along the X-axis and to sum the forces along the Y-axis. For equilibrium, these two sums must equal zero, and the magnitude and sense of the unknown are determined. It is proposed that the same approach be followed in the solution of closed traverses with one or two unknown elements.

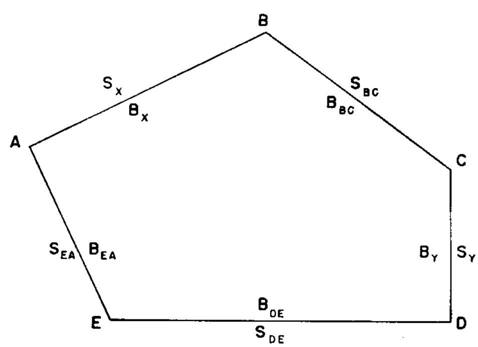

FIGURE 1
ACTUAL TRAVERSE

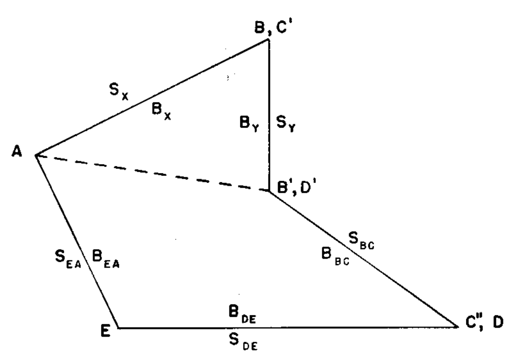

FIGURE 2
REARRANGED TRAVERSE

In the complete solution of a closed traverse, there are two equations: the algebraic sum of the latitudes (or Y-components) must equal zero; and the sum of the departures (or X-components) must equal zero. If these two equations are written for a closed traverse, which may have two lengths, two directions, or a length and a direction missing, two equations result with two unknowns; they may be solved simultaneously in approximately one-half the time required by the approach outlined above. Three cases are developed in detail below. In each case D represents the algebraic summation of the departures, or X-components, of all courses in which all elements are known and L represents the algebraic summation of the latitudes, or Y-components, of the same courses. S represents the length of a course, and B its azimuth or bearing.

In the first case to be considered, the lengths of two sides of a polygon are unknown. The remaining lengths and the directions of all sides are known, Figure 1. The two basic equations are:

$$S_y \sin B_y + S_x \sin B_x + D = 0 \qquad (1)$$
$$S_y \cos B_y + S_x \cos B_x + L = 0. \qquad (2)$$

In Equations (1) and (2) B_x and B_y are known values. Therefore, the sine and cosine of each direction can be evaluated. There remain in the two equations the two terms S_x and S_y, which are the only unknowns. The simultaneous solution for S_x and S_y is available from basic algebra. This particular type of problem was assigned a group of students with instructions that one half were to solve it by the common approach of shuffling and reshuffling; the other half were to solve it by the simultaneous equation approach. Those who solved the simultaneous equations completed the problem in approximately one-half the time required by the other group, with the same end results.

Next, consider the case of the polygon in which the length of one course and the direction of another course are unknown. Otherwise, all information is known with respect to the traverse polygon, Figure 1. In Equations (1) and (2) B_x and S_y are unknown. Since B_y is known, sin B_y and cos B_y can be evaluated and designated as M and N, respectively. Equations (1) and (2) can be written as

$$S_x \sin B_x + S_y M + D = 0$$
$$S_x \cos B_x + S_y N + L = 0.$$

Hence,

$$\sin B_x = -(S_y M + D)/S_x \qquad (3)$$
$$\cos B_x = -(S_y N + L)/S_x. \qquad (4)$$

Squaring each equation,

$$\sin^2 B_x = (S_y M + D)^2/S^2_x \qquad (5)$$
$$\cos^2 B_x = (S_y N + L)^2/S^2_x. \qquad (6)$$

Adding Equations (5) and (6),

$$\sin^2 B_x + \cos^2 B_x = [(S_y M + D)^2 + (S_y N + L)^2]/S_x^2$$

From trigonometric identities,

$$\sin^2 B_x + \cos^2 B_x = 1,$$

therefore

$$1 = [(S_y M + D)^2 + (S_y N + L)^2]/S^2_x$$

or

$$S^2_x = S^2_y (M^2 + N^2) + 2S_y(MD + NL) + D^2 + L^2 \qquad (7)$$

from which

$$S_y = \frac{-(MD + NL) \pm \sqrt{(MD + NL)^2 - (M^2 + N^2)(D^2 + L^2 - S^2_x)}}{M^2 + N^2}. \qquad (8)$$

As S_y is the solution of a quadratic equation, there will be two values of S_y which satisfy Equation (8). If any value of S_y is negative, it is not a solution to be considered. The positive values of S_y are substituted into either Equation (3) or Equation (4) to solve for B_x. The values thus obtained should be substituted into Equation (4) or Equation (3) (whichever was not used in the solution) as a check on arithmetic.

If by chance, as can happen, both values of S_y determined from Equation (8) are positive, then two values of B_x must be obtained. In this situation there must be an accurate sketch of the shape of the area under consideration. When both sets of values of S_y and B_x have been obtained, the proper pair are selected to fit the field conditions.

In the final case to be discussed, directions are missing in two sides of a closed traverse, Figure 1. In Equations (1) and (2), the values of B_x and B_y are unknown. Equations (1) and (2) may be rewritten as

$$S_x \sin B_x = -(S_y \sin B_y + D) \qquad (1a)$$
$$S_x \cos B_x = -(S_y \cos B_y + L). \qquad (2a)$$

Squaring each equation,

$$S^2_x \sin^2 B_x = (S_y \sin B_y + D)^2 \qquad (9)$$
$$S^2_x \cos^2 B_x = (S_y \cos B_y + L)^2. \qquad (10)$$

The sum of Equation (9) plus Equation (10) is

$$S^2_x = S^2_y + 2S_y(D \sin B_y + L \cos B_y) + D^2 + L^2$$

or

$$D \sin B_y + L \cos B_y = (S^2_x - S^2_y - D^2 - L^2)/2S_y. \qquad (11)$$

If both sides of Equation (11) are divided by

$$(D^2 + L^2)^{1/2}$$

and if

$$\sin \theta = D/(D^2 + L^2)^{1/2},$$

then

$$\cos \theta = L/(D^2 + L^2)^{1/2},$$

and

$$\sin \theta \sin B_y + \cos \theta \cos B_y = (S_x^2 - S_y^2 - D^2 - L^2)/[2S_y(D^2 + L^2)^{1/2}].$$

By the use of trigonometric identities this reduces to

$$\cos (B_y - \theta) = (S_x^2 - S_y^2 - D^2 - L^2)/[2S_y(D^2 + L^2)^{1/2}]$$

from which

$$(B_y - \theta) = \cos^{-1}\{(S_x^2 - S_y^2 - D^2 - L^2)/[2S_y(D^2 + L^2)^{1/2}]\}.$$

But

$$\theta = \cos^{-1}\{L/(D^2 + L^2)^{1/2}\}.$$

Therefore, the final result is:

$$B_y = \cos^{-1}\{L/(D^2 + L^2)^{1/2}\} + \cos^{-1}\{(S_x^2 - S_y^2 - D^2 - L^2)/[2S_y(D^2 + L^2)^{1/2}]\}. \qquad (12)$$

Substituting the value of B_y thus obtained into Equation (2a), the result is:

$$B_x = \cos^{-1}\{-(S_y \cos B_y + L)/S_x\}. \qquad (13)$$

As stated above this approach has been used in the classroom. The majority of the students to whom it was introduced could more readily visualize the procedures to be followed to obtain a solution. It naturally follows that if the procedures of computation are clear, there will be a real saving of time in the operation. Students using this approach were able to present complete and accurate solutions in one-third to one-half the time normally devoted to the common procedure. If students can save this amount of time using the above solution, the Professional should be able to save much more. Further, it appears to be a relatively simple problem to program any of the final equations for electronic computers.

Surveyor—101 Years Old—Surveys at Ninety Six

Thomas C. Anderson, Sr., of Ninety Six, South Carolina, become 101 years old on November 20, 1969, having been born in 1868.

Following studies in engineering at the University of South Carolina which were curtailed by hard times, Anderson was encouraged to take up surveying which was to become a lifetime career. Greenwood County records associate his surveys with virtually every major property in that county. He started surveying on the railroads, first in Georgia, then on a section of the Seaboard Airline R.R. from Atlanta to Birmingham. Later, in 1906, he not only surveyed a four-mile railroad between Donalds and Due West in Abbeville County, South Carolina, but built it under contract.

Still active as a land surveyor today, his services are contingent upon the client furnishing transportation and leg work.