# The Adjustment of Single Control Traverses

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## Abstract

*Methods available for the adjustment of single traverses are discussed; their theoretical limitations are assessed and the comparative soundness of the Smirnoff procedure is pointed out. A speciman traverse computation is included.*

## Introduction

EDM traversing with single-second theodolites is a well established procedure for the establishment of secondary and tertiary control for engineering and topographical purposes. Due to observational and instrumental errors, closed traverses always result in some linear closing error even after a distribution of the total angular misclosure. This must be distributed in some way, such adjustment being aimed not at increasing 'accuracy' but at greater internal consistency. Ideally the method should be simple, it should disturb the observed quantities by a minimal amount and should be 'correctly' related to the probable displacements caused by errors in angular and linear measurements. No such method exists, and all the various alternatives have their own pecularities which may lead to their being more, or less, suitable than others for use in any particular case. If the misclosures are large, then no method of adjustment can act as antidote to poor field work; if the misclosures are very small, then theoretical arguments for and against any method may be dismissed as academic, in favour of simplicity. The majority of traverses, however, fall into an intermediate category and in most cases adjustments are made without due regard to the principles on which they are based. In particular, the general use of the Bowditch method is almost unquestioned.

## Traverse Adjustment Methods

The methods which will be discussed are as follows:

1. Least Squares
2. Bowditch
3. Transit
4. Crandall
5. Smirnoff

## 1. Least Squares

Based on condition equations, this method should be employed for precise traverses. Provisional co-ordinates are adopted, these being in any case necessary for computation of the scale factors and arc-to-chord corrections required for such high order work. The adjusted values must fulfil three conditions – closures in bearings, eastings and northings. The equations so formulated may be expressed, in matrix form, by

$$C . \nu = q$$

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Where q is a vector matrix of misclosures
$\nu$ is a vector matric of residuals (adjustments to observations)
and C is a matrix of coefficients.

The least squares solution is obtained by minimising $(\nu^T.W.\nu)$, which is the same as minimising $(\nu^T.W.\nu - 2(C.\nu - q)^T.k)$, where W is the weight matrix and k the vector matrix of so-called 'correlates' The latter is an intermediate result obtained from

$$k = (C.W.^{-1}C^T)^{-1}q$$

and

$$\nu = W.^{-1}C^T.k.$$

If a traverse has n stations between the terminal points A and B, the matrix C has dimensions of [3 x (2n + 3)] and the weight matrix W[(2n + 3) x (2n + 3)]. Adjustment of most traverses using the rigorous least squares technique therefore involves large matrices with concomitant complex and lengthy arithmetical operations. This obviously militates against its use in all but work of high order. The method will therefore not be considered further in this paper.

## 2. Bowditch (Compass)

This was originally developed for compass traverses in which independent bearings were used at each station: when now used for conventional traverses, the bearings are first adjusted by distribution of the angular misclosure throughout the traverse. Provisional co-ordinates are then derived for all stations and the misclosures are then derived for all stations and the misclosures in easting and northing obtained algebraically by comparison of the computed and the known values for the easting and the northing of the closing station. The mathematical treatment due to Bowditch (reference 1) involves formation of condition equations followed by use of the least squares principle, but simplifying assumptions are made. These lead to the distribution of each misclosure according to the well known relationship –

$$\text{easting adjustment} = \frac{-\begin{bmatrix}\text{final misclosure}\\ \text{in easting}\end{bmatrix} \times \Sigma\,(\text{horizontal distances up to the station})}{\Sigma\,(\text{horizontal distances for all traverse})}$$

with an analogous relationship for the northing adjustment – to give the final adjusted co-ordinates.

The procedure is simple and easily programmable; it can give results close to those resulting from rigorous least squares adjustments. It has won such general acceptance, however, that it is used as a matter of course – often without regard to its disadvantages, which result from the assumptions made. These may be briefly stated as follows:

1. The displacement at the end of any leg due to the probable error in linear measurement is equal to the displacement at the end of the same leg caused by the probable error in bearing.

2. The probable errors in linear measurement are proportional to $S^{1/2}$ where S = measured distance.

3. The probable errors in bearings are proportional to $S^{-1/2}$.

The above assumptions would often appear difficult to justify (when distances are measured by EDM, 2 is particularly unlikely) and it is well known that following Bowditch adjustment it is necessary to recompute final bearings from the adjusted co-ordinates, thus often causing large changes in the previously adjusted bearings. The adjustment in fact produces the same linear "movement" of a station regardless of its bearing.

The Bowditch adjustment is not suitable for precise traverses and is not recommended for use in lower order work wherever angles are measured more accurately than distances. It is probably best in cases where the traverse follows a route more closely approaching a straight line.

## 3. Transit

In this method, bearings are adjusted as for the Bowditch method and provisional co-ordinates and misclosures obtained in a similar way. These misclosures are then used to adjust the co-ordinates of individual stations in a purely empirical way:

$$\text{eastings adjustment} = \frac{-\left[\text{final misclosure in easting}\right] \times}{\Sigma \; |(\Delta E \text{ in traverse})|} \; \Delta E \text{ of leg}$$

– and similarly for the northing adjustment. This is a procedure without any theoretical pretension and as such it is often disregarded in favour of the Bowditch method, although, as pointed out above, the latter has a theoretical basis having little validity in many present-day situations. One advantage of the Transit method of adjustment is that it results in smaller changes to previously adjusted bearings than does the Bowditch method. It probably is a better method to use where angular measurement is more accurate than linear measurement.

## 4. Crandall

This was formerly often preferred to the Bowditch method for reasons similar to those leading to the choice of the Transit method – viz. cases where angular measurements known to be more accurate than those of distance, and thus any large alterations in adjusted bearings produced by adjustment being regarded as unacceptable. The Crandall method (reference 1) goes to the opposite extreme in that it leaves preliminarily adjusted bearings totally unaltered. Since it is also based on an assumption that errors in linear measurements are proportional to $S^{1/2}$, the advent of EDM has rendered it harder to justify and the method is now little used. Such obsolescence seems both honourable and logical and may be contrasted with the apparent indestructibility of the Bowditch method which has no better theoretical rationale.

## 5. Smirnoff

This method is analysed below in some detail and a specimen computation is given (Appendix 1). In the latter, projection and scale factor effects are not included – since although obviously relevant they do not, per se, affect adjustments; however a comparison with co-ordinates produced by a Bowditch adjustment of the same field data is included.

Bearing adjustment before computation is not necessary. The method first quantifies contributions to the total closing error made separately by linear and angular measurements, the unadjusted forward bearings being used for this computation with adequate accuracy.

## Theoretical Basis

The separate and combined effects of small random errors in linear and angular measurements on the final misclosures can be expressed by the following argument:

In any traverse leg, gain in easting and northing is given by

$$\Delta E = S \sin \alpha$$
$$\Delta N = S \cos \alpha$$

$\Delta N$ is thus a product of measured linear distances S and cosine of bearing. Errors ds and $d\alpha$ will contribute separately to errors in northing:

$$dN = dS \cos \alpha + S\,d(\cos \alpha) \text{ by differentiating,}$$

$$\therefore \frac{dN}{\Delta N} = \frac{dS \cos \alpha}{\Delta N} + \frac{S\,d(\cos \alpha)}{\Delta N}$$

$$= \frac{dS \cos \alpha}{S \cos \alpha} + \frac{S\,d(\cos \alpha)}{S \cos \alpha}$$

$$= \frac{ds}{S} + \frac{d(\cos \alpha)}{\cos \alpha}$$

or

$$dN = \Delta N.\frac{dS}{S} + \Delta N \frac{d(\cos \alpha)}{\cos \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(A)}$$

The first term represents the effect due to the error in *linear* measurement and the second that due to error in *angular* measurement: dN if allotted appropriate sign represents the combined adjustment necessary at the station.

If this is applied throughout the traverse,

$$dN_1 = \Delta N_1 \frac{dS_1}{S_1} + \Delta N_1 \frac{d(\cos \alpha)}{\cos \alpha_1}$$

$$dN_2 = \Delta N_2 \frac{dS_2}{S_2} + \Delta N_2 \frac{d(\cos \alpha_2)}{\cos \alpha_2}$$

$$dNn = \Delta Nn.\frac{dSn}{Sn} + \Delta Nn \frac{d(\cos \alpha\, n)}{\cos \alpha\, n}$$

it is reasonable to assume that $\frac{dS}{S}$ will be more or less constant over a number of legs of a of a balanced traverse, i.e. that

$$\frac{dS_1}{S_1} = \frac{dS_2}{S_2} = \ldots = \frac{dSn}{Sn} = \frac{dS}{S}$$

and therefore $$dN_1 = \Delta N_1 \frac{dS}{S} + \Delta N_1 \frac{d(\cos\alpha_1)}{\cos\alpha_1} \qquad \text{(B)}$$

$$dNn = \Delta Nn.\frac{dS}{S} + \Delta Nn \frac{d(\cos\alpha n)}{\cos\alpha n}$$

The total closing error in northing becomes

$$dN = dN_1 + .. + dNn$$

$$dN = (\Sigma\Delta N)\frac{dS}{S} + \Sigma(\Delta N.\frac{d(\cos\alpha)}{\cos\alpha})$$

Similarly $$dE = (\Sigma\Delta E)\frac{dS}{S} + \Sigma(\Delta E.\frac{d(\sin\alpha)}{\sin\alpha})$$

$$\frac{dS}{S} = \frac{1}{\Sigma|\Delta N|}[\,|dN| - \Sigma|(\Delta N\frac{d\cos\alpha}{\cos\alpha})|\,] \qquad \text{(C)}$$

$$\alpha S = \frac{1}{\Sigma|\Delta E|}[\,|dE| - \Sigma|(\Delta E\frac{d\sin\alpha}{\sin\alpha})|\,] \qquad \text{(D)}$$

The terms $\frac{d(\cos\alpha)}{\cos\alpha}$ and $\frac{d(\sin\alpha)}{\sin\alpha}$ are important; each represents the relative change in the trigonometrical function involved for a small change in the angle, and can be termed "precision ratios". If field work is carried out with 1" theodolites the relevant quantity in each case is the relative change caused by an error of 1". Such ratios can easily be obtained from present day calculators without recourse to tables.

For each leg, $|\Delta N\frac{d(\cos\alpha)}{\cos\alpha}|$ and $|\Delta E\frac{d(\sin\alpha)}{\sin\alpha}|$ are derived.

The columns are then summed and the fractional linear error term $\frac{dS}{S}$ obtained from the equations C and D. (In a traverse having a large number of legs of course these two ratios will normally, as a result of balanced distribution of random error, be equal – but in a short traverse this will not be the case. Since the distribution is not predictable they should each be determined separately.)

Subsequently the contribution to the required adjustment made by the linear error is computed for each leg ($|\Delta N.\frac{dS}{S}|$ and $|\Delta E.\frac{dS}{S}|$) and this is added to the angular contribution to give the adjustment.

The necessary sign is allotted to the result by inspection of that of the final misclosure to be adjusted out.

It may be noted that the adjustment can be approximated to give the Transit adjustment – if in any traverse the angular accuracy is much greater than the linear accuracy then terms such as $(\Delta N\frac{d\cos\alpha)}{\cos\alpha}$ are neglected and, for example, equation C –

becomes $$\frac{dS}{S} = \frac{|dN|}{\Sigma|\Delta N|}$$

and substitution of this in equation B gives

$$dN_1 = \frac{\Delta N_1 . | dN |}{\Sigma | \Delta N |}$$

which is the Transit rule.

It may be argued that there is a subjective element in the selection of the precision ratio adopted, but in the opinion of the writer this is not a marked disadvantage. In any class of work the maximum random errors expected should be reasonably well known, determined as they are by equipment, experience, and observational technique. Thus if used for a precise traverse the adjustment may be calculated on the basis of precision ratios computed for say 0".25 differences, whilst if used for low order work a 10" difference could be employed.

## Specimen Computation

An example of traverse adjustment is given in Appendix 1 to which the following brief explanatory notes apply.

Column 2 – unadjusted bearings only are required – these give accuracies adequate for initial determination of $\Delta E$, $\Delta N$ and precision ratios.

Column 3 – reduced horizontal distances. Summation gives H.

Column 4 – $\Delta N$ for each leg. Algebraic summation, addition to the initial northing (X, at the top of column 10) and comparison with the known northing of the closing station gives the misclosures in northings (C).
Summation with regard to sign gives A.

Column 5 – precision ratios determined from the bearing of each leg.

Column 6 – obtained by multiplication of values in columns 4 and 5 ignoring all signs. Each term represents the bearing error contribution. Summation D enables $\frac{dS}{S}$ to be computed.

Column 7 – obtained by multiplication of terms in column 4 by $\frac{dS}{S}$ without regard to sign. This then represents the contribution of the linear errors.

Column 8 – simply the sum of values in columns 6 and 7 and the result given a sign opposite to that of C.

Column 9 – adjusted values of $\Delta N$ obtained by algebraic summation of values in columns 4 and 8.

Column 10 – adjusted northings obtained by adding values of adjusted $\Delta N$ down the column, starting with the initial value X.

Columns 11-17 – an exactly similar procedure in eastings.

The above computation procedure involves only a little more effort than a Bowditch adjustment; in any case, like the latter, it lends itself very obviously to programming on a hand calculator (Appendix 2).

For purposes of comparison the Bowditch adjusted co-ordinates are listed in addition to those produced by this adjustment. It will be seen that the eastings in particular are significantly different – a case which typifies the Bowditch characteristic of producing the same linear "movement" of the station regardless of its bearing. It is the writer's contention that the co-ordinates produced by the more reasoned Smirnoff adjustment are more likely to be "correct". It may also be noted that this traverse is one to which the Bowditch method is perhaps more suited than most in that it is not polygonal in pattern and follows a roughly NNW'ly quasi-linear route.

## Summary

Numerous methods are available for adjustment of a closed traverse survey. Each has peculiar advantages and disadvantages and generally, an associated theoretical rationale, yet none is completely satisfactory. Choice of method is probably not important in cases where misclosures are very small and has little relevance where misclosures are large; in other cases an automatic resort to the Bowditch method is often unjustified. In the opinion of the writer the method described in this paper deserves wider attention than it has apparently received.

## Appendix II

A simple program written for an HP 41CV computes values for each leg of the traverse, of $\Delta$N, $\Delta$E, precision ratios for 1" and appropriate products; it also summates horizontal distances, eastings and northings both algebraically and without regard to sign – i.e. it provides values for columns 5, 6, 12 and 13 and quantities A, B, H, J, K. Copies of the listing could be supplied by the author on request.

## References

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# Appendix 1
## Traverse by Smirnoff Method (Comparison with Bowditch)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Line | Bearing | Distance | $\Delta N$ | $\frac{d(\cos\alpha)}{\cos\alpha}$ | $\Delta N x \frac{d(\cos\alpha)}{\cos\alpha}$ | $\Delta N x \frac{ds}{s}$ | Adjustment (6 + 7) | Adjusted $\Delta N$ | Adjusted N Co-ordinate | $\Delta E$ | $\frac{d(\sin\alpha)}{\sin\alpha}$ | $\Delta E x \frac{d(\sin\alpha)}{\sin\alpha}$ | $\Delta E x \frac{ds}{s}$ | Adjustment | Adjusted $\Delta E$ | Adjusted E Co-ordinate |
| | | | | | (4 x 5) | $(4x \frac{ds}{s})$ | – | | x<br>1013.255 | | | (11 x 12) | $(11 x \frac{ds}{s})$ | – | | y<br>8502.655 |
| (Back) | 267 17 20 | | | x $10^{-6}$ | | | | | Bowditch<br>Smirnoff | | | | | | | Bowditch<br>Smirnoff |
| 1 | 334 54 38 | 315.773 | 285.9788 | 2.269 | 0.00065 | 0.0157 | 0.0164 | 285.9624 | 1299 216<br>1299 217 | –133.898 | .00001 | 0.00139 | .00444 | .00583 | –133.9038 | 8368.749<br>8368.751 |
| 2 | 032 37 07 | 99.744 | 84.0121 | 3.103 | .00026 | .0046 | 0049 | 84.0072 | 1383 223<br>1383 225 | 53.7664 | .00001 | 0.00041 | .00178 | .00219 | 53.7642 | 8422.513<br>8422.515 |
| 3 | 007 13 14 | 170.134 | 168.7848 | 0.614 | .00010 | .0093 | 0094 | 168.7754 | 1551 998<br>1552.000 | 21.3840 | .00004 | 0.00082 | .00071 | .00153 | 21.3824 | 8443.892<br>8443.898 |
| 4 | 346 45 40 | 105.628 | 102.8208 | 1.140 | .00012 | .0057 | 0058 | 102.8150 | 1654.813<br>1654.815 | – 24.1900 | .00002 | 0.00050 | .00080 | .00130 | – 24.1913 | 8419.700<br>8419.706 |
| 5 | 326 49 54 | 67.662 | 56.6376 | 3.168 | .00018 | .0031 | 0032 | 56.6340 | 1711.447<br>1711.449 | – 37.0179 | .00001 | 0.00027 | .00123 | .00150 | – 37.0194 | 8382.681<br>8382.687 |
| 6 | 338 44 52 | 121.659 | 113.3854 | 1.886 | .00021 | .0062 | 0064 | 113.3790 | 1824.827<br>1824.828 | – 44.0982 | .00001 | 0.00055 | .00146 | .00201 | – 44.1002 | 8338.581<br>8338.587 |
| 7 | 356 05 28 | 133.188 | 132.8782 | 0.331 | .00004 | .0073 | 0073 | 132.8709 | 1957.698<br>1957.699 | – 9.0794 | .00007 | 0.00064 | .00030 | .00094 | – 9.0803 | 8329.501<br>8329.507 |
| 8 | 331 13 07 | 119.661 | 104.8784 | 2.663 | .00028 | .0058 | 0061 | 104.8723 | 2062.571<br>2062.571 | – 57.6131 | .00001 | 0.00051 | .00191 | .00241 | – 57.6155 | 8271.887<br>8271.891 |
| 9 | 320 32 19 | 97.535 | 75.3022 | 3.991 | 00030 | .0041 | 0044 | 75.2978 | 2137.869<br>2137.869 | – 61.9892 | .00001 | 0.00037 | .00206 | .00243 | 61.9916 | 8209.898<br>8209.900 |
| 10 | 293 41 56 | 78.669 | 31.6194 | 11.046 | .00003 | .0017 | 0018 | 31.6176 | 2169.487<br>2169.487 | – 72.0349 | .00002 | 0.00015 | .00239 | .00254 | 72.0374 | 8137.862<br>8137.862 |
| (Fwd) | 285 49 22 | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | |
| | H Σ | 1309.653Σ | 1156.2978 | D Σ | .00217 | Σ | .0657 | 1156.2316 | K | –364.770 | G Σ | .00561 | Σ | .02268 | –364.7929 | |
| | | (+x →) | 2169.5528 | | | | (+x →) | 2169.4866 | (+Y →) | 8137.8847 | | | | (+Y →) | 8137.8621 | |
| | | True | 2169.4870 | | | | True | 2169.4870 | True | 8137.862 | | | | True | 8137.8621 | |
| | C | Misclose (N) | +.0658 | | | | (Misclose After Adjustment | .0004) | F. Misclose (E) | +0.0227 | | | | (Misclose After Adjustment | .0000) | |
| | A | $\Sigma \mid \Delta N \mid$ | 1156.2978 | | | | | | $B\ \Sigma \mid \Delta E \mid$ | 515.071 | | | | | | |
| | Relative Accuracy = | $\frac{\sqrt{C^2 + F^2}}{H}$ = | 1 : 18,815 | $\frac{ds}{s_N} = \frac{\mid C \mid - \mid D \mid}{A} = 0.000055$ | | | | | | | $\frac{ds}{S_E} = \frac{\mid F \mid - \mid G \mid}{B} = 0.000033$ | | | | | |