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Polaris and Solar Observations Reduction for Azimuth Without the Use of Ephemeris

by Sayed R. Hashimi

Abstract. In this highly technological era where computers are integrated in almost every walk of life, particular attention is paid to the techniques that maximize efficiency. Along the same vein this paper discusses a technique by which Polaris, or other stars for that matter, and solar observations reduction for azimuth can be performed without the use of star tables or solar ephemeris. The only required data in addition to the actual observations would be: the latitude and longitude of station, and the date and time of the observations.

Introduction

In general, most astronomic and solar observation reductions for azimuth require the use of either a star catalog of some sort or a solar ephemeris. This process in analagous to the use of trigonometric and logarithmic tables prior to the emergence of digital computers and hand-held programable calculators that provide these functions. Furthermore, unlike trigonometric and logarithmic tables, a star catalog and solar ephemeris change from year to year. This article discusses the step-by-step procedure for obtaining the apparent "true" position (i.e., right ascension and declination) of Polaris and those of the sun at any given epoch.

Using the following mean position (right ascension and declination) of Polaris for the year 1950:

a_o = 1.81355167 hours

Annual Proper Motion in a_o,

(da) = +0.1811 seconds of time

δ_o = +89.02881667 degrees

Annual Proper Motion in δ_o,

$(d\delta)$ = −0.004 seconds of arc

Note: Any other appropriate star given its mean position and annual proper motion can be used in place of Polaris.

Basically, for most ordinary work, the following four corrections need to be made to a mean position to obtain the apparent "true" position:

1. The Effect of Proper Motion

This is a small variation in the absolute position (right ascension and declination) of a star with respect to other stars in space.

$$a_1 = [((JD + UT/24 - JD_o)\, da)/(365.242199)(240)] + 15(a_o) \quad (1)$$

$$\delta_1 = [((JD + UT/24 - JD_o)\, d\delta)\, /(365.242199)(3600)] + \delta_o \quad (2)$$

where

JD = Julian days corresponding to zero hour Universal Time (UT) of the desired day (see Appendix A).

JD_o = Julian days corresponding to the year 1950.0.

a_1 = Right ascension in degrees corrected for proper motion.

δ_1 = Declination in degrees corrected for proper motion.

UT (the corresponding Universal Time in hours) = Local Standard Time − Standard Longitude (3)

Note: East longitude is positive and west longitude is negative.

2. The Effect of Precession

This is the motion of the equinoxes (points of intersection of the celestial equator and the

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ecliptic, path of the sun around the earth) in a westerly direction (opposite that of the motion of the stars) by about 50.″2 per year. This variation of the coordinate system is chiefly attributed to the gravitational forces of the sun and moon on the earth's equatorial bulge. The constants derived by Simon and Newcomb are:

$$\xi = (2304.948\,T + 0.302\,T^2 + 0.018\,T^3)/3600 \quad (4)$$

$$\zeta = (2304.948\,T + 1.093\,T^2 + 0.019\,T^3)/3600 \quad (5)$$

$$\theta = (2004.255\,T - 0.426\,T^2 - 0.042\,T^3)/3600 \quad (6)$$

ξ, ζ, and θ are all in degrees

where

$$T = (JD + UT/24 - JD_o)/36524.2199 \quad (7)$$

then

$$\tan(a_2 - \zeta) = A/B \quad (8)$$

and

$$\sin \delta_2 = D \quad (9)$$

where

$$A = \cos \delta_1 \sin(a_1 + \xi) \quad (10)$$

$$B = \cos\theta \cos\delta_1 \cos(a_1 + \xi) - \sin\theta \sin\delta_1 \quad (11)$$

$$D = \sin\theta \cos\delta_1 \cos(a_1 + \xi) + \cos\theta \sin\delta_1 \quad (12)$$

a_2 and δ_2 are right ascension and declination respectively corrected for proper motion and precession.

Note: For stars close to the pole such as Polaris equation (9) may be replaced by

$$\cos\delta_2 = \sqrt{(A^2 + B^2)} \quad (13)$$

3. The Effect of Nutation

This is a periodic change in the position of stars in space caused by the variation in the gravitational attraction between the sun, moon, and the earth due to the periodic changes in the distances between them. The combined effects in longitude ($\Delta\Psi$) and in obliquity ($\Delta\epsilon$), both in seconds of arc, are computed as follows:

$$\begin{aligned}\Delta\Psi = &- (17.2327 + 0.01737\,T')\sin O\\ &- (1.2729 + 0.00013\,T')\sin(2L) + 0.2088\sin(2O)\\ &- 0.2037\sin(2L') + (0.1261 - 0.00031\,T')\sin M\\ &+ 0.0675\sin M' - (0.0497 - 0.00012\,T')\sin(2L + M)\\ &- 0.0342\sin(2L' - O) - 0.0261\sin(2L' + M')\\ &+ 0.0214\sin(2L - M) - 0.0149\sin(2L - 2L' + M')\\ &+ 0.0124\sin(2L - O) - 0.0114\sin(2L' - M') \quad (14)\end{aligned}$$

$$\begin{aligned}\Delta\epsilon = &(9.2100 + 0.00091\,T')\cos O\\ &+ (0.5522 - 0.00029\,T')\cos(2L) + 0.0904\cos(2O)\\ &+ 0.0884\cos(2L') + 0.0216\cos(2L + M)\\ &+ 0.0183\cos(2L' - O) + 0.113\cos(2L' + M')\\ &+ 0.0093\cos(2L - M) - 0.0066\cos(2L - O) \quad (15)\end{aligned}$$

where

$$T' = (JD + UT/2415020.0)/36525.0 \quad (16)$$

L (the sun's mean longitude) in degrees is:

$$L = 279.6967 + 36000.7689\,T' + 0.000303\,T'^2 \quad (17)$$

L′ (the moon's mean longitude) in degrees is:

$$L' = 270.4342 + 481267.8831\,T' - 0.001133\,T'^2 \quad (18)$$

M (the sun's mean anomaly) in degrees is:

$$M = 358.4758 + 35999.0498\,T' - 0.000150\,T'^2 - 0.0000033\,T'^3 \quad (19)$$

M′ (the moon's mean anomaly) in degrees is:

$$M' = 296.1046 + 477198.8491\,T' + 0.009192\,T'^2 \quad (20)$$

O (the longitude of moon's ascending node) in degrees is:

$$O = 259.1833 - 1934.1420\,T' + 0.002078\,T'^2 \quad (21)$$

UT = as defined in equation (3)

$$a_3 = [((\cos E + \sin E \sin a_2 \tan\delta_2)\Delta\Psi - (\cos a_2 \tan\delta_2)\Delta\epsilon)/3600] + a_2 \quad (22)$$

$$\delta_3 = [((\sin E \cos a_2)\Delta\Psi + \sin(a_2)\Delta\epsilon)/3600] + \delta_2 \quad (23)$$

where a_3 and δ_3 are right ascension and declination (in degrees) corrected for proper motion, precession, and nutation, respectively.

E (the obliquity of the ecliptic) in degrees is:

$$E = 23.452294 - 0.13012\,T' - 0.00000164\,T'^2 + 0.0000005030\,T'^3 + 0.00256\cos O \quad (24)$$

4. The Effect of Annual Aberration

This is the apparent displacement of an object in space caused by the relative motion of the observer which is an appreciable fractional amount of the speed of light. Basically, in surveying and mapping, there are two components to aberration:

a. annual aberration which results from the displacement of the observer due to the rotational velocity of the Earth about its orbit.

b. diurnal aberration which results from the displacement of the observer due to the rotational velocity of the earth about its north-south diurnal axis of rotation. This is a relatively small amount and is only applicable to first-order work and therefore not presented here.

$$a_4 = [(-20.49\,(\cos a_3 \cos\lambda_s \cos E + \sin a_3 \sin\lambda_s))/(3600\,(\cos\delta_3))] + a_3 \quad (25)$$

$$\delta_4 = [-20.49\,(\cos\lambda_s \cos E\,(\tan E \cos\delta_3 - \sin a_3 \sin\delta_3) + \cos a_3 \sin\delta_3 \sin\lambda_s)]/3600 + \delta_3 \quad (26)$$

where

a_4, and δ_4 are final corrected apparent right ascension and declination, respectively, for the desired epoch.

λ_s (true apparent longitude of the sun) in degrees is computed as follows:

$$\lambda_s = L + C - 0.00569 - 0.00479 \sin O + 0.00134 \cos a + 0.00154 \cos b + 0.00200 \cos c + 0.00179 \sin d + 0.00178 \sin e \quad (27)$$

where

L = as defined in equation (17)

O = as defined in equation (21)

C (sun's equation of the center) in degrees is:

$$C = (1.919460 - 0.004789\,T' - 0.000014\,T'^2)\sin M + (0.020094 - 0.000100\,T')\sin(2M) + 0.000293 \sin(3M) \quad (28)$$

where

M = as defined in equation (19)

$$a = 153.23 + 22518.7541\,T' \quad (29)$$

$$b = 216.57 + 45037.5082\,T' \quad (30)$$

$$c = 312.69 + 32964.3577\,T' \quad (31)$$

$$d = 350.74 + 445267.1142\,T' - 0.00144\,T'^2 \quad (32)$$

$$e = 231.19 + 20.20\,T' \quad (33)$$

T′ = as defined in equation (16)

a, b, c, d, and e are all in degrees.

Right Ascension and Declination of the Apparent Sun

$$dT = 0.41 + 1.2053\,T' + 0.4992\,T'^2 \quad (34)$$

$$a_s = \tan^{-1}[(\cos E \sin\lambda_s)/(\cos\lambda_s)] + 0.002783\,dT/4 \quad (35)$$

where

a_s = the apparent right ascension of the sun in degrees

λ_s = as defined in equation (27)

O = as defined in equation (21)

Note: Proper care must be exercised in obtaining the appropriate quadrant for a_s in equation (35). a_s must be in the same quadrant as λ_s.

The apparent declination of the sun (δ_s) in degrees is computed as follows:

$$\sin\delta_s = \sin E \sin\lambda_s \quad (36)$$

Solving the PZS Spherical Triangle for Azimuth

In order to compute the Local Hour Angle (LHA) or "t" it is necessary to determine the Local Sidereal Time (LST).

$$LST = GST(15) + \text{longitude of station} \quad (37)$$

where GST is the corresponding Greenwich Sidereal Time in hours for the time of the observation. LST and longitude of state are both in degrees.

$$GST = 24[0.276919398 + 100.0021359\,T'' + 0.000001075\,T''^2] + 1.002737908\,UT + (\Delta\Psi \cos E)/54000 \quad (38)$$

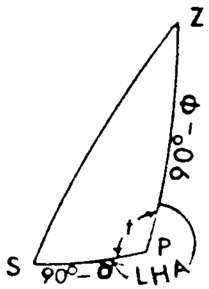
where

$$T'' = (JD - 2415020)/36525 \quad (39)$$

UT = as defined in equation (3)

$\Delta\Psi$ = as defined in equation (14)

E = as defined in equation (24)



Note: GST in equation (38) will be greater than 24 hours, to obtain GST between zero and 24 hours the result is divided by 24 and the fractional part multiplied by 24.

Finally,

$$LHA = LST - \text{Right Ascension} \quad (40)$$

The known elements of the PZS spherical triangle, at this point are: co-latitude ($90° - \phi$), which can be scaled from a reliable map for the station occupied, co-declination ($90° - \delta$) from equation (26) for Polaris or equation (36) for sun, and LHA or "t" from equation (40). Note that when LHA is greater than 180 degrees the angle at the pole designated as "t" is equal to 360 degrees minus LHA signifying that the star or the sun is east of north. When LHA is less than 180 degrees, then "t" = LHA signifying that the star or the sun is on the west side of north. Angle "Z" which is the azimuth of the star or the sun can be computed from

$$\tan Z = \sin t / (\cos \phi \tan \delta - \sin \phi \cos t) \quad (41)$$

Conclusion

The foregoing computations for azimuth are performed using the hour angle method which requires the knowledge of the latitude and longitude of the observation station along with an accurate knowledge of the time of the observation. Latitude and longitude of a station can be scaled from a reliable map within one second of arc. However, the knowledge of time requires a comparison of the timepiece before and after the observation with an accurate time service, such as those broadcasted by WWV, WCHU, etc., and correcting the recorded time for the drift of the timepiece, if any. The position of Polaris as computed above may be used for azimuth determinations where the accuracy requirements are equal to or lower than second order. Assuming no errors in time, declination, and latitude of the place exist, the combined uncertainty of 20 seconds of arc in right ascension and sidereal time will produce less than 0.6 seconds of arc in azimuth, in latitudes of up to 50 degrees north. This error is maximum when Polaris is observed near the observer's meridian. On the other hand, assuming no errors in time, latitude, and declination exist, the combined uncertainty of 20 seconds of arc in right ascension of the sun and sidereal time will produce an error of over one minute of arc in azimuth at 40 degrees latitude north. This error is maximum when the sun is observed at local apparent noon, and it has the maximum declination.

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Appendix A

Computing Julian Day from Gregorian Calendar

Given: YYY.MMDDdd

where

YYYY is the year
MM is the month
DDdd is the day and any fractional parts thereof
Example: 12:00 PM, June 8, 1984 = 1984.060850

Compute: The corresponding Julian Day (JD)

Solution:

If MM is greater than 2, then let y = YYYY, and m = MM
If MM is less than or equal to 2, then let y = YYYY − 1, and m = MM + 12
If YYYY.MMDDdd is greater than or equal to 1582.1015 (i.e., Gregorian Calendar), then let
A = Int(y/100), and B = 2 − A + Int(A/4)

If YYYY.MMDDdd is less than 1582.1015, then A and B above are not needed

$JD = Int(365.25\ y) + Int(30.6001(m + 1)) + DD.dd$
$+ 1720994.5$

Note: If Gregorian Calendar, then the quantity B is added to JD.

Example

Given: March 9, 1984

Compute: the corresponding Julian Day

Solution:

$y = 1984$, $m = 3$
$A = 19$, $B = -13$
$JD = 724656 + 122 + 9.0 + 1720994.5 - 13 = 2445768.5$ days

Appendix B

Example 1

Given: the following data with respect to a Polaris observation azimuth:

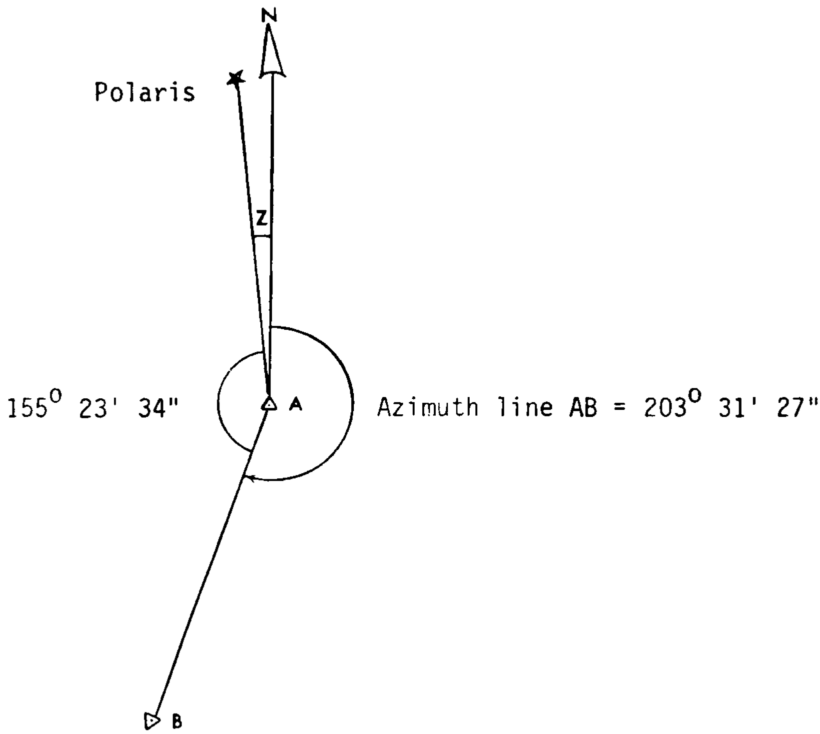
Station occupied: "A"
Azimuth mark: "B"
Date of observation: March 9, 1984 (1984.0309)
Latitude of station (ϕ): 43°32′15″.00 N.
Longitude of station (λ): 85°36′24″.00 W
Time of observation: $20^{h}\ 51^{m}\ 36^{s}.2$ E.S.T.
Clockwise angle from "B" to Polaris: 155°23′34″

Solution:

JD (Julian Day for the year 1950.0) = $2433282^{d}.5$ (Appendix A)
JD (Julian Day for March 9, 1984) = $2445768^{d}.5$ (Appendix A)

UT	$= 25^{h}.86005556$ from (3),	T	$= 0.3418848516$ from (7)
a_1	$= 27^{\circ}.22907311$ from (1),	δ_1	$= 89^{\circ}.02877867$ from (2)
ξ	$= 0^{\circ}.21890634$ from (4),	ζ	$= 0^{\circ}.21893203$ from (5)
θ	$= 0^{\circ}.19032582$ from (6),	A	$= 0.00781308$ from (10)
B	$= 0.01172070$ from (11),	D	$= 0.99990079$ from (12)
a_2	$= 33^{\circ}.90655690$ from (8),	δ_2	$= 89^{\circ}.19289781$ from (13) or (9)
T′	$= 0.8418775496$ from (16),	L	$= 30587^{\circ}.936020$ from (17)
L′	$= 405439^{\circ}.05952$ from (18),	M	$= 30665^{\circ}.267527$ from (19)
M′	$= 402039^{\circ}.108870$ from (20),	O	$= -1369^{\circ}.125955$ from (21)
$\Delta\Psi$	$= -15''.824464$ from (14),	$\Delta\epsilon$	$= 3''.530685$ from (15)
E	$= 23^{\circ}.4421775$ from (24),	a_3	$= 33^{\circ}.77549907$ from (22)
δ_3	$= 89^{\circ}.19199358$ from (23),	a	$= 19111^{\circ}.263522$ from (29)
b	$= 38132^{\circ}.637044$ from (30),	c	$= 28064^{\circ}.642685$ from (31)
d	$= 375211.126000$ from (32),	e	$= 248^{\circ}.195927$ from (33)
C	$= 1.75484698$ from (28),	λ_s	$= 30589^{\circ}.68523$ from (27)
a_4	$= 33.512852$ from (25),	δ_4	$= 89^{\circ}.195665$ from (26)
T″	$= 0.84184805$ from (39),	GST	$= 13^{h}.05514578$ from (38)

LST $= 110^{\circ}.2205193$ from (37)
LHA $= LST - a_4 = 76^{\circ}.7076673$ from (40)
t = LHA (star is west of north)
Z $= 1^{\circ}.0830836$ from (41) where $\delta = \delta_4$
Azimuth line AB = 203°31′27″



Example 2

Given: the following data with respect to a solar observation for azimuth:

Station occupied: "C"
Azimuth mark: "D"
Date of observation: April 26, 1984 (1984.0426)
Latitude of station (ϕ): 43°41'04".4 N.
Longitude of station (λ): 85°29'34".0 W.
Time of observation: $8^{h}\ 26^{m}\ 57^{s}$ E.S.T.
Clockwise angle from "D" to sun: 304°15'18"

Solution:

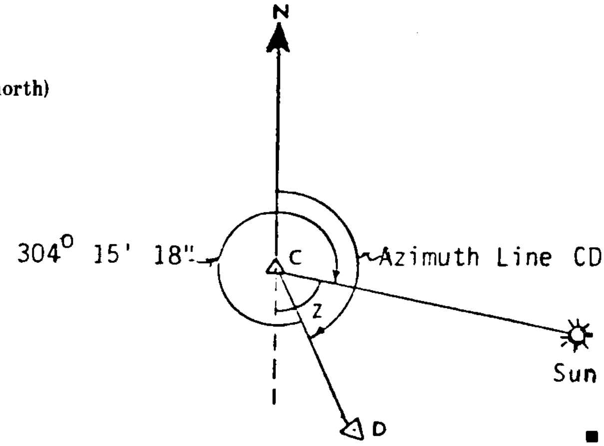
UT	$= 13^{h}.4491667$ from (3),	JD	$= 2445816^{d}.5$ (Appendix A)
T′	= 0.843177560 from (16),	O	$= -1371^{\circ}.640355$ from (21)
E	$= 23^{\circ}.44226592$ from (24),	L	$= 30634^{\circ}.737394$ from (17)
M	$= 30712^{\circ}.066666$ from (19),	C	$= 1^{\circ}.76105164$ from (28)
a	$= 19140^{\circ}.538136$ from (29),	b	$= 38191^{\circ}.186272$ from (30)
c	$= 28107^{\circ}.496692$ from (31),	d	$= 375789^{\circ}.97787$ from (32)
e	$= 248^{\circ}.222187$ from (33),	λ_s	$= 30636^{\circ}.48903$ from (27)
dT	= 1.7812 from (34),	a_s	$= 34^{\circ}.162597$ from (35)
T″	= 0.8431622 from (39)	L′	$= 406064^{\circ}.71277$ from (18)
M′	$= 402659^{\circ}.47235$ from (20),	$\Delta\Psi$	$= -16''.832096$ from (14)
GST	$= 3^{h}.76433141$ from (38),	LST	$= 330^{\circ}.9721934$ from (37)
LHA	$= 296^{\circ}.8095964$ from (40)		

t $= 360° - LHA$ (sun is east of north)

δ $= \delta_s = 13\overset{\circ}{.}684423$ from (36)

Z $= -81\overset{\circ}{.}37102$ from (41)

Azimuth line CD = 154°22′26″



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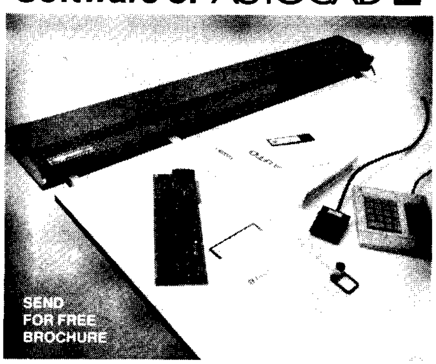

Comment and Discussion

The pages of SURVEYING AND MAPPING are open to free and temperate discussion of all matters pertaining to the interests of the Congress. It is the purpose of this department to encourage comments on published material or the presentation of new ideas in a formal or informal way.—EDITOR.

On: Comments on "Non-Iterative Solution of the ϕ Equation," by Mohammed Id Ozone

Published in Vol. 45, No. 3, p. 268

From: Dr. Mohammad Id Ozone, Assistant Professor, King Saud University, College of Engineering, P.O. Box 800, Riyadh 11421, Saudi Arabia. [Comments are re Bruce Hedquist's comments, published in Vol. 45, No. 3, p. 268, on author Ozone's paper, published in Vol. 45, No. 2, pp. 169-171.]

I appreciate the specific comments presented by Mr. Bruce Hedquist on my Non-Iterative Solution of the ϕ Equation and would like to draw his attention to the following:

a. Mr. Bruce says "I would like to point out that there are somewhat simpler equations available for performing this task. In fact, both the forward and inverse equations appear in the paper presented by Professor Laurila in the same issue."

Well, it is also a fact that Professor Laurila, as well as Mr. Meade, has applied the Bowring formula, which is an iterative solution as asserted by Mr. Bowring himself, while my solution is completely non-iterative as mentioned very specifically in the title of my paper.

b. Mr. Bruce also says "One thing that I can't understand in Dr. Ozone's formulas is how he arrives at a latitude value which is apparently not dependent upon a Y value."

The answer is that my non-iterative solution is not independent of Y as he says. In my solution (as well as in Bowring's), we take a parameter $X_1 = (X^2 + Y^2)^{1/2}$. In my paper this relation appears in the third line under Figure 2. Both Bowring and myself take this notation to manipulate the problem in the meridional plane of the geodetic point, after computing the longitude from $\tan^{-1}(Y/X)$ of course.

Taking this remark into consideration, Mr. Bruce won't find any discrepancies any more.

Re: Polaris and Solar Observations Reduction for Azimuth Without the Use of Ephemeris by Sayed R. Hashimi

Published in Vol. 45, No. 3, pp. 239–245

From: Sayed R. Hashimi, Associate Professor of Surveying, Ferris State College, Big Rapids, Michigan 49307. [Ed. Note: The Hashimi paper was one of two papers which were delayed enroute to the authors for review of page proofs by the postal service. Changes and corrigenda are presented here for the readers' convenience. We regret any inconvenience caused to the author as well as to the readership.]

p. 239, col. 1, line 4, sentence should read:

. . . This process is analagous to the use of trigonometric and logarithmic tables prior to the emergence of digital computers and hand-held programable calculators that provide these functions. . . .

p. 240, col. 1, line 7, sentence should read:

The constants derived by Simon Newcomb are: . . .

p. 240, cols. 1 and 2, under "The Effect of Nutation," equations (14) through (24) are car-

ried to make the several changes easier to assimilate:

$\Delta\Psi =$
$- (17.2327 + 0.01737\ T')\sin O$
$- (1.2729 + 0.00013\ T')\sin(2L) + 0.2088\sin(2O)$
$- 0.2037\sin(2L') + (0.1261 - 0.00031\ T')\sin M$
$+ 0.0675\sin M' - (0.0497 - 0.00012\ T')\sin(2L + M)$
$- 0.0342\sin(2L' - O) - 0.0261\sin(2L' + M')$
$+ 0.0214\sin(2L - M) - 0.0149\sin(2L - 2L' + M')$
$+ 0.0124\sin(2L - O) + 0.0114\sin(2L' - M')$ (14)

$\Delta\epsilon =$
$(9.2100 + 0.00091\ T')\cos O$
$+ (0.5522 - 0.00029\ T')\cos(2L) - 0.0904\cos(2O)$
$+ 0.0884\cos(2L') + 0.0216\cos(2L + M)$
$+ 0.0183\cos(2L' - O) + 0.0113\cos(2L' + M')$
$- 0.0093\cos(2L - M) - 0.0066\cos(2L - O)$ (15)

where

$T' = (JD + UT/24 - 2415020.0)/36525.0$ (16)

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$L = 279.6967 + 36000.7689\ T' + 0.000303\ T'^2$ (17)

L' (the moon's mean longitude) in degrees is:

$L' = 270.4342 + 481267.8831\ T' - 0.001133\ T'^2$ (18)

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$M = 358.4758 + 35999.0498\ T' - 0.000150\ T'^2 - 0.0000033\ T'^3$ (19)

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O (the longitude of moon's ascending node) in degrees is:

$O = 259.1833 - 1934.1420\ T' + 0.002078\ T'^2$ (21)

UT = as defined in equation (3)

$a_3 = [((\cos E + \sin E \sin a_2 \tan \delta_2)\Delta\Psi - (\cos a_2 \tan \delta_2)\Delta\epsilon)/3600] + a_2$ (22)

$\delta_3 = [((\sin E \cos a_2)\Delta\Psi + \sin(a_2)\Delta\epsilon)/3600] + \delta_2$ (23)

where a_3 and δ_2 are right ascension and declination (in degrees) corrected for proper motion, precession, and nutation, respectively.

E (the obliquity of the ecliptic) in degrees is:

$E = 23.452294 - 0.013012\ T' - 0.00000164\ T'^2 + 0.0000005030\ T'^3 + 0.00256\cos O$ (24)

p. 241, col. 2, the last sentence of the paragraph after equation (37 should read:

. . . LST and longitude of station are both in degrees.

p. 242, under "Conclusion", line 12 ff. should read:

. . . broadcasted by WWV, CHU, etc., and correcting the recorded time for the drift of the timepiece, if any. . . .

p. 242, under "Appendix A, Computing Julian Day from Gregorian Calendar," line 1 should read:

Given: YYYY.MMDDdd

p. 243, under "Appendix B, Example 1, line 1 should read:

Given: the following data with respect to a Polaris observation for azimuth:

From: Joseph H. Senne, Professor Emeritus of Civil Engineering, University of Missouri-Rolla, Missouri. It is a pleasure to see that there is a growing awareness of the advantages of developing algorithms to compute astro azimuths without the use of external ephemerides. Such procedures have been around for a long time but have usually required the use of mainframe computers for their execution. The present-day challenge, as I see it, is to reduce these programs so that they will fit in a hand-held computer or programmable calculator and still retain a reasonable accuracy.

I note that the author has drawn extensively on the formulas given by Meuss in his book on *Astronomical Formulas for Calculators*, however Meuss presents these formulas in separate chapters and since each understandably has its own particular use, he does not always tie them together into an efficient package but leaves this exercise up to the user. It is, therefore, important that to establish a workable practical algorithm, each expression must be carefully examined for consistency and not used verbatim. For example, formula (33) of the paper, when combined with its trigonometric part is a correction term for the sun's longitude with a maximum amplitude of 1.4 arc-seconds and a period of 1782 years. Since most present-day surveyors would be interested only in the sun's position during the last 25 years of this century, using a value of 0.9 for T would make this term equivalent to -6.0 arc-seconds during that period. This is also true of many of the T^2 and T^3 terms that are used to determine the fundamental arguments.

Also formula (34) which is based on the gradual slowing of the earth's rotation is designed for long-term changes and should not be used to calculate current variations in dT which represents the difference between Universal and Ephemeris time. For 1985 this formula is in error by as much as the value it is designed to correct. At present dT is 55 seconds of time, while formula (34) gives 107. Again, a value of 60 seconds should be sufficient. Also the dT term should be applied to T at the beginning of the program since all time calculations except Sidereal should be in Ephemeris or Terrestrial Dynamic time.

An accuracy of 20 arc-seconds, as stated by the author, in the sun's position may not be acceptable to modern-day surveyors who, with access to accurate time and precise theodolites, are routinely using solar observations for azimuth determinations with accuracies in the order of from 10 to 15 arc-seconds.

Most of this error in the sun's position is due to using an insufficient number of planetary perturbation terms to correct the sun's longitude. The author used only four, while it requires at least 16 to increase accuracy to a usable amount. Granted, this increases program size, but the number of nutation terms can be reduced to five each with little loss in accuracy. In 1980 (September issue of *The Australian Surveyor*) G. G. Bennett published a short algorithm which calculated the sun's position to an accuracy of about four arc-seconds. While this is probably on the borderline of acceptable accuracy today, it was nevertheless certainly a great step in the right direction.

Finally, it should be pointed out that in 1984, based on the recommendations of the International Astronomical Union, the almanac offices in most countries changed from the standard epoch reference system of the year 1900 to the year 2000, which is known as the epoch J2000.0. In doing this, most of the fundamental arguments and planetary masses have been revised; and, although accuracies using the old system are still satisfactory, I would strongly recommend that any new algorithms be based on the epoch J2000.0. In addition to some increased accuracy and consistency, there are simplifications that make the use of this system attractive for programs in small computers. Most of those changes are discussed in the U.S. Naval Observatory's Circular No. 163, including the new nutation series and how to update star position from 1950 to the year 2000. A new catalog (FK5), based on this system and containing positions of over 1500 stars, will soon be available.

During the past several years I have developed algorithms for the sun, Polaris, and other stars, based on the new epoch, that fit into the HP-41 and HP-71B calculators. Accuracies for the sun are within 2.2 and 1.7 arc-seconds for the GHA and declination, respectively, and within 0.2 arc-second in absolute position for stars. These programs which include the use of the HP time module for automatic input of time and date have been incorporated into the ASTRO*ROM module for the HP-41.

To me, it is fascinating to see the recent progress that has been made in the development of microcomputers and to realize that less than 10 years ago precise ephemerides had to be generated using mainframe computers. I would predict that within the next decade, we will be able to compute, using hand-held computers, ephemerides nearly to the same accuracy as those now done by the nautical almanac offices.

From: Michael R. Craymer, Survey Science, University of Toronto, Erindale Campus, Mississauga, Ontario L5L 1C6, Canada. I am a firm believer in giving credit where credit is due and in Mr. Hashimi's paper I feel that Dr. Jean Meuss should be *explicitly* acknowledged within the text for having developed the material presented.... This paper ... offers nothing more (actually less) than Meuss (1982).

Meuss actually derived many of these algorithms from the original work by the Canadian astronomer, Simon Newcomb (cf. Newcomb, 1898). Meuss' book first published in 1978 and later re-released in 1982, has been in wide use by amateur astronomers and may be obtained from *Astronomy* or *Sky and Telescope* magazines. In addition to being free of major errors, the book also gives more detailed explanations of the concepts and algorithms.

[Here Craymer notes corrigenda which have already been printed above. In addition, he comments as follows.].

Eq. (34)—This equation gives an estimate of the difference between ephemeris (or dynamic) time and universal time only for epochs *prior to 1710* and is thus incorrectly applied here (cf. Meuss, p. 31). Using this equation for 1980 gives 1.69 min., whereas the *Astronomical Almanac* gives the observed value of 0.84 min. (a factor of two smaller).

Eq. (35)—The coefficient of -0.002783, accounting for effect of the difference between dynamic and universal times, should be -0.002738. In actual fact, this last term (about 0.16 sec. in magnitude) is well within the level of accuracy of the algorithm and may therefore be omitted along with eq. (34) with no loss of accuracy.

Eq. (38)—Although not actually in error, this expression may be simplified. Because one actually requires Greenwich Apparent Sidereal Time (GST) for the time of observation and not for Ohr UT, T′ can be used in place of T″ and the term "+1.002737908 UT" may then be omitted (cf. Mueller, 1977, p. 153, eq. (5.41)); note that T″ is no longer needed in any other expressions.

Appendix B, Ex. 1—The Julian date for 1950.0 is incorrect. The notation 1950.0 refers to the beginning of the Bessilian year and not to January 0, Ohr UT (cf. Mueller, 1977). The actual Julian date for the beginning of the Bessilian years is tabulated in the *Astronomic Almanac* (e.g., Julian date for 1950.0 is 2433282.423). Nevertheless , the effect of the epochs on the coordinates of polaris is insignificant here (about 1 sec. of arc). Thus, either epoch may be used with no loss of accuracy.

I also have a few general questions abouth the algorithms. First, what is the level of accuracy of both the solar and polaris update? Although Mr. Hashimi does not explicitly state the accuracy, he implies in the "Conclusion" that it is of the order of 20 sec. of arc. I have actually compared Meuss' algorithm with the *Astronomic Almanac* for 1982 and have found agreement to better than 7.5 sec. of arc in right ascension and 3.2 sec. of arc in declination.

Second, why has the 1950.0 FK4 epoch been used instead of the closer 1975.0 FK4 or the new FK5 epoch. The latter is now in official use by the *Astronomic Almanac*. Thus, comparisons of α, δ, and GST from the FK4 system will not agree with the FK5 system because of a new theory of nutation (not yet implemented) and an adjustment in the equinox. The correction $\triangle E$ to the FK4 equinox and thus to right ascension and Greenwich sidereal time is (Stein, 1982):

$$\triangle E = 0.035 + 0.085(T - 19.50) \text{ sec.}$$

Note that although there will be differences between the right ascensions and Greenwich sidereal times of the FK4 and FK5 systems due to the equinox change, these will cancel in the local hour angle and thus will not affect azimuth determinations. However, significant discrepancies may occur when the new theory of nutation is finally implemented.

Third, why has the semi-diameter correction for the sun not been included? Land surveyors often observe the trailing edge of the sun and therefore need such a correction. The expressions for this may be found in Meuss (1982) or Newcomb (1898). In Mr. Hashimi's notation, the necessary expressions are:

$$ec = \text{eccentricity} = 0.01675104 - 0.0000418\,T' - 0.000000126\,T'^2$$

$$h = 353.40 + 65928.7155\,T' \text{ (deg)}$$

$$R_m = \text{mean radius vector} = [1.0000002(1 - ec^2)] / [1 + ec\cos(M + C)] \text{ (AU)}$$

$$\triangle R = (0.543 \sin a + 1.575 \sin b + 1.627 \sin c + 3.076 \sin d + 0.9027 \sin h)\, 10^{-5} \text{ (AU)}$$

$$SD = \text{sun's semi-diameter} = 959.63 / (R_m + \triangle R) \text{ (sec. of arc).}$$

Finally, I would also like to mention the algorithms of Bennett (1980) and Craymer (1984). The former provides very concise algorithms for the sun with improved accuracy over Meuss while the latter gives *all* of the necessary expressions to reproduce exactly the astronomical tables based on the FK4 system. Microcomputer programs employing the algorithms of Bennett (1980) and Craymer (1984) for azimuth determination have been developed at the University of Toronto for the Apple II and IBM PC. These have been in use by many practicing Ontario land surveyors since 1984 and may be obtained at nominal cost by writing to me personally.

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