A Least Squares Resection Programme

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Introduction

Trig stations are exposed places by necessity and observing conditions at the stations are seldom ideal. Access is usually difficult. Resection will often give a more reliable fix and will always be much quicker than intersection. When more than three beacons are observed the best analysis is by the least squares method. This assumes that the position taken for the observing station which reduces the sum of the squares of the residuals (the difference between the observed and the calculated angles) to a minimum is the correct one. The programme has been written for a Texas T159 calculator and occupies less than half a programme card. Listing will be available from the author after he has returned to Hobart in January 1983. As written, up to eight sights can be analysed in a minute of running time. You are unlikely to be able to see as many co-ordinated marks as this from a single station.

Method

The co-ordinates of the observing station shown in Figure 1, E8 and N8 are guessed, taken from a map or calculated using the best three observations. The observed angles then have 'arc to chord' corrections applied so that the angles H become plane angles. The operation of the P→R function on the calculator makes it necessary for θ_0 to be chosen so that θ-θ_0 is always positive (Figure 2).

The residual e for each sight after the first one will be $(\theta - \theta_0 - H)$.

Our aim is to minimise $U = \Sigma e^2 = \Sigma(\theta - \theta_0 - H)^2$ by shifting the assumed position of the observing station in the E or x direction, and in the N or y direction. We use x and y only because the mathematical symbols are familiar.

Differentiating U with respect to θ and noticing that H is constant

$$\frac{dU}{dx} = \Sigma\, 2(\theta - \theta_0 - H)\left(\frac{d\theta}{dx} - \frac{d\theta_0}{dx}\right)$$

$$= \Sigma\, 2(\theta - \theta_0 - H)\left(\frac{n_0}{r_0^2} - \frac{n}{r^2}\right) \qquad \text{see Appendix 1}$$

Similarly

$$\frac{dU}{dy} = \Sigma\,(\theta - \theta_0 - H)\left(\frac{e}{r^2} - \frac{e_0}{r_0^2}\right)$$

The second differential is also required for the analysis

$$\frac{d^2U}{dx^2} = 2\,\Sigma\left[(\theta - \theta_0 - H)\left(\frac{d^2\theta}{dx^2} - \frac{d^2\theta_0}{dx^2}\right) + \left(\frac{d\theta}{dx} - \frac{d\theta_0}{dx}\right)^2\right]$$

$$= 2\,\Sigma\left[(\theta - \theta_0 - H)\left(\frac{en}{r^4} - \frac{e_0 n_0}{r_0^2}\right) + \left(\frac{n_0}{r_0^2} - \frac{n}{r^2}\right)^2\right]$$

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Now $(\theta - \theta_0 - H)$ is a small quantity compared with the values in the other brackets if the original estimate of position has been at all accurate, especially as it is calculated in radians, and it is good enough to take

$$\frac{d^2U}{dx^2} = 2\,\Sigma\left(\frac{n_0}{r_0^2} - \frac{n}{r^2}\right)^2$$

Similarly $$\frac{d^2U}{dy^2} = 2\,\Sigma\left(\frac{e}{r^2} - \frac{e_0}{r_0^2}\right)^2$$

The assumed value of E8 is then reduced by $\frac{dU}{dx} \div \frac{d^2U}{dx^2}$ and the assumed value of N8 is reduced by $\frac{dU}{dy} \div \frac{d^2U}{dy^2}$. The calculations are repeated until a stable value of E8 and N8 are reached. The procedure usually converges sufficiently after four or five iterations.

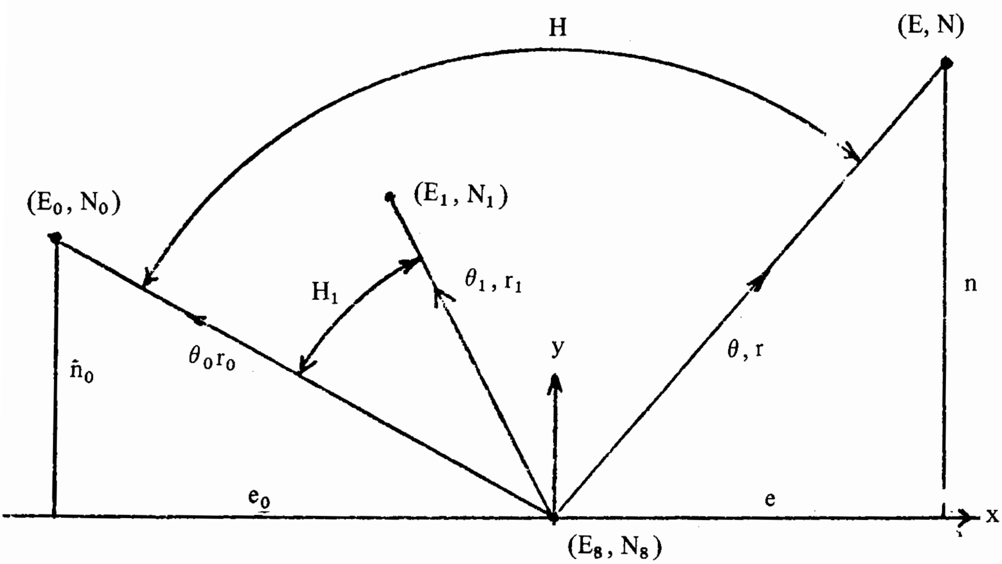


FIGURE 1

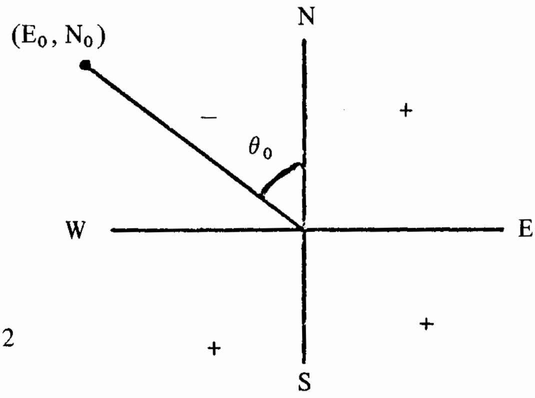
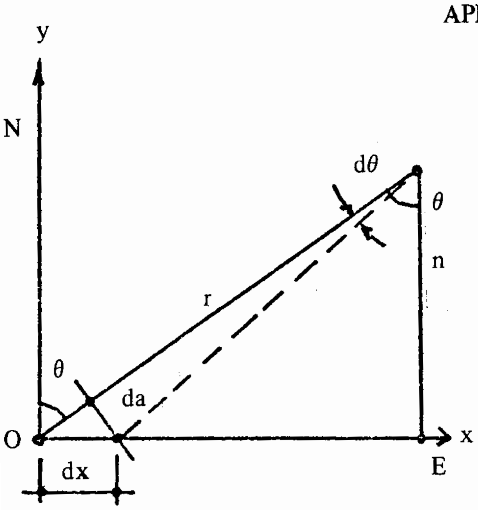
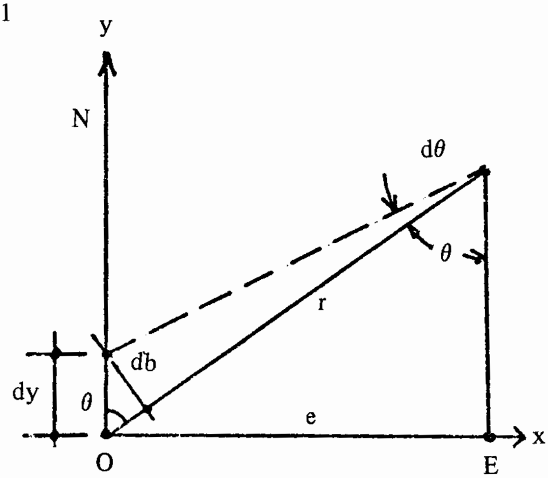


FIGURE 2

APPENDIX 1





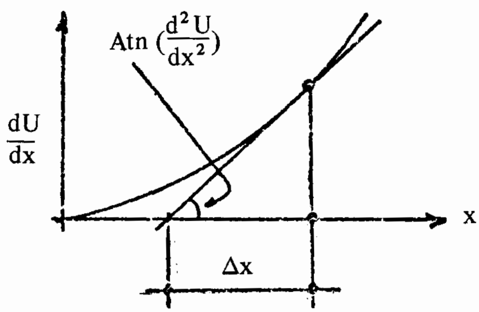
$$\frac{da}{dx} = \frac{n}{r} = \cos\theta$$

$$d\theta = -\frac{da}{r} = -\frac{n}{r^2}\,dx$$

The negative sign for dθ on the left hand side is because θ is decreased by a positive shift of 0.

$$\frac{db}{dy} = \frac{e}{r} = \sin\theta$$

$$d\theta = \frac{db}{r} = \frac{e}{r^2}\,dy$$



$$\Delta x = \frac{dU}{dx} \div \frac{d^2U}{dx^2}$$ deduct from E_8

Similarly

$$\Delta y = \frac{dU}{dy} \div \frac{d^2U}{dy^2}$$ deduct from N_8

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