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Coordinates and Azimuth Determination from an Inaccessible Baseline

by Walter H. Treftz

Abstract. Often there is a lack of control points that can be occupied by an instrument in the survey support of geophysical exploration—a lack prevalent in remote areas of the United States and many times in foreign countries. Coordinates of these points, however, have often been determined by intersections or photogrammetric methods, in which cases the survey can establish a baseline with a known azimuth and the known coordinates of at least one end of the line.

Described in this paper is a modification of the technique developed by the British Royal Engineers, which is suited for control of lower-order surveys, such as hydrographic, topographic, photogrammetric, gravity and seismograph lines. This technique requires two points with known coordinates that can be seen from both ends of the unknown baseline.

The technique is particularly useful for work in harsh environment and when meteorological conditions rule out celestial observations for azimuth control and satellite or other systems are not available. In these instances, the surveyor can establish a baseline even though he cannot occupy the control station.

In the survey support of geophysical explorations, especially overseas or in the more remote regions of the United States, the situation is frequently encountered where there is a lack of control points that can be occupied by an instrument, i.e., intersection stations may be available but not points which can be occupied. The coordinates of these "unoccupiable" points have usually been determined by intersections or photogrammetric methods. In these cases, the surveyor must establish a baseline with a known azimuth and known coordinates of at least one end of this line.

Currently much of the petroleum exploration work is being conducted in harsh environments, such as high latitudes (sub-polar) and tropical (jungles and desert) regions. Meteorological conditions in many of these areas rule out celestial observations for azimuth control and satellite or other systems may not be available. In these instances, given the right circumstances, the surveyor can establish a baseline even though he cannot occupy the control stations.

The technique described in this paper is a modification that was developed by the British Royal Engineers and is especially suited for control of lower-order surveys, such as hydrographic, topographic, photogrammetric, gravity, and seismograph lines. All that is required are two points with known coordinates that can be seen from both ends of the unknown baseline. The coordinates of these points could have been determined by any acceptable means, such as intersections, traverse, photogrammetry, etc. The points usually will be in the form of intersection stations, i.e., radio towers, water tanks, church spires. [Note: For simplicity, the inversed line between the two known stations will be referred to as the "known baseline," while that line being established from it, the "unknown baseline."]

They can also be towers constructed over ground stations, such as on mountain tops or on the far side of a river or bay where they are inaccessible to the survey to be controlled. The unknown baseline should be as long as possible, while maintaining the conditions that the control points must be visible from both ends. In practice, a length of about 2,000 ft. works nicely; although this is a field decision on the part of the surveyor. Three other elements of the layout are: (1) The two baselines should skew at a reasonable angle

Walter H. Treftz, R.L.S., is survey supervisor (control) with Geophysical Service, Inc., Survey Support Group, P.O. Box 225621 (M/S 3954), Dallas, Texas 75265. His personal address is 830 Kentucky Avenue, New Port Richey, Florida 33552.

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to each other; (2) both control stations should be on the same side of the unknown baseline; and (3) the length of the unknown baseline should be measured—if coordinates on the new baseline are desired.

An example of this situation is illustrated in Figure 1. The points with known coordinates are A and B, while the unknown baseline, with an unknown azimuth, terminates at X and Y. The instrument occupies points X and Y and angles AXB, AXY, BXY, XYA, XYB, and AYB are observed a suitable number of times. As in good survey practice, these angles should be observed an equal number of times direct and reversed; the horizon should be closed with each set and the errors adjusted. The measured length of the unknown baseline must be reduced to a horizontal distance on the same datum as the known coordinates.

The theory behind the solution to this problem is based on the fact that if two baselines are not parallel, extensions of the lines will intersect at some point. Some mathematicians will argue that even if baselines are parallel they will still meet at infinity, but the angle between two parallel lines at an infinite distance away is too small for the practical surveyor to deal with, consequently the two baselines should skew a suitable amount. To guard against the lines being parallel, or too close to it, the approximate alignment of the baseline should be laid out with a compass and designed to differ from the inversed azimuth between the two known stations. Determining the intersecting angle ($\emptyset$) between the two baseline extensions is where the rubber meets the road and is the heart of the problem.

The relationship of the numbered angles in Figure 1 are:

Angle		Relation to Other Angles
1	=	$180° - (a_1 + a_2)$
2	=	$(a_1 - b_1)$
3	=	b_1
4	=	a_2
5	=	$(b_2 - a_2)$
6	=	$180° - (b_1 + b_2)$
7	=	$b_1 + \emptyset$
8	=	$a_2 - \emptyset$

The intersecting angle between the two baseline extensions, i.e., $\emptyset$, is computed by the following equation:

$$\operatorname{Tan} \emptyset = \frac{\operatorname{Cot} a_1 + \operatorname{Cot} a_2 - \operatorname{Cot} b_1 - \operatorname{Cot} b_2}{(\operatorname{Cot} b_1 + \operatorname{Cot} a_2) - (\operatorname{Cot} a_1 - \operatorname{Cot} b_2)}$$

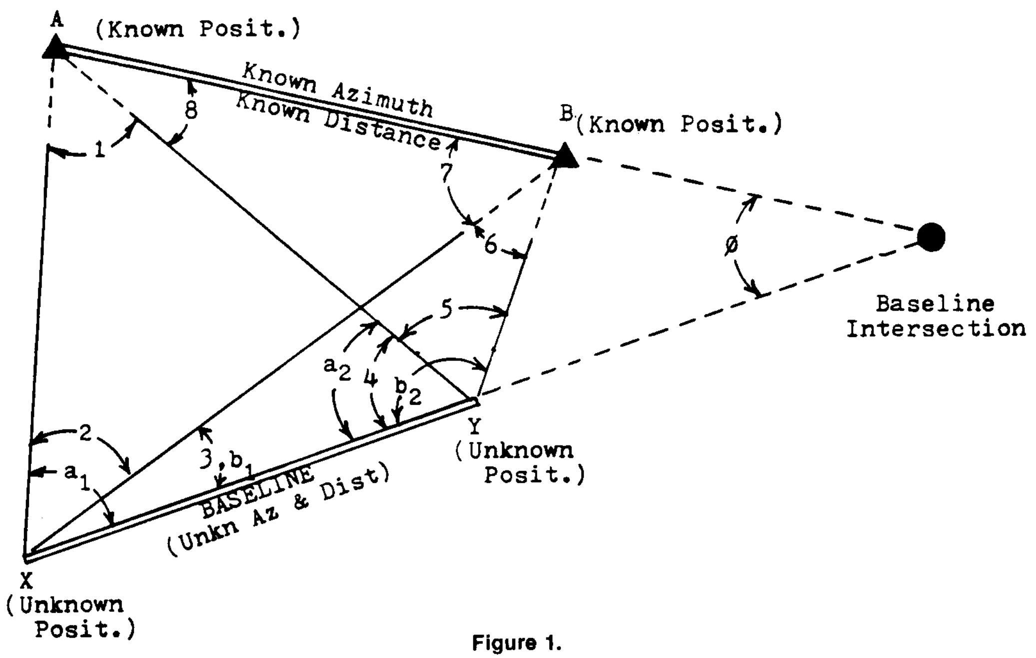

Figure 1.

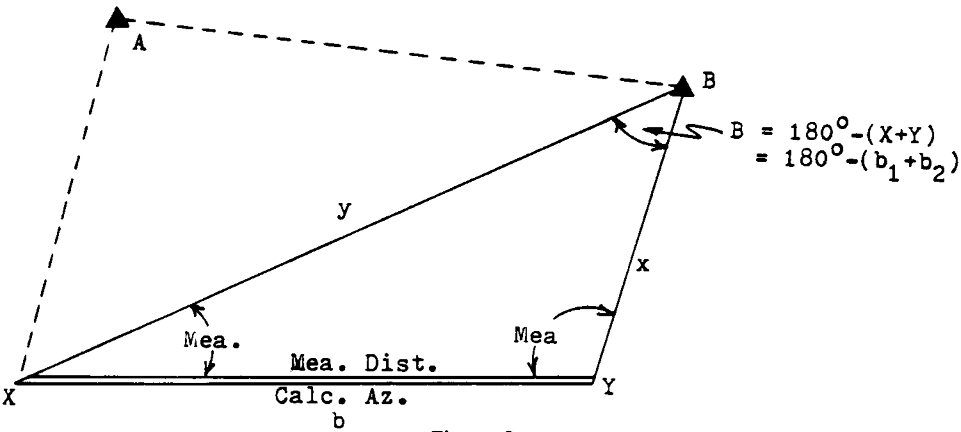

Figure 2.

Strict attention must be given to the signs of all angle functions. If the skew relationship is as shown in Figure 1, the intersection point is on the side of points B and Y, the angle $\emptyset$ will be negative ($-\emptyset$). Angle $\emptyset$ will be positive ($+\emptyset$) when the intersection is on the same side as points A and X.

Once angle $\emptyset$ has been computed, using the inversed azimuth between the known points, the azimuth of the unknown baseline can be computed. The coordinates of points X and Y can be computed by the Law of Sines, based on the known data, and is summarized as follows:

(1) Distances for sides x and y can be computed by the Law of Sines. Angle B = 180° − (X + Y).

$$x = (b \text{ Sin } X)/(\text{Sin } B)$$
$$y = (b \text{ Sin } Y)/(\text{Sin } B)$$

(2) Using the azimuth of XY obtained by applying $\emptyset$ to the azimuth of AB at the baseline intersection point, and the angles b_1 and b_2, measured at X and Y, respectively, the azimuth of XB and YB is computed. Using these azimuths and the previously computed distances for x and y, coordinates for X and Y are calculated.

(3) As shown in Figure 3, the same basic procedure is repeated for the other half of the problem based on a different set of observed angles from X and Y.

$$x' = (a \text{ Sin } x)/(\text{Sin } A)$$
$$y' = (a \text{ Sin } Y)/(\text{Sin } A)$$

(4) The coordinates of X and Y are then recomputed using the azimuth of XY, the measured angles AXY and XYA and the calculated distances for x′ and y′. If this second set of coordinates for X and Y are within acceptable agreement, the mean of the two sets should be taken. If there is an unacceptable disagreement or to strengthen the solution, the following two additional solutions should

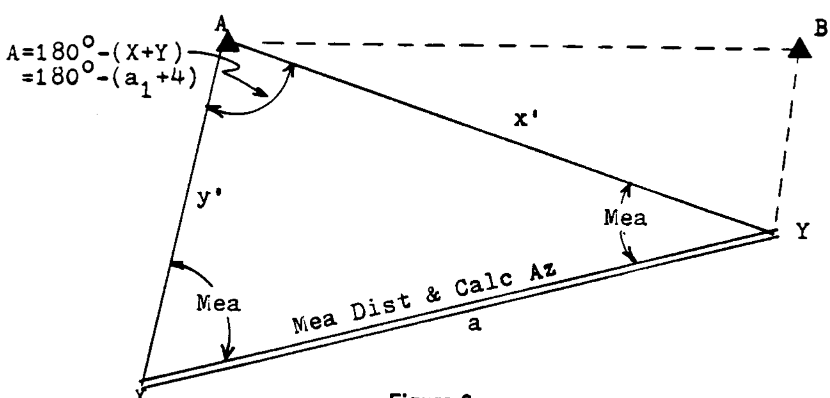

Figure 3.

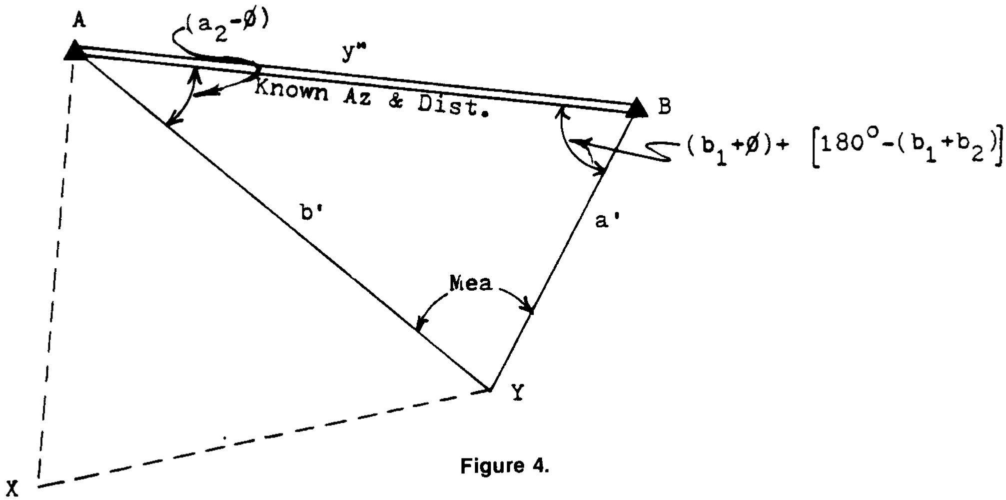

Figure 4.

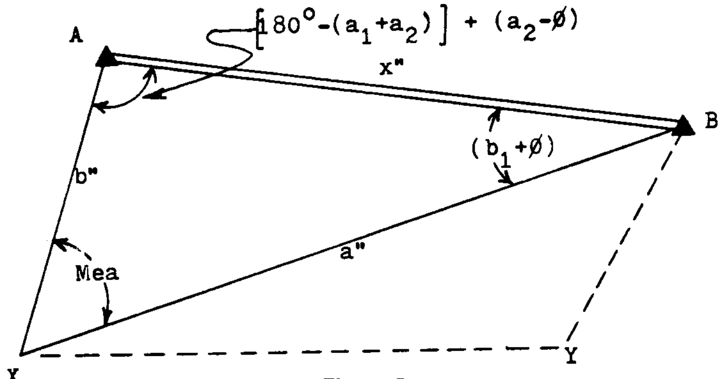

Figure 5.

be performed, as shown in Figures 4 and 5, based on the inversed azimuth and the distance between the two known stations (A and B) along with different measured angles at X and Y.

(5) Again, using the Law of Sines to solve for distances a′ and b′:

$$a' = (y'' \text{ Sin A})/(\text{Sin Y})$$
$$b' = (y'' \text{ Sin B})/(\text{Sin Y})$$

(6) Using the distances a′ and b′, the known azimuth AB, the coordinates and calculated angles of A and B, two new sets of coordinates are computed for point Y.

(7) The final two sets of coordinates for X are computed as shown in Figure 5, using the Law of Sines:

$$a'' = (x'' \text{ Sin A})/(\text{Sin X})$$
$$b'' = (x'' \text{ Sin B) 09}/(\text{Sin X})$$

When all of the conditions outlined above have been computed, there will be four sets of coordinates for points X and Y. The decision is left to the surveyor to choose between meaning all values, discarding any "sour" values, and meaning those that are within his limits of acceptability or to apply a statistical analysis to obtain the final coordinate values for X and Y.

In conclusion, it can be seen that under a given set of field conditions, this technique might make the difference between a successful or unsuccessful survey; also, it should be always kept in mind that the coordinates of X and Y can never exceed the order of accuracy of the known station (A and B) and will in all probability be one order lower than the control stations. As a result, this should not deter its use in topographic or geophysical use if targets are built over otherwise inaccessible first-order or second-order stations or if a sufficient number of repetitions in the angles are made on intersection stations—normally third-order. ■