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# Azimuth Determination by Solar Observation: New Perspectives on an Old Problem

**by Paul Boucher**

## Introduction

Solar observations have a long history in surveying. Eratosthenes determined the radius of the earth in approximatley 220 B.C., using solar observations. Surveyors today regularly observe the sun to determine azimuth, both as a starting reference and a check on field work. Dr. R. Ben Buckner in "Reasons and Methods for Accurate Directions in Land Surveying" (1975) provides a good review of the reasons for its regular use. Recent advances in timekeeping and hand-held calculators allow for the use of new solution methods. This paper will review: the general solution theory, the five possible solution techniques, derive the formulas for each, and discuss the advantages and disadvantages of each. It should be noted that the solution techniques apply equally well to star observations. The sun is used here since it is available during normal working hours and is most frequently used.

## General Solution Theory

The solution of surveying problems using astronomical observation is based on a spherical geocentric universe. This model assumes a stationary (nonrotating, nonrevolving) earth located at the center of a sphere. The sphere is of infinite radius and all celestial bodies are located on it. The sphere is used as a calculating surface for azimuth and position problems. This model dates at least to Eratosthenes. It was used by western science until the Middle Ages when it fell from favor with the work of Copernicus, Kepler, and Galileo. Although the model does not represent reality, it does provide simple mathematical solutions for azimuth and positional problems. It has, therefore, been retained in surveying and geodesy.

A spherical model of the universe allows the use of spherical geometry and trigonometry in solving azimuth and position problems. Although a general knowledge of spherical geometry and trigonometry is assumed (Ayers, 1954 (for review)), a quick review of some of the major properties is appropriate.

Spherical geometry is non-Euclidian. There are some important consequences of this. First, there are no parallel lines in spherical geometry. Any three great circles (lines) will form a triangle. There are, therefore, no intractable azimuth or positional problems. Second, the sum of the interior angles of a spherical triangle is not constant. It can vary from 180° to 360°. Third, all parts of a spherical triangle can be expressed in angular units. Arc distance of a side can be divided by the radius to yield radians. Finally, given any three parts of a spherical triangle, it is possible to solve for those remaining.

The solution of the solar azimuth problem is based on the solution of a spherical triangle (PZS). The triangle is formed by passing great circles through the celestial pole (P), the observer's zenith (Z), and the celestial body, sun, or star, (S). The celestial pole is the point at which the extension of the earth's axis intersects the celestial sphere. The observer's zenith is the point of intersection of the sphere and a radian line from the center of the earth through the observer's station. The sides are the co-altitude, co-declination, and co-latitude. The three angles are the local hour angle (t), north angle (z), and parallactic angle (p). (See Figure 2.)

Of the six parts of the PZS triangle, the parallactic angle (p) is unobservable and the north angle (z) is solved for, to determine the azimuth. This leaves four observable parts. Since only three are needed to solve the tri-

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angle, there are multiple solution techniques. The number of independent techniques can be found from combination theory

$$C_r^n = \frac{n!}{r!(n-r)!} \quad (1)$$

Four parameters taken three at a time yield four independent combinations

$$C_3^4 = \frac{4!}{3!(4-3)!} = 4 \quad (2)$$

The techniques and PZS parts used are listed in Table 1. The field data needed for each are listed in Table 2.

## Derivation of Equations

The equations used in these solution techniques are easily derived. Three of the four use either the spherical law of cosines or spherical law of sines. The fourth is somewhat more complex using the solutions for a spherical right triangle and sum of angles. The four will be derived in detail. All the derivations use Figures 1 and 2.

The altitude method uses the three sides as parameters and is solved by the law of spherical cosines (sides). Note, there is also a law of spherical cosines for angles.

Given a, b, c, find B.

Using the law of spherical cosines (sides)

$$\cos(b) = \cos(a)\cos(c) + \sin(a)\sin(c)\cos(B) \quad (3)$$

$$\cos(b) - \cos(a)\cos(c) = \sin(a)\sin(c)\cos(B) \quad (4)$$

$$\cos(B) = \frac{\cos(b) - \cos(a)\cos(c)}{\sin(a)\sin(c)} \quad (5)$$

Substituting from Figure 1

$$\cos(z) = \frac{\cos(90-\delta) - \cos(90-h)\cos(90-\phi)}{\sin(90-h)\sin(90-\phi)} \quad (6)$$

**Table 1.**

| Solution Method | Solution Technique | PZS Triangle Parts |
|---|---|---|
| Altitude | Three sides | Co-altitude<br>Co-declination<br>Co-latitude |
| Hour angle | Two sides and included angle | Co-declination<br>Co-latitude, local Hour angle |
| Non-Ephemeris | Two sides and non-includes angle | Co-altitude<br>Co-latitude<br>Local hour angle |
| Non-latitude | Two sides and non-included angle | Co-altitude<br>Co-declination<br>Local hour angle |

Reducing to first quadrant functions

$$\cos(z) = \frac{\sin(\delta) - \sin(h)\sin(\phi)}{\cos(h)\cos(\phi)} \quad (7)$$

Equation (7) is for vertical angles. If the instruments measure zenith angles the equation is (8)

$$\cos(z) = \frac{\sin(\delta) - \cos(ZA)\sin(\phi)}{\sin(ZA)\cos(\phi)} \quad (8)$$

The hour angle method uses two sides and the included angle as parameters and is solved using the law of spherical cosines (sides) and the law of spherical sines.

Given A, b, c, find B

$$\cos(b) = \cos(a)\cos(c) + \sin(a)\sin(c)\cos(B) \quad (9)$$

$$\cos(B) = \frac{\cos(b) - \cos(a)\cos(c)}{\sin(a)\sin(c)} \quad (10)$$

Using the law of spherical cosines (sides) to solve for cos(a)

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A) \quad (11)$$

**Table 2.**

| Field Data | Solution Method<br>Latitude | Hour Angle | Non-Ephemeris | Non-Latitude |
|---|---|---|---|---|
| Horizontal angle | X | X | X | X |
| Vertical angle | X | | X | X |
| Time of Observation | X | X | X | X |
| Data and year | X | X | X | X |
| Atmospheric pressure | X | | X | X |
| Atmospheric temperature | X | | X | X |
| Station latitude | X | X | X | |
| Station longitude | | X | X | X |

Substituting (11) into (10)

$$\cos(B) = \frac{\cos(b) - (\cos(b)\cos(c) + \sin(b)\sin(c)\cos(A))\cos(c)}{\sin(a)\sin(c)} \qquad (12)$$

Solving for sin(a), using the law of spherical sines

$$\frac{\sin(a)}{\sin(A)} = \frac{\sin(b)}{\sin(B)} \qquad (13)$$

$$\sin(a) = \frac{\sin(A)\sin(b)}{\sin(B)} \qquad (14)$$

Substituting (14) into (12)

$$\cos(B) = \frac{\cos(b) - (\cos(b)\cos(c) + \sin(b)\sin(c)\,\mathrm{Cos}(A))\cos(C)}{\frac{\mathrm{Sin}(A)\sin(b)}{\sin(B)}\sin(C)} \qquad (15)$$

Multiplying both sides by 1/sin(B)

$$\frac{\cos(B)}{} = \frac{\frac{\cos(b) - (\cos(b)\cos(c) + \sin(b)\sin(C)\cos(A))\cos(c)}{\sin(B)}}{\frac{\sin(A)\sin(b)\sin(c)}{\sin(B)}} \qquad (16)$$

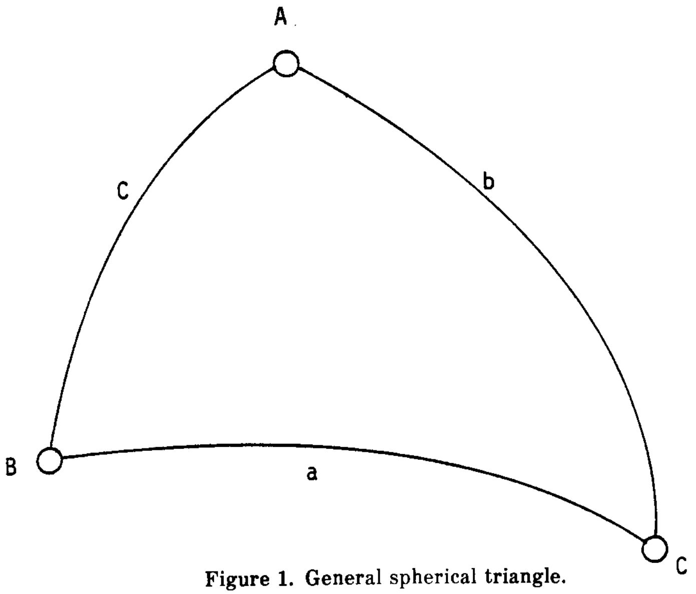


**Figure 1.** General spherical triangle.

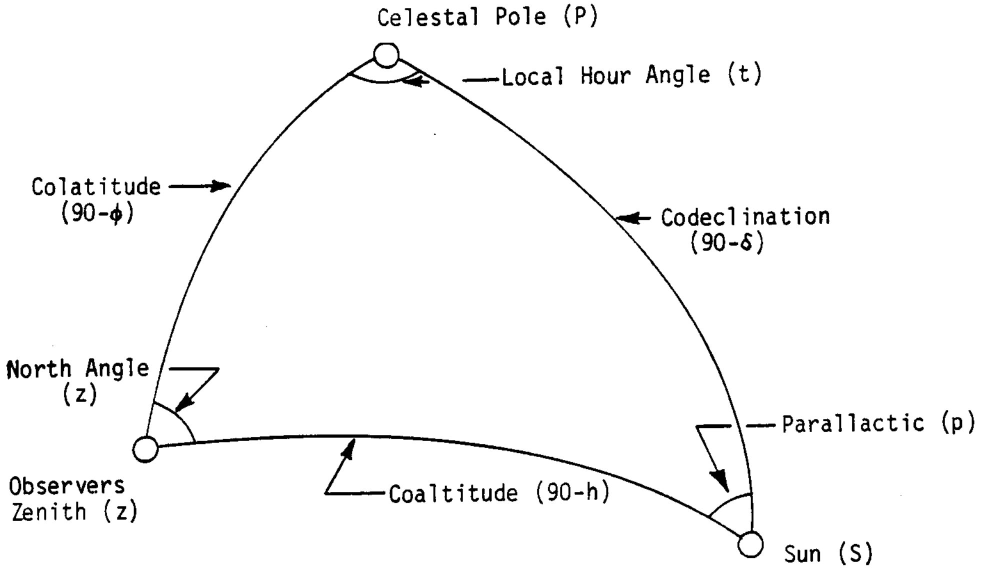


**Figure 2.** PZS triangle.

Combining and reducing terms

$$\frac{\cos(B)}{\sin(B)} = \frac{\cos(b) - \cos(b)\cos^2(c) - \sin(b)\sin(c)\cos(c)\cos(A)}{\sin(a)\sin(b)\sin(c)} \quad (17)$$

Factoring

$$\frac{\cos(B)}{\sin(B)} = \frac{(1 - \cos^2(c))\cos(b) - \sin(b)\sin(c)\cos(c)\cos(A)}{\sin(A)\sin(b)\sin(c)} \quad (18)$$

Substituting $\sin^2(x) = 1 - \cos^2(x)$

$$\frac{\cos(B)}{\sin(B)} = \frac{\sin^2(c)\cos(b) - \sin(b)\sin(c)\cos(c)\cos(A)}{\sin(a)\sin(b)\sin(c)} \quad (19)$$

Dividing sin(c) out

$$\frac{\cos(B)}{\sin(B)} = \frac{\sin(C)\cos(b) - \sin(b)\cos(c)\cos(A)}{\sin(b)\sin(c)} \quad (20)$$

Multiplying left side by 1/sin(b)/1/sin(b)

$$\frac{\cos(B)}{\sin(B)} = \frac{\sin(c)\frac{\cos(b)}{\sin(b)} - \cos(c)\cos(A)}{\frac{\sin(a)\sin(b)}{\sin(b)}} \quad (21)$$

Inverting and reducing terms

$$\mathrm{Tan}(B) = \frac{\sin(A)}{\sin(c)\cot(b) - \cos(c)\cos(A)} \quad (22)$$

Substituting from Figure 1

$$\mathrm{Tan}(Z) = \frac{\sin(t)}{\sin(90-\phi)\cot(90-\delta) - \cos(90-\phi)\cos(t)} \quad (23)$$

Reducing to first quadrant function

$$\mathrm{Tan}(z) = \frac{\sin(t)}{\cos(\phi)\mathrm{Tan}(\delta) - \sin(\phi)\cos(t)} \quad (24)$$

The non-ephemeris method uses two sides and the non-included angles as parameters. It uses an indirect method of solving the PZS triangle. A right spherical triangle can be formed by passing the proper great circle through any other two great circles. A great circle is passed through the observer's zenith (Z), so it intersects the sun's meridian (of which the co-declination side is part), to form two right spherical triangles. (See Figure 3.) The altitude of the right triangles (h′) can be found using the formulas for solving right spherical triangles. There are ten formulas and their derivation can be found in Ayres (1954). The north angle (z) is either the sum or difference of angles X and y.

The needed formulas for solving the right spherical triangles are:

$$\mathrm{Cot}(X) = \cos(c)\,\mathrm{Tan}(A) \quad (25)$$

$$\mathrm{Cos}(Y) = \mathrm{Tan}(h')\,\mathrm{Cot}(a) \quad (26)$$

$$\mathrm{Sin}(h') = \mathrm{Sin}(c)\,\mathrm{Sin}(A) \quad (27)$$

$$B = Y \pm Y \quad (28)$$

Substituting from Figure 1

$$\mathrm{Cot}(X) = \mathrm{Cos}(90-\phi)\,\mathrm{Tan}(t) \quad (29)$$

$$\mathrm{Cos}(Y) = \mathrm{Tan}(h')\,\mathrm{Cot}(90-h) \quad (30)$$

$$\mathrm{Sin}(h') = \mathrm{Sin}(90-\phi)\sin(t) \quad (31)$$

Reducing to first quadrant functions

$$\mathrm{Cot}(X) = \sin(\phi)\,\mathrm{Tan}(t) \quad (32)$$

$$\mathrm{Cos}(Y) = \mathrm{Tan}(h')\,\mathrm{Tan}(h) \quad (33)$$

$$\mathrm{Sin}(h') = \mathrm{Cos}(\phi)\,\mathrm{Sin}(t) \quad (34)$$

$$z = X \pm Y \quad (35)$$

Equations (32), (33), (34), are for vertical angles. If the instrument measures zenith angles, the equations are (36), (37), and (38).

$$\mathrm{Cot}(x) = \sin\phi\,\tan(t) \quad (36)$$

$$\mathrm{Cos}(y) = \mathrm{Tan}(h')\,\mathrm{Cot}(za) \quad (37)$$

$$\mathrm{Sin}(h') = \mathrm{Cos}(\phi)\,\mathrm{Sin}(t) \quad (38)$$

This method has two possible solutions. For $\phi > \delta$, the solution is X + Y. This is the most common case in North America. For $\phi > \delta$, the solution is X – Y. A rare but possible solution is $\phi = \delta$. For $\phi = \delta$, the solution can be either X + Y or X – Y. The solution depends on the value of $\phi$, $\delta$, and t. This method should not be used when $\phi$ and $\delta$ are equal or nearly so.

The non-latitude method uses two sides and the non-included angle. The solution uses the spherical law of sines and is computationally the simplest solution.

Given A, a, b, find B.

Using the law of spherical sines

$$\frac{\sin(a)}{\sin(A)} = \frac{\sin(b)}{\sin(B)} \quad (39)$$

$$\mathrm{sub}(B) = \frac{\sin(b)\sin(A)}{\sin(a)} \quad (40)$$

Substituting from Figure 1

$$\mathrm{Sin}(z) = \frac{\sin(90-\delta)\sin(t)}{\sin(90-h)} \quad (41)$$

Reducing to first quadrant functions

$$\mathrm{Sin}(z) = \frac{\cos(\delta)\sin(t)}{\cos(h)} \quad (42)$$

Equation (42) is for vertical angles. If the instruments measure zenith angles, the equation is (43).

$$\mathrm{Sin}(z) = \frac{\cos\delta\,\sin(t)}{\sin(ZA)} \quad (43)$$

The final technique is the special case of meridian transit ($z = 0$). Meridian transit occurs when the sun crosses the observer's meridian. The co-declination and co-latitude coincide and the PZS triangle does not exist. By precalculating the time of transit and tracking the sun until the moment of transit, the instrument will be aligned with the observer's meridian, either north or south. The time of transit can be calculated by:

$$\text{Time of transit} = \lambda/15 + 12 + \text{equation of time.} \quad (44)$$

The answer will be in hours, minutes, and seconds U.T. The equation of time can be found in an ephemeris and is explained in Kissam (1956) or Mueller (1969).

The main drawback with the solution is that it is non-repeatable. It provides no check for blunders and does not eliminate instrumental errors. It does provide an easy check on field work if a setup occurs near local noon. The solution is undefined for $\phi = \delta$, since a horizontal angle cannot be measured in a vertical plane.

The field data needed for each method is listed in Table 2. By obtaining the data needed for the altitude solution plus the observer's longitude, it is possible to use any of the four triangular methods. A standard field procedure eliminates confusion on the data to be gathered and allows for error analysis by using multiple solution techniques.

As noted above, a minimum of three parameters is needed to define a spherical triangle. Since optical instruments measure only one parameter of z, altitude (h), it is necessary to obtain the others, local hour angle (t), declination ($\delta$), Latitude ($\phi$), by indirect methods. Time of observation provides two, local hour angle and declination. A look at the importance of time of observation is appropriate.

The path of the sun is assumed to be a great circle. The position of the sun can be defined by two angular coordinates, hour angle and declination. (See Figure 4.) The sun's path on the celestial sphere, however, is not constant. The sun's great circle oscillates between $+23.5°$ declination over the course of a year. This results from the fact that the earth's equatorial plane is inclined to its orbital plane. Remember, the spherical universe model describes relative motions, not reality.

By reducing the sun's motion into two components, one in the plane of the celestial equator and one in declination, it is possible to describe the motion and position of the sun

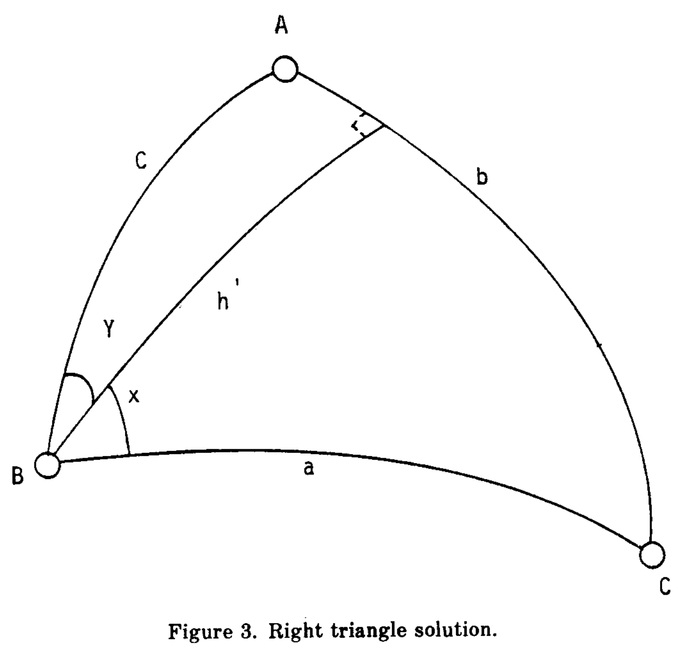


Figure 3. Right triangle solution.

as a function of time of observation (date and year are part of the time observation). The solar declination table in an ephemeris is a listing of the two coordinates for a given year. By fixing the observer's position (latitude and longitude), it is possible to solve for local hour angle (t), north angle (z), and solar altitude (h). The minimum solution appears to involve only two independent parameters (time of observation and observer's position), not the three mentioned above.

The minimum solution still uses three parameters, but there are more than six parts to a spherical triangle. The coordinates of the vertices are also parts of the PZS triangle. In order to solve the solar azimuth problem completely, it is necessary to fix the orientation of the PZS triangle on the celestial sphere. Fixing two vertices still allows for two solutions (A.M., P.M.). Only by fixing all vertices is the problem uniquely defined.

It is obvious that the field solution technique differs from the theoretical ones above. The positions of the vertices are fixed and coordinate geometry is used to determine the values of the parameters used in the triangle solution formulas. Since fixing the position of the vertices completely defines the PZS triangle, any measurement of the sides or angles is therefore redundant. This explains the reports of differing values of z obtained from the altitude and hour angle methods using the same data set. The hour angle method uses the minimum data necessary (coordinates of P, Z, S). The altitude method adds the solar altitude (h), which is a redundant measurement. The difference in z is a measure of the relative error in local hour angle (t) and solar altitude (h).

By way of explanation, the spherical triangle methods require a closed figure. Errors in the measurement of the parameters usually will not produce a closed figure. The altitude method forces a closed figure by not fixing the angles. The hour angle method allows the co-altitude to absorb the "error of closure." The non-emphasis and non-latitude methods put the "error of closure" in the co-declination and co-latitude, respectively.

This non-independence or more precisely, redundancy of data, does not mean that the solution techniques cannot or should not be used. It does mean, however, the choice of techniques should be based on an analysis of the error expected in the field data.

## Advantages and Disadvantages

The altitude method is the traditional method

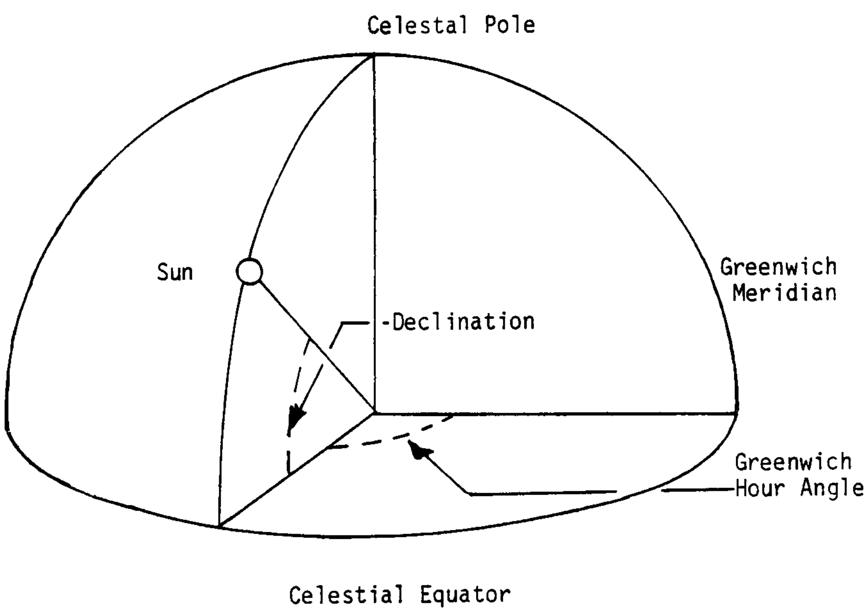


Figure 4. Celestial coordinator system.

of choice. The main advantage is the insensitivity to timing errors. In the United States to cause a 1 sec. error in z, acceptable timing errors range from 20 sec. to 1 hr., depending on date and latitude. (Smith, 1978) Timing errors affect the north angle (z) through the declination. The rate of change of the declination is slow (58′/hr. maximum), so a relatively large error in time of observation has little effect on the declination ($\delta$) and north angle (z).

The disadvantages are in the use of the solar altitude. For a given set of observations the solar altitude is the only parameter which changes appreciably. It is the main source of error.

The north angle (z) is obtained through the arc cosine function (equations (7) and (8)). The rate of change of the cosine is large, near 180° and 0°. These are the possible values of z at local noon (meridian transit). As a result, near local noon, a small error in the solar altitude (h) will cause a large error in the north angle (z). The recommended standard is not to observer from 2 hr. before till 2 hr. after local noon.

As noted above, the sun has velocities relative to the observer's horizontal and vertical planes. These velocities are relatively high and vary during the day. The need for simultaneous horizontal and vertical pointings increases the likelihood of error. Since the sun is a moving target, it is not possible to eliminate instrumental errors (i.e., collimation errors) by direct, reverse pointings.

The solar altitude must be corrected for vertical refraction. Refraction correction is based on Snell's Law and assumes a constant density medium. It is only an approximation. Refraction correction is a function of the vertical angle and will tend to increase errors in the measurement of h. The refraction correction varies from 6 min. 53 sec. at 7° to 0 at 90°. The recommended minimum solar altitude is 20° for observation because of the magnitude and rate of change of the refraction correction.

Combining the two restrictions, minimum altitude and local noon, it is not possible to use the altitude method during certain periods of the year. At a latitude of 40°, it is not possible to meet both criteria on the winter solstice. At 45° it is not possible to meet both from approximately November 15 till January 25. At a latitude of 35° the solar altitude does not reach 20° 3 hr. before local noon (allowing a 1-hr. work period) between approximately November 22 and January 21. At 45° latitude the period runs from October 25 till February 18. The altitude method is of limited value during these periods.

The hour angle method is the modern method of choice. It is the minimum solution technique (no redundant measurements). There are no morning or afternoon restrictions and it can be used throughout the year, since it does not use the solar altitude. The local noon restriction still applies, but the error is reported by Cothren (1982) to be less than 1 min. of arc, so observation near local noon could be used to check field work.

The main disadvantage is its sensitivity to timing errors. An error of 1 sec. in time of observation will cause an error in the north angle (z) of from 6 to 14 sec. of arc depending on date and latitude (Smith, 1978). Quartz stop watches provide 1/100-sec. readout. Portable shortwave radios provide accurate time reference. Time of observation can now be obtained to accuracies of 0.1 sec. or better at a low cost.

The non-ephemeris method does not use the sun's declination. It is advantageous on those days the ephemeris is left in the office or at the beginning of the year when the new ephemeris is not yet in house. It also overcomes the problem of low precision ephemeris. A 0.1-min. ephemeris is not mathematically compatible with good horizontal control (i.e., second or third order), 1 sec. instruments and accurate time, (0.1 sec. or better). This method eliminates the need for a second more precise ephemeris (i.e., BLM ephemeris).

The disadvantages are the same as the altitude method and the same restrictions apply. In addition, it requires accurate timekeeping, like the hour angle method. It is also computationally the most complex.

The non-latitude method has two advantages. First, it is computationally the simplest. It can be done quickly on a non-programmable calculator. Second, it uses the sine function for the local hour angle (t) and the north angle (z), (equation (41)). Near local noon, these parameters approach 0 and/or 180°. The rate of change of the sine function is a minimum at these values. An error of 1 sec. in time of observation 15 min. before local noon causes an error in the north angle (z) of from 1 to 4 sec. of arc, depending on date at 40° latitude. At 30 min. the error is 2 sec. of arc or less.

In addition, errors in the solar altitude will be minimized. Near local noon, the sun's vertical velocity will be minimum, which makes tracking and pointing easier. It is also highest in the sky, minimizing refraction.

The disadvantages are the same as the non-ephemeris method.

## Conclusions

1. Modern equipment makes it possible to solve the solar azimuth problem five distinct ways.

2. The field solution techniques are modifications of the theoretical one and are not totally independent of each other.

3. Choice of solution techniques should be based on date and time of day plus an analysis of errors to be expected in the field data.

4. Given accurate timekeeping, the hour angle method is the accurate and usable solution.

5. The non-latitude method allows for observation close to local noon (±15 min. of noon). Errors in the north angle (z) are less than 5 sec. of arc.

6. By obtaining the field data for the altitude method plus station latitude, it is possible to solve for the solar azimuth by all four spherical triangle techniques. This allows for analysis of errors in the field data.

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## LIST OF ABBREVIATIONS

| | |
|---|---|
| cos | cosine function |
| cot | co tangent function |
| sin | sine function |
| h | solar altitude |
| δ | solar declination |
| φ | latitude |
| λ | longitude |
| P | celestial pole |
| p | parallactic angle |
| S | sun |
| t | local hour angle |
| U.T. | Universal time |
| Z | observer's zenith |
| z | north angle, solar azimuth |
| ZA | zenith angle |

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are largely overkills. Developing specifications, once the standard(s) is established for use at the local level is not a tremendous chore; deciding on the cadastral standards might cause problems however, i.e., in some places 1:20,000 will fill the bill, in other parts of the country something between 1:2500 and 1:20,000 could satisfy the needs. A manual will take considerable more effort.

I think a start might be for NSPS and AAGS (or NSPS alone) to get people really interested in the subject together and get something started. Unfortunately, people really interested are rarely major participants in such things. Having someone such as yourself pushing and getting directly involved is half the battle. Good luck!

Sincerely, /s/ Joseph F. Dracup

## Re: "Azimuth Determination by Solar Observation: New Perspectives on an Old Problem," by Paul Boucher

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**From: Prof. Rafael N. Sanchez, 823 Moreau, Ste-Foy, Quebec GIV 3BS [professor at University Laval, Quebec, Canada].** According to the author the solution of survey problems using astronomical observations and computations is only a practical one because it allows the use of spherical trigonometry. He says that reality is not modeled by an earth-centered sphere. In his opinion, the spherical solutions suppose a non-rotating, non-orbiting earth.

When, as a geodesist, I ask for the apparent right ascension and declination of a star I require that the effects of annual aberration and parallax be taken into account, which means to accept the Copernican point of view. Afterward I feel free to use these data in a spherical computation because it is a legal and exact means of solving for some elements of a trihedral angle when three of them are known. Reality is that trihedral angle "en tout cas!". It is defined by the direction of the axis of the earth, and those of the principal axis of the theodolite and the line aimed to a star at a given instant.

Things are slightly more complicated in the case of zenith distance observations and a fourth direction is implied in the azimuth of a line problem, indeed.

The use of the word "position" for latitude and longitude, a nautical tradition, is very bad for surveying and geodesy. Satellite Doppler gives position. Latitude and longitude are only—and not less than—the spatial direction of the plumb line.

## Corrigendum

In the article "Geometrical Parameters of the Geodetic Reference System 1980," by Earl F. Burkholder, appearing in *Surveying and Mapping*, Vol. 44, No 4, December 1984, p. 340, right column, equation (10) is in error. The parenthesis, which appeared at the end of the first line of the equation was extraneous and would make no sense. The author and the editor are certain that anyone using the equation would note there was no opening parenthesis to complete the pair; however, the equation is shown here in its correct form.

$$R_2 = [a(1-e^2)^{1/2}/\sqrt{2}][1/(1-e^2) + (1/2e)\ln\{(1+e)/(1-e)\}]^{1/2}. \quad (10)$$

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