

# TRAVERSING AND TRAVERSE ADJUSTMENT

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## COMPUTING TRUE VALUE

- True value is recorded value plus the correction:

$$T = R + C$$

– Always added to the reading

- Error:

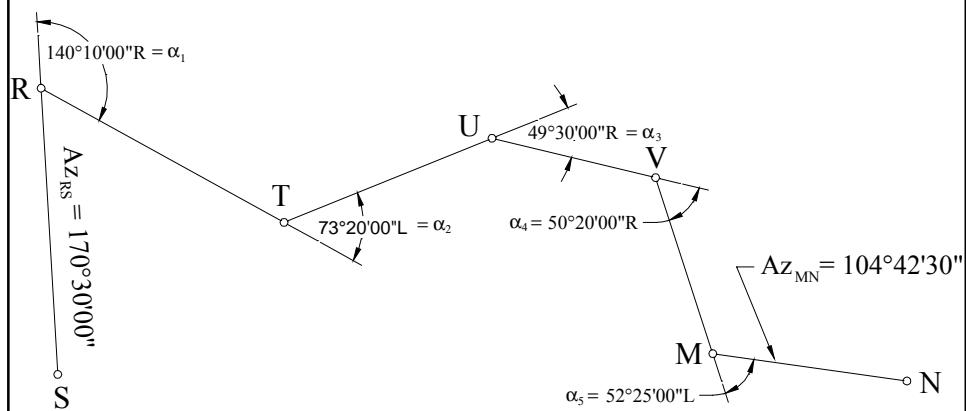
$$E = R - T$$

- Always subtracted from the reading

# TRAVERSES

- Open v. closed traverses
- Open loop v. closed loop traverses
- Types of angles measured
  - Deflection angles
  - Interior angles of traverse
  - Exterior angles of traverse
  - Angles to the right

## DEFLECTION ANGLE TRAVERSE



$$Az_1 + \sum_{i=1}^n \alpha_{R_i} - \sum_{i=1}^n \alpha_{L_i} - Az_2 - 360^{\circ} = 0$$

$$\begin{aligned}\sum \alpha_R &= \alpha_1 + \alpha_3 + \alpha_4 \\ &= (140^\circ 10' 00'') + (49^\circ 30' 00'') + (50^\circ 20' 00'') \\ &= 240^\circ 00' 00''\end{aligned}$$

$$\begin{aligned}\sum \alpha_L &= \alpha_2 + \alpha_5 \\ &= (73^\circ 20' 00'') + (52^\circ 25' 00'') \\ &= 125^\circ 45' 00''\end{aligned}$$

$$Az_{S-R} + \sum \alpha_R - \sum \alpha_L - Az_{M-N} - 360^\circ = E_{\angle}$$

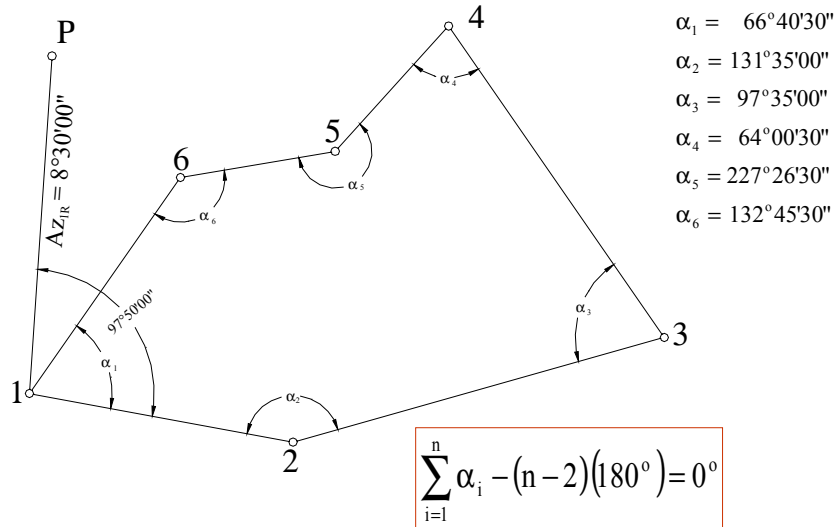
$$(350^\circ 30' 00'') + (240^\circ 00' 00'') - (125^\circ 45' 00'') - (104^\circ 42' 30'') - 360^\circ = +0^\circ 02' 30''$$

$$cpa = -\frac{E_{\angle}}{n} = -\frac{0^\circ 02' 30''}{5} = -30''$$

$\alpha_1$	=	+140° 10' 00"	- 30"	= +140° 09' 30"
$\alpha_2$	=	- 73° 20' 00"	- 30"	= - 73° 20' 30"
$\alpha_3$	=	+ 49° 30' 00"	- 30"	= + 49° 29' 30"
$\alpha_4$	=	+ 50° 20' 00"	- 30"	= + 50° 19' 30"
$\alpha_5$	=	- 52° 25' 00"	- 30"	= - <u>52° 25' 30"</u>
				+114° 12' 30"

$$(350^\circ 30' 00'') + (114^\circ 12' 30'') - (104^\circ 42' 30'') - 360^\circ = 0^\circ \quad \checkmark$$

## INTERIOR ANGLE TRAVERSE



$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 - (6-2)(180^\circ) = E_\angle$$

$$(720^\circ 03' 00'') - (720^\circ) = +3' 00''$$

The correction per angle (cpa) is next determined.

$$\text{cpa} = -\frac{30' 00''}{6} = -30''$$

The corrected angles are found by adding the cpa to each observation.

$$\alpha_1 = (66^\circ 40' 30'') + (-30'') = 66^\circ 40' 00''$$

$$\alpha_2 = (131^\circ 35' 00'') + (-30'') = 131^\circ 34' 30''$$

$$\alpha_3 = (97^\circ 35' 00'') + (-30'') = 97^\circ 34' 30''$$

$$\alpha_4 = (64^\circ 00' 30'') + (-30'') = 64^\circ 00' 00''$$

$$\alpha_5 = (227^\circ 26' 30'') + (-30'') = 227^\circ 26' 00''$$

$$\alpha_6 = (132^\circ 45' 30'') + (-30'') = 132^\circ 45' 00''$$

$$\sum \alpha_i = 720^\circ 00' 00''$$

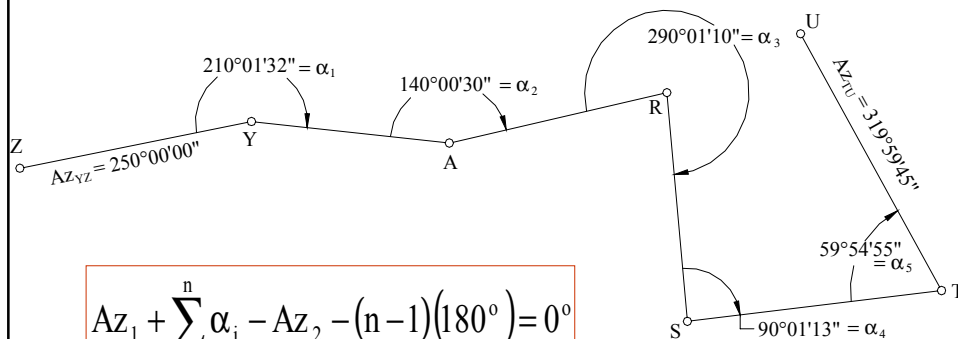
## EXTERIOR ANGLE TRAVERSE

- Special case of interior angle traverse

$$\sum_{i=1}^n \alpha_{i_{\text{Ext}}} - (n + 2)(180^\circ) = 0^\circ$$

- Note that interior and exterior angle traverses are closed loop traverses.

## ANGLES TO THE RIGHT



$$Az_1 + \sum_{i=1}^n \alpha_i - Az_2 - (n - 1)(180^\circ) = 0^\circ$$

From the example, the angular error of closure is found as

$$Az_{YZ} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 - Az_{TU} - (5-1)(180^\circ) = E_{\angle}$$

$$(250^\circ 00' 00'') + (789^\circ 59' 20'') - (319^\circ 59' 45'') - 720^\circ = -00' 25''$$

The correction per angle is

$$cpa = -\frac{25''}{5} = 5''$$

The corrected angles are found by adding the cpa to each measurement.

$$\alpha_1 = (210^\circ 01' 32'') + 5'' = 210^\circ 01' 37''$$

$$\alpha_2 = (140^\circ 00' 30'') + 5'' = 140^\circ 00' 35''$$

$$\alpha_3 = (290^\circ 01' 10'') + 5'' = 290^\circ 01' 15''$$

$$\alpha_4 = (90^\circ 01' 13'') + 5'' = 90^\circ 01' 18''$$

$$\alpha_5 = (59^\circ 54' 55'') + 5'' = \underline{59^\circ 55' 00''}$$

$$\sum \alpha_i = 789^\circ 59' 45''$$

Check using the general relationship for the angles to the right.

$$(250^\circ 00' 00'') + (789^\circ 59' 45'') - (319^\circ 59' 45'') - 720^\circ = 0^\circ \quad \checkmark$$

## STEPS IN TRAVERSE ADJUSTMENT

1. Balance/adjust angles
2. Compute azimuths/bearings of lines
3. Compute latitudes and departures

$$\text{Lat} = s \cdot \cos \alpha = \Delta Y$$

$$\text{Dep} = s \cdot \sin \alpha = \Delta X$$

4. Determine closure error

$$e_D = dX = \sum_{i=1}^{n-1} \text{Lat}_{i,i+1} - (X_n - X_1)$$

$$e_L = dY = \sum_{i=1}^{n-1} \text{Dep}_{i,i+1} - (Y_n - Y_1)$$

## STEPS IN TRAVERSE ADJUSTMENT

5. Compute linear error of closure, accuracy ratio, direction of closing line

$$E_c = \sqrt{e_D^2 + e_L^2}$$

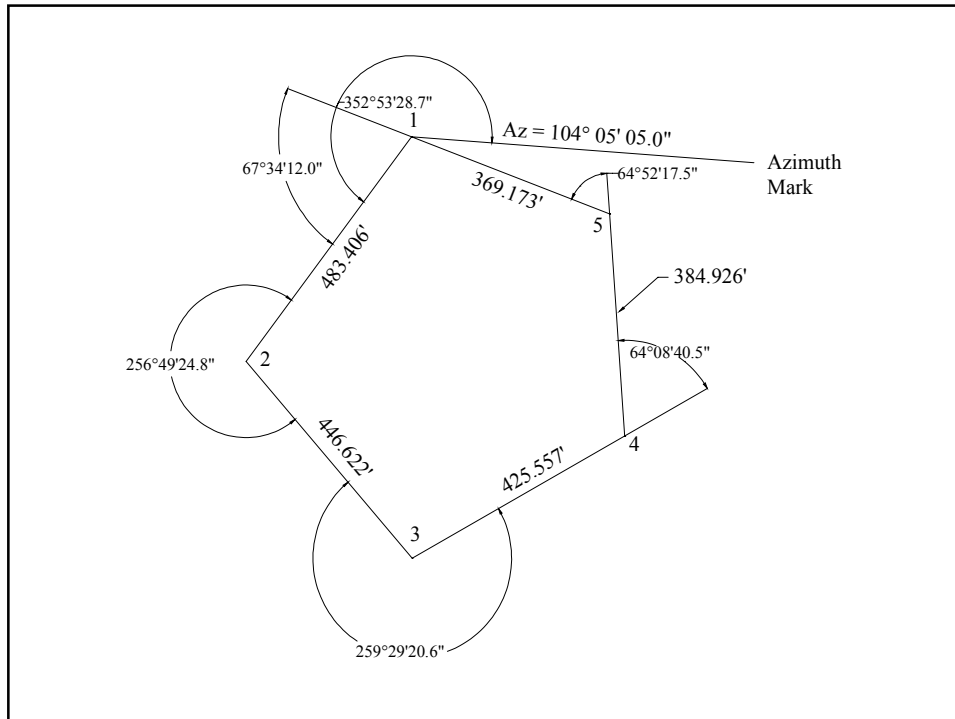
$$\text{Accuracy} = E_c / P$$

$$\beta_c = \tan^{-1} \left( \frac{e_D}{e_L} \right)$$

6. Adjust traverse and compute adjusted distances, directions and coordinates
7. Compute area of traverse

# EXAMPLE

Station	Angle	Distance
Az. Mark		
1	-67° 34' 12.0" D	483.406'
2	256° 49' 24.8" L	446.622'
3	259° 29' 20.6" L	425.557'
4	-64° 08' 40.5" D	384.926'
5	-64° 52' 17.5" D	369.173'
1	352° 53' 28.7" R	
Az. Mark		



Azimuth	1 ⇒ Az. Mark	104° 05' 05.0"
	+180°	180° 00' 00.0"
	Back Azimuth	284° 05' 05.0"
	-∠ <sub>1</sub>	-67° 34' 12.0"
Azimuth	1 - 2	216° 30' 53.0"
	-∠ <sub>2</sub>	-256° 49' 24.8"
		-40° 18' 31.8"
	+ 180°	180° 00' 00.0"
Azimuth	2 - 3	139° 41' 28.2"
	+ 180°	180° 00' 00.0"
	-∠ <sub>3</sub>	-259° 29' 20.6"
Azimuth	3 - 4	60° 12' 07.6"
	-∠ <sub>4</sub>	-64° 08' 40.5"
	+ 360°	360° 00' 00.0"
Azimuth	4 - 5	356° 03' 27.1"
	-∠ <sub>5</sub>	-64° 52' 17.5"
Azimuth	5 - 1	291° 11' 09.6"
	-180°	-180° 00' 00.0"
	+ ∠ to Az. Mark	356° 53' 28.7"
	-360°	-360° 00' 00.0"
Azimuth	1 - Az. Mark	104° 04' 38.2"

The angular error of closure is

$$\begin{aligned}
 E_{\angle} &= (\text{Az. } 1 \rightarrow \text{Az Mark}_{\text{Calc}}) - (\text{Az. } 1 \rightarrow \text{Az Mark}_{\text{True}}) \\
 &= (104^{\circ} 04' 38.3'') - (104^{\circ} 05' 05.0'') \\
 &= -26.7''
 \end{aligned}$$

The correction per angle is found using

$$\text{cpa} = -\frac{E_{\angle}}{n} = -\frac{-26.7''}{6} = 4.45''$$

## ADJUSTED AZIMUTHS

Line	Preliminary Azimuth	Correction	Adjusted Azimuth
1-2	216° 30' 53.0"	+04.5"	216° 30' 57.5"
2-3	139° 41' 28.2"	+08.9"	139° 41' 37.1"
3-4	60° 12' 07.6"	+13.4"	60° 12' 21.0"
4-5	356° 03' 27.1"	+17.8"	356° 03' 44.9"
5-1	291° 11' 09.6"	+22.3"	291° 11' 31.9"
1-Az Mk	104° 04' 38.3"	+26.7"	104° 05' 05.0"

## LATITUDES AND DEPARTURES

Traverse Adjustment Program						
		Azimuth				
Sta	Dist	Deg	Min	Sec	Departure	Latitude
1						
	483.406	216	30	57.5	-287.649	-388.509
2						
	446.622	139	41	37.1	288.908	-340.592
3						
	425.557	60	12	21	369.305	211.453
4						
	384.926	356	3	44.9	-26.432	384.017
5						
	369.173	291	11	31.9	-344.207	133.455
1						
	2109.684				-0.075	-0.176

Error in closure:

$$e_D = dX = \sum_{i=1}^{n-1} \text{Lat}_{i,i+1} = -0.075'$$

$$e_L = dY = \sum_{i=1}^{n-1} \text{Dep}_{i,i+1} = -0.176'$$

Linear error of closure

$$E_c = \sqrt{(-0.075)^2 + (-0.176)^2} = 0.19$$

Azimuth of the closing line

$$\beta_c = \tan^{-1}\left(\frac{-0.075'}{-0.176'}\right) = 203^\circ 04' 50''$$

Accuracy ratio

$$\text{Accuracy} = \frac{0.191'}{2109.684} = \frac{1}{11,045}$$
$$\approx 1 : 11,000$$

## COMPASS RULE ADJUSTMENT

$$c_L = \left(\frac{L}{P}\right)cl_L$$

$$c_D = \left(\frac{L}{P}\right)cl_D$$

where:  $c_L, c_D$  = corrections for latitudes and departures respectively,  
 $L$  = length of the line,  
 $P$  = perimeter distance around the traverse, and  
 $cl_L, cl_D$  = closure correction in latitude and departure, respectively, for the traverse.

From the example,  
For line 1-2:

$$c_L = \left( \frac{0.176'}{2109.684'} \right) 483.406' = +0.040'$$

$$c_D = \left( \frac{0.075'}{2109.684'} \right) 483.406' = +0.017'$$

Line 2-3:

$$c_L = \left( \frac{0.176'}{2109.684'} \right) 446.622' = +0.037'$$

$$c_D = \left( \frac{0.075'}{2109.684'} \right) 446.622' = +0.016'$$

Traverse Adjustment Program						
Sta	Departure	Latitude	Corrections to		Corrected	Corrected
			Departure	Latitude	Departure	Latitude
1	-287.649	-388.509	0.017	0.040	-287.632	-388.469
2	288.908	-340.592	0.016	0.037	288.924	-340.555
3	369.305	211.453	0.015	0.035	369.320	211.489
4	-26.432	384.017	0.014	0.032	-26.419	384.049
5	-344.207	133.455	0.013	0.031	-344.194	133.486
1	-0.075	-0.176	0.075	0.176	0.000	0.000

## DETERMINE COORDINATES

$$X_{n+1} = X_n + \text{Dep}_{n-(n+1)}$$

$$Y_{n+1} = Y_n + \text{Lat}_{n-(n+1)}$$

Traverse Adjustment Program				
	Corrected	Corrected		
Sta	Departure	Latitude	X	Y
1			7885.572	7097.635
	-287.632	-388.469		
2			7597.940	6709.166
	288.924	-340.555		
3			7886.864	6368.611
	369.320	211.489		
4			8256.185	6580.100
	-26.419	384.049		
5			8229.766	6964.149
	-344.194	133.486		
1			7885.572	7097.635
	0.000	0.000		

## COMPUTE ADJUSTED DISTANCES AND DIRECTIONS

$$D_{n-(n+1)} = \sqrt{(X_{n+1} - X_n)^2 + (Y_{n+1} - Y_n)^2}$$

$$= \sqrt{\text{Dep}_{n-(n+1)}^2 + \text{Lat}_{n-(n+1)}^2}$$

$$\alpha_{n-(n+1)} = \tan^{-1} \left[ \frac{X_{n+1} - X_n}{Y_{n+1} - Y_n} \right] = \tan^{-1} \left[ \frac{\text{Dep}_{n-(n+1)}}{\text{Lat}_{n-(n+1)}} \right]$$

## FROM EXAMPLE

Line 1-2:

$$D_{1-2} = \left[ (5172.813' - 5460.445')^2 + (5849.543' - 6238.012')^2 \right] = 483.364'$$

$$\alpha_{1-2} = \tan^{-1} \left[ \frac{5172.813' - 5460.445'}{5849.543' - 6238.012'} \right] = 216^\circ 31' 01.8''$$

Line 2-3:

$$D_{2-3} = \left[ (5461.737' - 5172.813')^2 + (5508.988' - 5849.543')^2 \right] = 446.604'$$

$$\alpha_{2-3} = \tan^{-1} \left[ \frac{5461.737' - 5172.813'}{5508.988' - 5849.543'} \right] = 139^\circ 41' 20.4''$$

etc....

Traverse Adjustment Program						
				Adjusted		
			Adjusted	Azimuth		
Sta	X	Y	Distance	Deg	Min	Sec
1	7885.572	7097.635				
			483.363	216	31	1.8
2	7597.940	6709.166				
			446.604	139	41	20.4
3	7886.864	6368.611				
			425.588	60	12	9.7
4	8256.185	6580.100				
			384.957	356	3	53.4
5	8229.766	6964.149				
			369.172	291	11	50.6
1	7885.572	7097.635				

## TRANSIT RULE

$$c_L = \frac{cl_L}{\sum |Lat|} S_L$$

$$c_D = \frac{cl_D}{\sum |Dep|} S_D$$

where:  $c_L, c_D$  = corrections to the latitudes and departures of the line,  
 $cl_L, cl_D$  = closure corrections for the latitudes and departures for the traverse,  
 $\sum |Lat|, \sum |Dep|$  = sum of the absolute values of the latitudes and departures, and  
 $S_L, S_D$  = latitude and departure length of the line (absolute values).

## FROM EXAMPLE

Line 1-2:

$$c_L = \left( \frac{0.176'}{1458.026'} \right) 388.509' = +0.047'$$

$$c_D = \left( \frac{0.075'}{1316.501'} \right) 287.649' = +0.016'$$

Line 2-3:

$$c_L = \left( \frac{0.176'}{1458.026'} \right) 340.592' = +0.041'$$

$$c_D = \left( \frac{0.075'}{1316.501'} \right) 288.908' = +0.016'$$

etc....

Traverse Adjustment Program						
	Transit Rule					
			Corrections to		Corrected	Corrected
Sta	Departure	Latitude	Departure	Latitude	Departure	Latitude
1						
	-287.649	-388.509	0.016	0.047	-287.633	-388.462
2						
	288.908	-340.592	0.016	0.041	288.925	-340.551
3						
	369.305	211.453	0.021	0.025	369.326	211.479
4						
	-26.432	384.017	0.002	0.046	-26.431	384.064
5						
	-344.207	133.455	0.020	0.016	-344.187	133.471
1						
	-0.075	-0.176	0.075	0.176	0.000	0.000

Traverse Adjustment Program								
						Adjusted		
	Corrected	Corrected			Adjusted	Azimuth		
Sta	Departure	Latitude	X	Y	Distance	Deg	Min	Sec
1			7885.572	7097.635				
	-421.071	-153.909			448.318	249	55	18.5
2			7464.501	6943.726				
	-159.696	-411.834			441.712	201	11	40.7
3			7304.805	6531.893				
	285.578	-380.184			475.494	143	5	15.7
4			7590.383	6151.708				
	430.139	49.312			432.957	83	27	36.2
5			8020.522	6201.020				
	187.862	339.346			387.877	28	58	8.2
6			8208.385	6540.367				
	-229.663	374.666			439.454	328	29	32.9
7			7978.722	6915.032				
	-93.150	182.603			204.989	332	58	22.2
1			7885.572	7097.635				

## CRANDALL METHOD

$$A = \frac{e_D \left( \sum_{i=1}^n \frac{\text{Lat}_i \cdot \text{Dep}_i}{100s_i} \right) - e_L \left( \sum_{i=1}^n \frac{\text{Dep}^2}{100s} \right)}{\left( \sum_{i=1}^n \frac{\text{Dep}^2}{100s} \right) \left( \sum_{i=1}^n \frac{\text{Lat}^2}{100s} \right) - \left( \sum_{i=1}^n \frac{\text{Lat}_i \cdot \text{Dep}_i}{100s_i} \right)^2}$$

$$B = \frac{e_L \left( \sum_{i=1}^n \frac{\text{Lat}_i \cdot \text{Dep}_i}{100s_i} \right) - e_D \left( \sum_{i=1}^n \frac{\text{Lat}^2}{100s} \right)}{\left( \sum_{i=1}^n \frac{\text{Dep}^2}{100s} \right) \left( \sum_{i=1}^n \frac{\text{Lat}^2}{100s} \right) - \left( \sum_{i=1}^n \frac{\text{Lat}_i \cdot \text{Dep}_i}{100s_i} \right)^2}$$

## CORRECTION APPLIED TO PARTICULAR LINE

$$c_{\ell_1} = \text{Lat}_1 A + \text{Dep}_1 B$$

$$c_{\ell_2} = \text{Lat}_2 A + \text{Dep}_2 B$$

$$c_{\ell_3} = \text{Lat}_3 A + \text{Dep}_3 B$$

⋮

$$c_{\ell_n} = \text{Lat}_n A + \text{Dep}_n B$$

## CORRECTIONS TO LATITUDES AND DEPARTURES

- General form of correction

$$c_L = c_\ell \frac{\text{Lat}}{100s}$$

$$c_D = c_\ell \frac{\text{Dep}}{100s}$$

- Substituting into previous equation:

$$c_L = A \frac{\text{Lat}^2}{100s} + B \frac{\text{Lat} \cdot \text{Dep}}{100s}$$

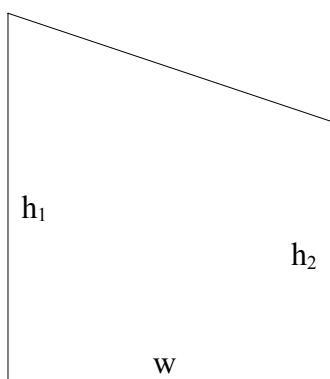
$$c_D = A \frac{\text{Lat} \cdot \text{Dep}}{100s} + B \frac{\text{Dep}^2}{100s}$$

## FROM EXAMPLE

Traverse Adjustment Program										
Point	Distance	Departure	Latitude	1	2	3	Correction to Departure	Latitude	Adjusted Departure	Latitude
1	483.406	-287.6492	-388.5091	3.122412	1.711648	2.311811	0.048	0.064	-287.602	-388.445
2	446.622	288.9084	-340.5924	2.597346	1.868874	-2.203205	-0.022	0.025	288.887	-340.567
3	425.557	369.3052	211.4531	1.05068	3.20489	1.835024	0.050	0.029	369.356	211.482
4	384.926	-26.43238	384.0174	3.831109	0.018151	-0.2637	-0.004	0.058	-26.436	384.075
5	369.173	-344.207	133.4551	0.482437	3.209293	-1.2443	0.002	-0.001	-344.204	133.454
1										
Sum =	2109.684	-0.075009	-0.175764	11.08398	10.01286	0.435631	0.075	0.176	0.000	0.000

Traverse Adjustment Program								
	Adjusted			Adjusted				
Point	Departure	Latitude	X	Y	Distance	Azimuth		
						Deg	Min	Sec
1	-287.602	-388.445	5460.445	6238.012	483.326	216	30	57.5
2	288.887	-340.567	5172.843	5849.567	446.589	139	41	37.1
3	369.356	211.482	5461.730	5509.000	425.615	60	12	21
4	-26.436	384.075	5831.086	5720.483	384.984	356	3	44.9
5	-344.204	133.454	5804.649	6104.558	369.17	291	11	31.9
1			5460.445	6238.012				

## AREA CALCULATIONS

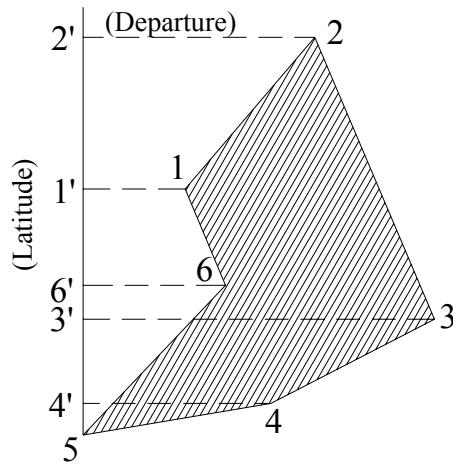


$$\text{Area} = \frac{h_1 + h_2}{2} \cdot w$$

Double area:

$$2A = (h_1 + h_2)w$$

## DOUBLE MERIDIAN DISTANCE



$$2A = 2A_{2,2',3,3'} + 2A_{3,3',4,4'} + 2A_{4,4',5} - 2A_{5,6,6'} - 2A_{1,1',6,6'} - 2A_{1,1',2,2'}$$

## DMD

1. DMD of first line equal departure of that line
2. DMD of each subsequent line equals the DMD of preceding line + departure of previous line + departure of line in question
3. DMD of last line should equal departure of last line, with opposite sign.

## DMD

$$\begin{aligned}
 2A = & \text{DMD}_{5-6} * \text{Lat}_{5-6} + (\text{DMD}_{5-6} + \text{Dep}_{5-6} + \text{Dep}_{6-1}) * \text{Lat}_{6-1} \\
 & + (\text{DMD}_{6-1} + \text{Dep}_{6-1} + \text{Dep}_{1-2}) * \text{Lat}_{1-2} \\
 & + (\text{DMD}_{1-2} + \text{Dep}_{1-2} + \text{Dep}_{2-3}) * \text{Lat}_{2-3} \\
 & + (\text{DMD}_{2-3} + \text{Dep}_{2-3} + \text{Dep}_{3-4}) * \text{Lat}_{3-4} \\
 & + (\text{DMD}_{3-4} + \text{Dep}_{3-4} + \text{Dep}_{4-5}) * \text{Lat}_{4-5}
 \end{aligned}$$

## FROM EXAMPLE

2A:	(-287.632) (-388.469)	=	111,736.1154
	(-287.632 - 287.632 + 288.924) (-340.555)	=	97,514.5187
	(-286.340 + 288.924 + 369.320) (211.489)	=	78,653.6051
	(371.904 + 369.320 - 26.418) (384.049)	=	274,520.5295
	(714.806 - 26.418 - 344.194) (133.486)	=	45,945.0803
		2A =	608,369.8489
		A =	304,184.82 sq. ft.
		=	6.98 ac.

## AREA BY COORDINATES

$$2A = X_1(Y_2 - Y_n) + X_2(Y_3 - Y_1) + \dots + X_{n-1}(Y_n - Y_{n-2}) + X_n(Y_1 - Y_{n-1})$$

2A:	5460.445 (5849.543 - 6104.526)	=	-1,392,320.6474
	+ 5172.813 (5508.988 - 6238.012)	=	-3,771,104.8245
	+ 5461.737 (5720.477 - 5849.543)	=	-704,924.5476
	+ 5831.057 (6104.526 - 5508.988)	=	3,472,616.0237
	+ 5804.639 (6238.012 - 5720.477)	=	3,004,103.8449
		2A =	608,369.8489
		A =	304,184.82 sq. ft.
		=	6.98 ac.

Point	X	Y
1	5460.445	6238.012
2	5172.813	5849.543
3	5461.737	5508.988
4	5831.057	5720.477
5	5804.639	6104.526
1	5460.445	6238.012

Sum 1 =	163,487,060.260	
- Sum 2 =	<u>162,878,690.411</u>	
2A =	608,369.849	
A =	304,184.92 sq. ft. or 6.98 ac.	