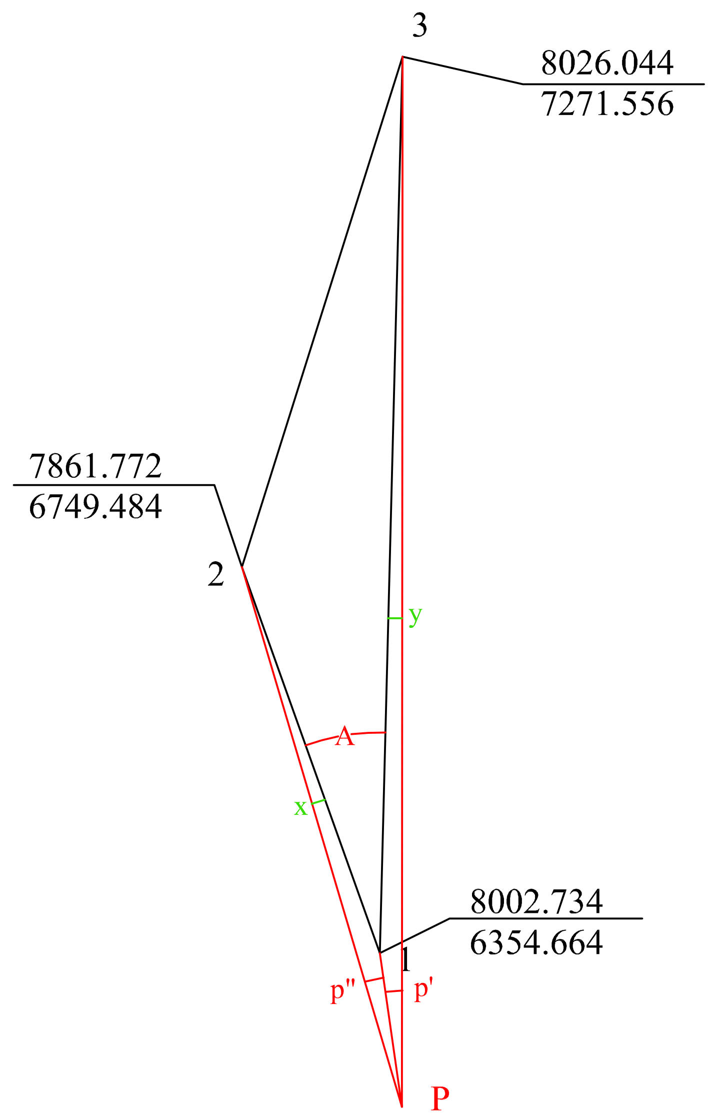
3-Point Resection Problem

Given points and coordinates of points 1, 2 and 3 given in black.

Identify point P from the angles measured from P to points 1, 2 and 3. The lines from P to the points are shown in red.

This method uses the approach presented by the U.S. Coast and Geodetic Survey



p" = 8° 28' 21.0"
p' = 8° 15' 46.8"

3-POINT RESECTION SOLUTION - USC&GS METHOD

DISTANCE & AZIMUTH FROM 2-3

$$D_{23} = \left[(X_3 - X_2)^2 + (Y_3 - Y_2)^2 \right]^{1/2} = 547.3066$$

$$Az_{23} = \tan^{-1}\left[\frac{(X_3 - X_2)}{(Y_3 - Y_2)} \right] = 17^\circ 27' 59.0''$$

DISTANCE & AZIMUTH FROM 1-2

$$D_{12} = \left[(X_2 - X_1)^2 + (Y_2 - Y_1)^2 \right]^{1/2} = 419.2292$$

$$Az_{12} = \tan^{-1}\left[\frac{(X_2 - X_1)}{(Y_2 - Y_1)} \right] = 340^\circ 21' 07.2''$$

DISTANCE & AZIMUTH FROM 3-1

$$D_{31} = \left[(X_3 - X_1)^2 + (Y_3 - Y_1)^2 \right]^{1/2} = 917.1883$$

$$Az_{31} = \tan^{-1}\left[\frac{(X_3 - X_1)}{(Y_3 - Y_1)} \right] = 181^\circ 27' 22.7''$$

LET $a = D_{23}$, $b = D_{31}$, $c = D_{12}$

$$A = Az_{13} - Az_{12} = 21^\circ 06' 15.6''$$

$$S = \tfrac{1}{2}(x + y) = \tfrac{1}{2}(A - P' - P'') = 2^\circ 11' 03.9''$$

$$Z = \tan^{-1}\left[\frac{c \sin P'}{b \sin P''} \right] = 24^\circ 01' 48.3''$$

$$\epsilon = \tan^{-1}\left[\frac{\tan S}{\tan(Z + 45^\circ)} \right] = 0^\circ 50' 15.2''$$

$$x = S + \epsilon = 3^\circ 01' 19.0''$$

$$y = S - \epsilon = 1^\circ 20' 48.7''$$

IN TRIANGLE P-1-3

$$\measuredangle_{P13} = 180^\circ - (y + P') = 170^\circ 23' 24.5''$$

$$Az_{3P} = Az_{31} - y = 180^\circ 06' 34.0''$$

$$D_{3P} = \frac{b}{\sin P'} \sin \measuredangle_{P13} = 1065.3844$$

THE COORDINATES OF POINT P:

$X_P = X_3 + D_{3P} \sin Az_{3P}$ $= \boxed{8024.009}$

$Y_P = Y_3 + D_{3P} \cos Az_{3P}$ $= \boxed{6206.174}$

CHECKING RESULTS

$\measuredangle_{P12} = 180° - (P'' + \gamma)$ $= 168°30'20.0''$

$D_{2P} = \dfrac{r}{\sin P''} \sin \measuredangle_{P12}$ $= 567.0159$

$Az_{2P} = Az_{21} + \gamma$ $= 163°22'26.2''$

THE COORDINATES OF POINT P:

$X_P = X_2 + D_{2P} \sin Az_{2P}$ $= \boxed{8024.009}$ ✓

$Y_P = Y_2 + D_{2P} \cos Az_{2P}$ $= \boxed{6206.174}$ ✓

Example 3-Point Resection Problem:

$$\mathrm{dd(ang)} := \left| \begin{array}{l} \mathrm{degree} \leftarrow \mathrm{floor(ang)} \\ \mathrm{mins} \leftarrow (\mathrm{ang} - \mathrm{degree}) \cdot 100.0 \\ \mathrm{minutes} \leftarrow \mathrm{floor(mins)} \\ \mathrm{seconds} \leftarrow (\mathrm{mins} - \mathrm{minutes}) \cdot 100.0 \\ \mathrm{degree} + \dfrac{\mathrm{minutes}}{60.0} + \dfrac{\mathrm{seconds}}{3600.0} \end{array} \right.$$

$$\mathrm{radians(ang)} := \left| \begin{array}{l} d \leftarrow \mathrm{dd(ang)} \\ d \cdot \dfrac{\pi}{180.0} \end{array} \right.$$

$$\mathrm{dms(ang)} := \left| \begin{array}{l} \mathrm{degree} \leftarrow \mathrm{floor(ang)} \\ \mathrm{rem} \leftarrow (\mathrm{ang} - \mathrm{degree}) \cdot 60 \\ \mathrm{mins} \leftarrow \mathrm{floor(rem)} \\ \mathrm{rem1} \leftarrow (\mathrm{rem} - \mathrm{mins}) \\ \mathrm{secs} \leftarrow \mathrm{rem1} \cdot 60.0 \\ \mathrm{degree} + \dfrac{\mathrm{mins}}{100} + \dfrac{\mathrm{secs}}{10000} \end{array} \right.$$

$$\mathrm{trad} := \frac{\pi}{180}$$

$$\mathrm{tdeg} := \frac{180}{\pi}$$

Given the following values:

$X_1 := 8002.734$ $\quad Y_1 := 6354.664$

$X_2 := 7861.772$ $\quad Y_2 := 6749.484$

$X_3 := 8026.044$ $\quad Y_3 := 7271.556$

α is the angle between points 1 and 2 (designated by the USC&GS as p") while β is the angle between points 2 and 3 (designated by the USC&GS as p').

$\alpha := 8.2821$ $\quad \beta := 8.15468$

Solution:

Distances and azimuths between the control points

$$D_{23} := \sqrt{(X_3 - X_2)^2 + (Y_3 - Y_2)^2} \qquad D_{23} = 547.30655$$

$$Az_{23} := \mathrm{atan2}\left[(Y_3 - Y_2), (X_3 - X_2)\right] \qquad \mathrm{dms}(Az_{23} \cdot \mathrm{tdeg}) = 17.275899$$

$$D_{12} := \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} \qquad D_{12} = 419.22919$$

$$Az_{12} := \mathrm{atan2}\left[(Y_2 - Y_1), (X_2 - X_1)\right] + 2 \cdot \pi \qquad \mathrm{dms}(Az_{12} \cdot \mathrm{tdeg}) = 340.210715$$

$$D_{31} := \sqrt{(X_3 - X_1)^2 + (Y_3 - Y_1)^2}$$ $D_{31} = 917.18826$

$a := D_{23}$ $b := D_{31}$ $c := D_{12}$

$$Az_{31} := \text{atan2}[(Y_3 - Y_1), (X_3 - X_1)] + \pi$$ $\text{dms}(Az_{31} \cdot \text{tdeg}) = 181.272271$

$$A := (Az_{31} + \pi) - Az_{12}$$ $\text{dms}(A \cdot \text{tdeg}) = 21.06156$

$$S := \frac{(A - \text{radians}(\alpha) - \text{radians}(\beta))}{2}$$ $\text{dms}(S \cdot \text{tdeg}) = 2.11039$

$$Z := \text{atan2}(b \cdot \sin(\text{radians}(\alpha)), c \cdot \sin(\text{radians}(\beta)))$$ $\text{dms}(Z \cdot \text{tdeg}) = 24.01483$

$$\varepsilon := \text{atan2}\left(\tan\left(Z + \frac{\pi}{4}\right), \tan(S)\right)$$ $\text{dms}(\varepsilon \cdot \text{tdeg}) = 0.50152$

$$x := S + \varepsilon$$ $\text{dms}(x \cdot \text{tdeg}) = 3.01190$

$$y := S - \varepsilon$$ $\text{dms}(y \cdot \text{tdeg}) = 1.20487$

In triangle P-1-3

$$A_{P13} := \pi - (y + \text{radians}(\beta))$$ $\text{dms}(A_{P13} \cdot \text{tdeg}) = 170.23245$

$$Az_{3P} := Az_{31} - y$$ $\text{dms}(Az_{3P} \cdot \text{tdeg}) = 180.06340$

$$D_{3P} := \frac{b}{\sin(\text{radians}(\beta))} \cdot \sin(A_{P13})$$ $D_{3P} = 1065.38438$

The coordinates of point P:

$$X_P := X_3 + D_{3P} \cdot \sin(Az_{3P})$$ $X_P = 8024.009$

$$Y_P := Y_3 + D_{3P} \cdot \cos(Az_{3P})$$ $Y_P = 6206.174$

Checking the results:

$$A_{P12} := \pi - (\text{radians}(\alpha) + x)$$ $\text{dms}(A_{P12} \cdot \text{tdeg}) = 168.30200$

$$D_{2P} := \frac{c}{\sin(\text{radians}(\alpha))} \cdot \sin(A_{P12})$$ $D_{2P} = 567.0159$

$$Az_{2P} := (Az_{12} - \pi) + x$$ $\text{dms}(Az_{2P} \cdot \text{tdeg}) = 163.222620$

The coordinates of point P:

$$X_P := X_2 + D_{2P} \cdot \sin\left(Az_{2P}\right)$$ $X_P = 8024.009$ check

$$Y_P := Y_2 + D_{2P} \cdot \cos\left(Az_{2P}\right)$$ $Y_P = 6206.174$ check