

Ferris State University
Surveying Engineering
Surveying Computation - SURE 215
PROBLEM NUMBER 999
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4-Parameter Coordinate Transformation Problem

Write your answers in the spaces provided

Given Data:

	X - Y Coordinate System		X' - Y' Coordinate System	
POINT NO.	X	Y	X'	Y'
1	6567.816	6504.555	6172.620	6444.403
2	6798.660	12271.950	11940.019	6675.247
3	6741.049	6719.063	_____	_____
4	6584.981	6661.367	_____	_____
5	6628.913	6603.671	_____	_____
6	6666.569	6597.075	_____	_____

Compute:

The following transformation parameters for the two coordinate systems given above:

- (a). The rotation angle = _____
- (b). The X translation = _____
- (c). The Y translation = _____
- (d). The scale factor = _____
- (e). The coordinates of points 3 to 6 in the X' - Y' system

SOLUTION

$$Az_{1-2_{xy}} = \tan^{-1} \left[\frac{X_2 - X_1}{Y_2 - Y_1} \right] = \tan^{-1} \left[\frac{230.844}{5,767.395} \right] = 2^\circ 17' 31.5''$$

$$D_{1-2_{xy}} = \left[(X_2 - X_1)^2 + (Y_2 - Y_1)^2 \right]^{1/2} = 5,772.0130$$

$$Az_{1-2_{x'y'}} = \tan^{-1} \left[\frac{X'_2 - X'_1}{Y'_2 - Y'_1} \right] = \tan^{-1} \left[\frac{5,767.399}{230.844} \right] = 87^\circ 42' 28.5''$$

$$D_{1-2_{x'y'}} = \left[(X'_2 - X'_1)^2 + (Y'_2 - Y'_1)^2 \right]^{1/2} = 5,772.0170$$

ROTATION ANGLE:

$$\begin{aligned} \alpha &= Az_{1-2_{xy}} - Az_{1-2_{x'y'}} = (2^\circ 17' 31.5'') - (87^\circ 42' 28.5'') \\ &= \underline{\underline{274^\circ 35' 03''}} \end{aligned}$$

SCALE FACTOR:

$$C = \frac{D_{1-2_{x'y'}}}{D_{1-2_{xy}}} = \frac{5772.0170}{5772.0130} = \underline{\underline{1.00000}}$$

X & Y TRANSLATIONS:

~~$$\Delta X' = X'_1 - X_1$$~~

$$\begin{aligned} \Delta X' &= X'_1 - X_1 \cdot C \cdot \cos \alpha + Y_1 \cdot C \cdot \sin \alpha = \underline{\underline{-836.0536}} \\ \Delta Y' &= Y'_1 - X_1 \cdot C \cdot \sin \alpha - Y_1 \cdot C \cdot \cos \alpha = \underline{\underline{12,471.3472}} \end{aligned}$$

TRANSFORMED COORDINATES:

$$\begin{aligned} X'_3 &= X_3 \cdot C \cdot \cos \alpha - Y_3 \cdot C \cdot \sin \alpha + \Delta X' = \underline{\underline{6,400.208}} \\ Y'_3 &= X_3 \cdot C \cdot \sin \alpha + Y_3 \cdot C \cdot \cos \alpha + \Delta Y' = \underline{\underline{6,208.869}} \end{aligned}$$

$$\begin{aligned} X'_4 &= X_4 \cdot C \cdot \cos \alpha - Y_4 \cdot C \cdot \sin \alpha + \Delta X' = \underline{\underline{6,330.303}} \\ Y'_4 &= X_4 \cdot C \cdot \sin \alpha + Y_4 \cdot C \cdot \cos \alpha + \Delta Y' = \underline{\underline{6,439.827}} \end{aligned}$$



$$X'_5 = X_5 \cdot C \cdot \cos \alpha - Y_5 \cdot C \cdot \sin \alpha + \Delta X' = \underline{\underline{6,276.303}}$$

$$Y'_5 = X_5 \cdot C \cdot \sin \alpha + Y_5 \cdot C \cdot \cos \alpha + \Delta Y' = \underline{\underline{6,391.424}}$$

$$X'_6 = X_6 \cdot C \cdot \cos \alpha - Y_6 \cdot C \cdot \sin \alpha + \Delta X' = \underline{\underline{6,272.738}}$$

$$Y'_6 = X_6 \cdot C \cdot \sin \alpha + Y_6 \cdot C \cdot \cos \alpha + \Delta Y' = \underline{\underline{6,353.361}}$$

AN ALTERNATIVE SOLUTION - USING LINEAR FORM

$$\bar{X} = X_2 - X_1 = 230.844$$

$$\bar{Y} = Y_2 - Y_1 = 5,767.395$$

$$\bar{X}' = X'_2 - X'_1 = 5,767.399$$

$$\bar{Y}' = Y'_2 - Y'_1 = 230.844$$

$$a = \frac{\bar{X}'\bar{X} + \bar{Y}'\bar{Y}}{\bar{X}^2 + \bar{Y}^2} = \frac{1,331,369.4548 + 1,331,368.5314}{33,316,134.0383}$$

$$= 0.079923$$

$$b = \frac{\bar{Y}'\bar{X} - \bar{X}'\bar{Y}}{\bar{X}^2 + \bar{Y}^2} = \frac{53,200.9523 - 33,262,868.1556}{33,316,134.0383}$$

$$= -0.996802$$

$$c = \frac{-836.0536}{\quad} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \Delta X' \text{ \& } \Delta Y' \text{ VALUES AS CALCULATED} \\ \text{ON PREVIOUS PAGE} \end{array}$$

$$d = \underline{\underline{12,471.3472}}$$

$$\text{SCALE FACTOR} = \sqrt{a^2 + b^2} = \underline{\underline{1.00000}}$$

$$\alpha = \tan^{-1}(b/a) = \underline{\underline{84^\circ 27' 35.03''}}$$



TRANSFORMED COORDINATES:

$$X'_3 = aX_3 - bY_3 + c = \underline{\underline{6400.207}}$$

$$Y'_3 = bX_3 + aY_3 + d = \underline{\underline{6208.068}}$$

$$X'_4 = aX_4 - bY_4 + c = \underline{\underline{6330.302}}$$

$$Y'_4 = bX_4 + aY_4 + d = \underline{\underline{6439.826}}$$

$$X'_5 = aX_5 - bY_5 + c = \underline{\underline{6276.302}}$$

$$Y'_5 = bX_5 + aY_5 + d = \underline{\underline{6391.423}}$$

$$X'_6 = aX_6 - bY_6 + c = \underline{\underline{6272.737}}$$

$$Y'_6 = bX_6 + aY_6 + d = \underline{\underline{6353.360}}$$

