EVERYMAN'S GUIDE TO LEVELLING

by H. Solenthaler

Everyman's Guide to Levelling

WILD HEERBRUGG

Levelling

Levelling is the process of determining the difference in height between two or more points. The accuracy with which the measurements are made depends on the purpose of the survey and the means available.

Simple measurement of difference in height

With a straight-edge and spirit level

When only a moderate degree of accuracy is needed – for example on a steep bank – a wooden straight-edge and a simple spirit level can be used with a folding rule (fig. 1). The measurements are taken in steps. A levelling staff can be used instead of a straight-edge.

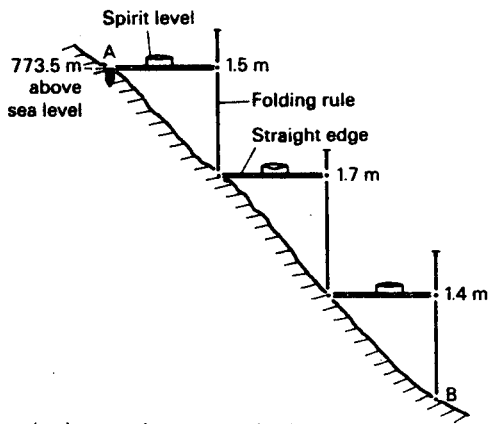

Fig. 1: Measuring in steps down a steep bank.
Starting at point A, 773.5 m above sea level, the height of the final point B is
H_B = 773.5 − 1.5 − 1.7 − 1.4 = **768.9 m above sea level**

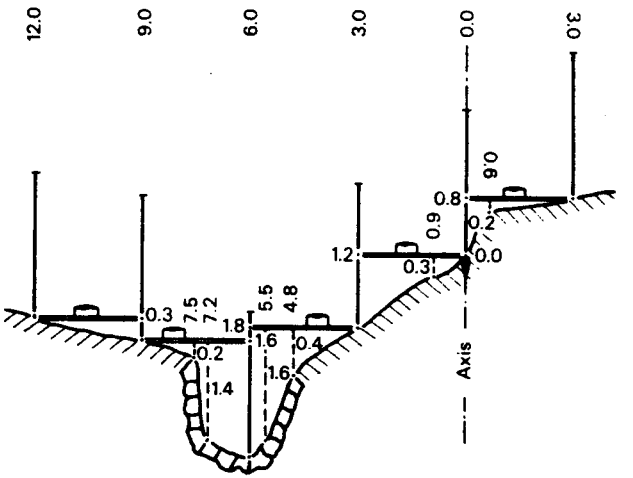

Fig. 2: Cross-section with a wooden straight-edge and simple spirit level

With an optical square and a plumb-bob

Simple height measurements can be made using a Wild optical square (pentaprism) and a plumb-bob. Wind the plumb-bob string several times around the shaft of the Wild optical square. Hold the optical square horizontal in the left hand so that the plumb-bob string is hanging from below the centre of the shaft. The prism nearest the string will turn the line of sight downwards.

Look into the prism at the plumb-bob string which will appear to be "horizontal". At the same time look straight through the gap in the optical square at the staff. Read the staff against the "horizontal" plumb-bob string.

The optical square corresponds to the levelling instrument, the height of the observer's eye to the height of the instrument, and the "horizontal" plumb-bob string to the horizontal line of sight.

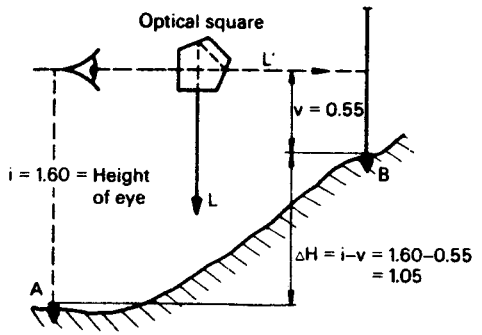

Fig. 3: Simple height measurements with optical square and plumb-bob.

Measuring heights with a levelling instrument

For an accurate determination of difference in height, particularly over longer distances, a **levelling instrument** is essential.

Difference in height between two points
There are three methods, depending on where the instrument is set up.

Method 1:
Set the instrument over the point of known height, A (fig. 4a). Measure the height of the instrument i, i.e. the distance from the ground point to the centre of the telescope. Instruct an

assistant to hold a levelling staff vertically on the point of unknown height, B. Centre the tubular bubble (unless it's an automatic level) and read the staff where it is cut by the horizontal cross hair. The reading is called the foresight, v. The difference in height between A and B is $+i-v$.

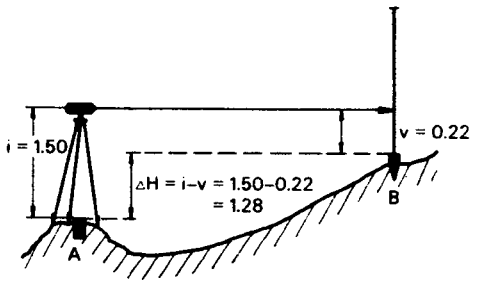

Fig. 4a: Height of B = Height of A + i – v

If the instrument is set up over the point of unknown height B (fig. 4b) and the staff is on the point of known height A, the reading is known as the backsight r. The difference in height between A and B is $+r-i$.

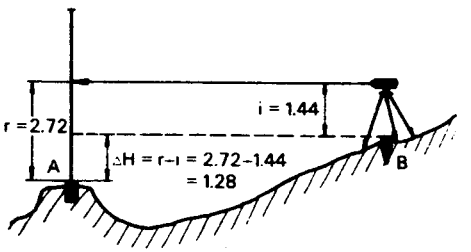

Fig. 4b: Height of B = Height of A + r – i

Rule: Backsights r (to known points) are always + positive. Foresights v (to unknown points) are always – negative.

Note: The height of the instrument i cannot usually be measured to a high degree of accuracy. Method 1 should only be used when high accuracy is not required.

Method 2:
Set up the instrument I between the point of known height A and the point of unknown height B so that distances **IA and IB are roughly equal.** Note that the instrument does not need to be on the straight line between A and B.

Hold the staff on A, set the line of sight horizontal by centring the tubular bubble (unless it's an automatic level), and take the backsight reading r. Hold the staff on B, set the line of sight horizontal, and take the foresight reading v. Note that the position of the instrument must not be changed. The difference in height between A and B is + r – v.

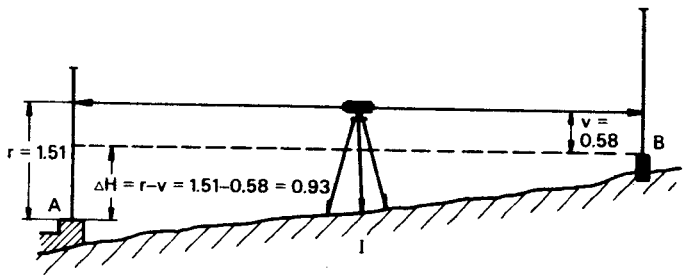

Fig. 5: Height of B = Height of A + r – v

Note: This is the best and most accurate method of levelling and should be used whenever possible.

Method 3:
If the instrument cannot be set up between the two points (Method 2), it has to be set up as shown in fig. 6. Read the backsight r to the staff on the known point A, and the foresight v to the staff on the unknown point B. The difference in height between A and B is + r – v.

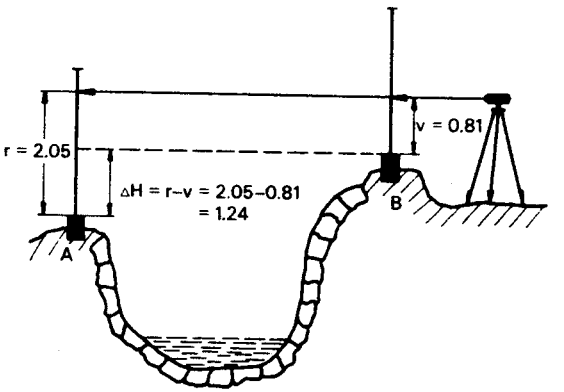

Fig. 6: Height of B = Height of A + r – v

Line levelling

Exactly as the straight edge has to be used several times for simple height measurements, so too is line levelling carried out in stages. The average sighting distance is usually about 50 m. In steep terrain distances will have to be shorter.

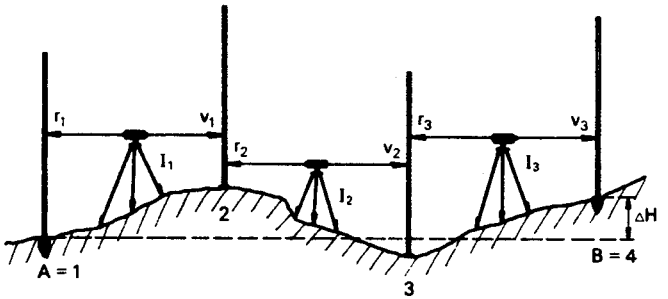

Fig. 7: Line levelling

Hold a staff on the starting point A = 1. Set up the instrument at I_1. Set the line of sight horizontal by centring the tubular level (unless it's an automatic level) and take the backsight reading r_1 to the staff at 1. The staffman now picks up the staff and paces the distance to the instrument. He then proceeds approximately the same number of paces past the instrument and sets up the staff at the next point 2. Turn the instrument towards the staff, set the line of sight horizontal, and take the foresight reading v_1. Move the instrument to I_2. The staff remains on point 2 but is turned to face the instrument at I_2. Take the backsight r_2. Move the staff to point 3 and take the foresight v_2. Proceed in this manner until the last reading is the foresight to the end point B = 4.

Note: At each instrument set-up, keep the backsight and foresight distances roughly equal.
Note: Staff and instrument points must be chosen so that the line of sight will not pass above or below the staff.
Note: For a long line of levels, it is quicker to use two staffs.

The readings are entered in a **level book** as shown in example 1.

Example 1: Line levelling

Point No.	Backsight r	Foresight v	Remarks
A = 1	*+ 2.50*		*r_1*
2		*− 1.80*	*v_1*
2	*+ 0.90*		*r_2*
3		*− 1.90*	*v_2*
3	*+ 3.10*		*r_3*
B = 4		*− 0.90*	*v_3*
	+ 6.50	*− 4.60*	*Sum*
	− 4.60 ←		
ΔH	*+ 1.90*	*= Difference in height A to B*	

If only the difference in height ΔH between the starting point A and the end point B is needed, all that is necessary is to subtract the sum of all the foresights (− 4.60) from the sum of all the backsights (+ 6.50): ΔH = + 6.50 − 4.60 = + 1.90.

Rule: Backsights r are always + positive.
Foresights v are always − negative.

If the heights of the ground at the staff stations 2 and 3 are required, readings are booked and reduced as shown in example 2.
The height of the starting point A = 1 (e.g. the height above sea level) is entered on the first line. The backsight r_1 is entered on the second line and **added** to the height of A = 1 to give

the height of the instrument $\otimes_1$ at I_1. The backsight v_1 is entered and **subtracted** from $\otimes_1$ to obtain the ground height of point 2. r_2 is **added** to the height of point 2 to give the height of the instrument $\otimes_2$ at I_2. The foresight v_2 is **subtracted** to give the height of point 3. r_3 is **added** to give $\otimes_3$ and v_3 **subtracted** to obtain the height of the end point B = 4.
Note: As backsights r are + and foresights v are −, the signs change.

Example 2: Line levelling

Point No.		Remarks
A = 1	*650.00*	*Starting point*
r_1	*+ 2.50*	
$\otimes_1$	*652.50*	
v_1	*− 1.80*	
2	*650.70*	*Curb*
r_2	*+ 0.90*	
$\otimes_2$	*651.60*	
v_2	*− 1.90*	
3	*649.70*	*Ditch*
r_3	*+ 3.10*	
$\otimes_3$	*652.80*	
v_3	*− 0.90*	
B = 4	*651.90*	*End point*
A = 1	*650.00*	
ΔH	*+ 1.90*	*(see example 1)*

⊗ = Height of instrument

Area levelling

If several points have to be levelled from one set-up, the method is as follows.

Set the staff on known point A and take the backsight reading r_1. Enter the height of A and the reading r_1 as show in example 3. Add together to obtain the height of the instrument ⊗.

The staff is now held at the other points, called intermediate points. The readings are treated as foresights and are negative. Enter the readings as shown in example 3 and subtract each reading from the height of the instrument ⊗ to get the ground height of each intermediate point.

Example 3: Area levelling (see fig. 8)

Point No.		Intermediate Points Reading	Height	Remarks
A	*592.00*			*Known point*
r_1	*+ 2.20*			
⊗	*594.20*			
v_1		*– 1.80*	*592.40*	*Big rock*
v_2		*– 1.90*	*592.30*	
v_3		*– 2.50*	*591.70*	
v_4		*– 2.30*	*591.90*	*Stake*
v_5		*– 2.70*	*591.50*	
v_A	*– 2.21*			*Check*
A	*591.99* √			*Check*

After levelling several intermediate points, read the staff again at A. Enter the reading as a foresight v_a and subtract from the height of the instrument ⊗. The result should be the ground height at A. This checks that the instrument has not moved, for example due to sinking on soft ground or touching the tripod leg.

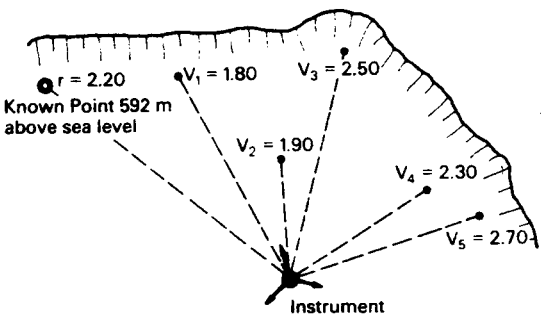

Fig. 8: Area levelling (see example 3)

Checking and adjusting the levelling instrument

From time to time, and particularly before each important job, the instrument must be checked and, if necessary, adjusted. It must also be checked if it has been dropped or subjected to jolts and shocks.

Checking the horizontality of the line of sight

A stretch AD of about 45 to 60 m is divided into three equal lengths d. Bang in a peg at B and at C. Set up the instrument at A. Hold the staff on peg B. Centre the tubular level (unless it's an automatic level) and read the staff, a'_1. Hold the staff on peg

C, centre the tubular level, and take the reading, a'_2. Move the instrument to D. Take the readings a'_3 to the staff on C and a'_4 to the staff on B.

If the line of sight is absolutely horizontal the readings will be the correct readings a_1, a_2, a_3, a_4 and

$$a_4 - a_1 = a_3 - a_2 \quad \text{(see fig. 9)}$$

If this is not so, the line of sight is inclined to the horizontal by the small angle δ. If a line is drawn through a'_3, parallel to $a'_2 - a'_1$, it will cut the staff at B at the reading a_4. a_4 is the required staff reading for a true horizontal line of sight from D.

$$a_4 - a'_1 = a'_3 - a'_2 \quad \text{(see fig. 9)}$$

Therefore the required reading a_4 is given by

$$a_4 = a'_1 - a'_2 + a'_3.$$

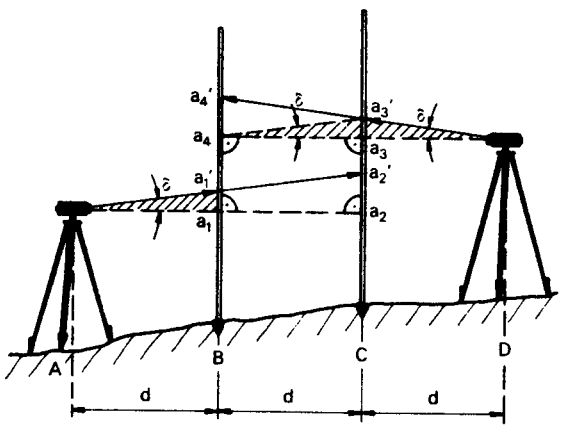

Fig. 9: Checking the horizontality of the line of sight

If the actual reading a'_4 differs from the calculated, correct value a_4 by an unacceptable amount (say by more than 3 mm in 30 m), the whole procedure must be repeated. If the difference is confirmed, the instrument must be adjusted.

Adjusting

The instrument is still at D.

With tilting levels, turn the tilting screw to set the horizontal cross hair to the calculated correct staff reading a_4. Then turn the tubular level adjusting screw to centre the bubble.

With a Dumpy Level, centre the tubular level exactly. Then shift the cross hairs by means of the adjusting screw until the correct reading a_4 is obtained.

Automatic levels are also usually adjusted by shifting the cross hairs with an adjustment screw until the correct reading a_4 is obtained.

In any case, the instrument's instruction booklet should be consulted.

After adjusting the instrument as described above, repeat the checking procedure. See example 4.

Example 4: Checking and adjusting

	Check before adjusting	Check after adjusting
a'_1	+ 1.613	+ 1.573
a'_2	− 1.515	− 1.456
a'_1-a'_2	+ 0.098	+ 0.117
a'_3	+ 2.434	+ 2.424
a_4	2.532 calculated	2.541 calculated
a'_4	2.571 staff reading	2.543 staff reading
Diff.	0.039 unacceptable	0.002 acceptable √