

**COMPUTATION OF TRAVERSE
BY PLANE COORDINATES**

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COMPUTATION OF TRAVERSE BY PLANE COORDINATES

In making available for local use the plane coordinate system adopted for a State or part of a State, it is necessary to have the coordinates of all the control stations within the area computed on that system. After the triangulation network of an area is adjusted, the plane coordinates of each triangulation station in that area are computed together with the grid azimuth from the triangulation station to a mark which may be a special azimuth mark or another triangulation station in the vicinity. These coordinates are being made available for the use of field engineers as rapidly as the facilities of the Coast and Geodetic Survey permit.

A traverse can be started from any control station by turning off an angle from the azimuth mark to the first station of the traverse. Then, when the lengths of the traverse are measured together with the angles at the various stations, the whole can be computed directly on the plane with x and y coordinates for each of the stations, using the x and y coordinates of the triangulation station as the origin. To eliminate the uncertainty of the coordinates of these stations due to the possibility of error or blunder in the measurement either of the distances or the angles it is important to end the traverse on another control point or run the traverse in a loop closing on the starting point. The discrepancy due to the failure to check, if within allowable limits, can then be distributed through the traverse.

The method of distributing this closing error can best be explained by going through an example of the computation of a traverse run between two first-order control stations in Nebraska. This example is on the Lambert conformal projection, but the method of adjustment holds true for the transverse Mercator projection except as noted. In this example the traverse starts at station *Prosser* and ends on station *Wanda*. The coordinates of the triangulation stations were furnished, together with the grid azimuths to azimuth marks, by the Coast and Geodetic Survey. At *Prosser* the angle was turned from *Insane Asylum standpipe*, the azimuth mark, to the first traverse station, No. 325; the angle at this traverse station was turned from *Prosser* to the next traverse station, No. 326; and so on through the traverse to *Wanda* where the angle was turned from the last traverse station No. 333 to *Insane Asylum standpipe*, the azimuth mark for *Wanda*.

TRAVERSE, PROSSEER TO WANDA

List of angles

Station	From station	To station	Angle
			° ' "
Prosser.....	Insane Asylum standpipe.	325.....	50 32 09.3
325.....	Prosser.....	326.....	220 25 02.3
326.....	325.....	327.....	173 07 38.3
327.....	326.....	328.....	166 48 27.8
328.....	327.....	329.....	161 23 23.1
329.....	328.....	330.....	242 47 27.7
330.....	329.....	331.....	149 08 56.5
331.....	330.....	332.....	149 08 56.7
332.....	331.....	333.....	167 17 47.7
333.....	332.....	Wanda.....	162 52 38.8
Wanda.....	333.....	Insane Asylum standpipe.....	130 12 41.0

The foregoing table and the other tables given on the following pages are taken from Coast and Geodetic Survey Special Publication No. 194, Manual of Traverse Computation on the Lambert Grid, to which the reader is referred for a more complete discussion of methods.

The first step necessary in the adjustment is to distribute the angle discrepancy through the traverse so that the azimuth carried through the various stations from the fixed grid azimuth *Prosser* to *Insane Asylum standpipe* will agree with the fixed grid azimuth *Wanda* to *Insane Asylum standpipe*. To the fixed grid azimuth *Prosser* to *Insane Asylum standpipe* of $315^{\circ}37'27.8''$ is added the angle at *Prosser* from *Insane Asylum standpipe* to No. 325 which gives an azimuth from *Prosser* to No. 325 of $6^{\circ}09'37.1''$. Then adding 180° to this azimuth gives $186^{\circ}09'37.1''$, the azimuth from No. 325 to *Prosser*. To $186^{\circ}09'37.1''$ is added the angle at No. 325 between *Prosser* and No. 326 which gives an azimuth of $46^{\circ}34'39.4''$ the azimuth from No. 325 to No. 326, and so on through the traverse to the line *Wanda* to *Insane Asylum standpipe*. The grid azimuth of the line *Wanda* to

Computation of grid azimuths

Stations	Azimuth and angle			Correction for closure	Seconds of corrected azimuth		
	°	'	''		°	'	''
Prosser to Insane Asylum standpipe	315	37	27.8				27.8
∠ Insane Asylum standpipe to 325	50	32	09.3	+1.8			11.1
Prosser to 325	6	09	37.1				38.9
325 to Prosser	186	09	37.1				38.9
∠ Prosser to 326	220	25	02.3	+1.7			04.0
325 to 326	46	34	39.4				42.9
326 to 325	226	34	39.4				42.9
∠ 325 to 327	173	07	38.3	+1.8			40.1
326 to 327	39	42	17.7				23.0
327 to 326	219	42	17.7				23.0
∠ 326 to 328	166	48	27.8	+1.7			29.5
327 to 328	26	30	45.5				52.5
328 to 327	206	30	45.5				52.5
∠ 327 to 329	151	23	23.1	+1.8			24.9
328 to 329	357	54	08.6				17.4
329 to 328	177	54	08.6				17.4
∠ 328 to 330	242	47	27.7	+1.7			29.4
329 to 330	60	41	36.3				46.8
330 to 329	240	41	36.3				46.8
∠ 329 to 331	149	03	56.5	+1.8			58.3
330 to 331	29	46	32.8				45.1
331 to 330	209	45	32.8				45.1
∠ 330 to 332	149	03	56.7	+1.7			58.4
331 to 332	358	49	29.5				43.5
332 to 331	178	49	29.5				43.5
∠ 331 to 333	157	17	47.7	+1.8			49.5
332 to 333	336	07	17.2				33.0
333 to 332	156	07	17.2				33.0
∠ 332 to Wanda	162	52	38.8	+1.7			40.5
333 to Wanda	318	59	56.0		319	00	13.5
Wanda to 333	138	59	56.0		139	00	13.5
∠ 333 to Insane Asylum standpipe	130	13	41.0	+1.8			42.8
Wanda to Insane Asylum standpipe	269	13	37.0				56.3
Fixed azimuth	269	13	56.3				
Discrepancy			-19.3				

Insane Asylum standpipe is found to be $269^{\circ}13'37.0''$ as carried through the traverse, failing to check the fixed grid azimuth of $269^{\circ}13'56.3''$ by $-19.3''$. This discrepancy of $19.3''$ is then divided by the number of angles through the traverse and the result applied to each of the observed angles. The azimuths are then recomputed through the traverse, as shown in the last column of the example. This makes the azimuth of the line *Wanda to Insane Asylum standpipe* as carried through the traverse stations agree with the fixed grid azimuth. The azimuth adjustment is now complete for use in the computation of the traverse.

The next step in the adjustment is to reduce the corrected tape distances between the traverse stations to grid distances. In the example this is started with the taped distances after they have been corrected for tape standardization, inclination, and temperature. It is first necessary to reduce the corrected taped distances to the geodetic distances at sea level as the distance between the control stations *Prosser* and *Wanda* is a sea-level distance. To do this an elevation factor for the line is first computed. In the equation for elevation factor as given just below the following table,

$$\log R_a = 5.8304093 - \frac{1}{2} \log A - \frac{1}{2} \log B,$$

$\log A$ and $\log B$ are taken from the table given in U. S. Coast and Geodetic Survey Special Publication No. 8. The mean latitude of the traverse loop is the argument. The latitude may be taken as the mean of the latitudes of the control stations. (R_a is the mean radius of the spheroid for the given region.)

The grid factors are taken from the projection tables. These projection tables, of which there is one or more for each state, have been prepared by the U. S. Coast and Geodetic Survey and may be obtained upon request to the Director. In the example the argument used in taking out the grid factors from the tables is the mean latitude of the line between adjacent traverse stations. The latitudes were found by plotting the traverse line on the U. S. Geological Survey topographic sheet of the area in which the traverse lies and scaling the latitude of the midpoint of each section. This method can be used, however, only when an adequate map of the area is available. When a map is not available it is necessary to make a preliminary computation of the traverse using the geodetic lengths as the distances between traverse stations. The example does not show this preliminary computation, but it is made in a manner similar to that shown for the final computation. The form, computation of coordinates, is self-explanatory. The latitude and departure of each distance is computed and, starting with the coordinates of the control station, the latitude and departure is added to or subtracted from the coordinates of the preceding station. The latitude and departure are added if north and east respectively, and subtracted if south and west. Then, from table I of the projection tables, take out the scale factor, using the mean of the values of the y coordinates of the traverse stations at each end of the section concerned as the argument. The geodetic distance is then multiplied by this scale or grid factor to give the grid distance. As the example is on the Lambert projection the y coordinates of the stations are used in taking out the grid factors.

Reduction of lengths

[Average elevation=2,055 feet. Elevation factor=0.99990175]

Section	Corrected taped length	Geodetic length	Grid factor	Grid length
	<i>Feet</i>	<i>Feet</i>		<i>Feet</i>
Prosser-325	3,958.752	3,958.363	0.9999363	3,958.111
325-326	7,192.436	7,191.729	.9999372	7,191.277
326-327	4,156.066	4,155.688	.9999385	4,155.432
327-328	5,806.934	5,806.358	.9999400	5,806.006
328-329	3,056.766	3,056.466	.9999411	3,056.286
329-330	6,037.526	6,036.933	.9999420	6,036.583
330-331	5,333.693	5,333.079	.9999432	5,332.776
331-332	6,304.189	6,303.569	.9999452	6,303.224
332-333	6,495.734	6,495.096	.9999472	6,494.753
333-Wanda	4,092.182	4,091.780	.9999478	4,091.566

Mean latitude=40°38.5'

$$\text{Elevation factor} = \frac{R_a}{R_a + \text{average elevation}}$$

$$\log R_a = 5.8304093 - \frac{1}{2} \log A - \frac{1}{2} \log B$$

$$= 5.8304093 - \frac{1}{2}(8.5091022 - 10) - \frac{1}{2}(8.5108029 - 10)$$

$$= 5.8304093 - (9.2545511 - 10) - (9.2554014 - 10)$$

$$= 7.3204568$$

$$R_a = 20,914,950 \quad \text{Elevation factor} = \frac{20,914,950}{20,917,005} = 0.99990175$$

In the formula the constant 5.8304093 is the sum of the colog of the sin of 1" and the log of the reduction factor for meters to feet or 3937/1200. An explanation of this formula is shown in Special Publications No. 194 and No. 195.

Due to the divergence of the origin of the y coordinates and the parallel of latitude through that origin on the central meridian of the Lambert projection, the grid factors obtained by using the y coordinates instead of the latitude are exact only when the traverse lies along the central meridian, the error in the factor increasing with the distance away from that meridian. However, the discrepancy between the true grid factor and that obtained by using the y coordinates is so small that it can be neglected. In the example, the grid factor found in the projection tables, by using the mean latitude of the line as the argument, for the distance between traverse stations No. 325 and No. 326 is 0.9999372, while that found by using the mean of y coordinates of the two stations is 0.9999369, an error of about 1 part in 3,300,000. The distance out from the central meridian in this case is approximately 50 miles.

For computations on the transverse Mercator projection the x' coordinates are always used as the argument in taking out the grid factors. These are found in table III of the projection tables. In using the table for the transverse Mercator projection the x' value used should be the difference between the x coordinate and 500,000 (except for the State of New Jersey where it is 2,000,000) as the central meridian is arbitrarily given that value so that none of the coordinates of the stations on the projection will have a negative sign.

For traverses up to 10 miles in length it is usually unnecessary to obtain the scale factor for each section of the traverse in order to reduce the lengths to grid distances. The scale factor found by using the mean of the coordinates of each end of the line as the argument in the projection tables is sufficiently accurate.

The next step is the final computation of the coordinates and the distribution of the length discrepancies. The adjusted azimuths and the grid distances as found in the preceding paragraph are used. The x and y coordinates were computed, starting with the coordinates of *Prosser* through the traverse to *Wanda*. The difference between the computed coordinates of *Wanda* and the fixed coordinates were found to be +0.41 foot for y and +1.08 feet for x . These discrepancies are to be distributed through the y and x coordinates for the various stations. For the y coordinates, the closing error +0.41 is divided by 52,486, the length of the traverse, the result being a factor to be used in correcting each y coordinate.

The distance from the initial control station to each traverse station, shown by the figures in the third column of the computation form marked with an asterisk (*), is multiplied by this factor and the result applied to the y coordinate for that particular station. In this way the closing error is distributed proportionally to the length of each section of the traverse. The discrepancy for the x coordinate is distributed through the traverse by dividing the closing error +1.08 by 52,486 and using the result as a factor for obtaining the correction to the x coordinates. The corrections are then applied in the same way as was done to the y coordinates. The adjusted coordinates for each of the stations are shown on the form.

It can readily be seen that in distributing the closing error through the coordinates of the traverse stations that the azimuths between adjacent stations would be changed from that used in computing the traverse. Therefore, when using these traverse stations as starting or closing points for other traverses, it is necessary to use a grid azimuth computed from the corrected coordinates of the stations and not that used in computing the traverse.

In the example the difference between the geodetic length and the measured length of a line at about 2,000 feet elevation is of the order of 1 part in 10,000; the difference between the grid length and the geodetic length is of the order of 1 part in 16,000. The difference between the geodetic length and the measured length would be of the order of 1 part in 7,000 if the elevation were about 3,000 feet. The coordinate projections for the various States are such that the difference between the grid length and the geodetic length will seldom exceed 1 part in 10,000. From this it may be seen that, except for areas of considerable elevation, the effect of applying corrections other than that of distributing the closing error of the traverse through the various stations, is of little practical importance. For control surveys, such as executed by the Federal Government and some of the larger cities where an accuracy of better than 1 part in 10,000 is desired, these corrections should be applied to determine whether the desired accuracy is being obtained. For farm surveys and the like, where such a standard of accuracy is unnecessary, the measured distances may be used in computing the coordinates of the traverse stations, all corrections being considered as closing error and distributed as such through the traverse line.

Computation of coordinates

Traverse line: Prosser to Wanda.
 State: Nebraska, south.
 Year: 1894.

Initial station: Prosser.
 Closing station: Wanda.

County: Hall and Adams.
 Month: January-March.

Station	Azimuth (plane)	Grid distance	Log. latitude, Log. cos azimuth, Log. distance, Log. sin azimuth, Log. departure	Latitude	Departure	Grid coordinates	
						<i>y</i>	<i>z</i>
Prosser	" "	Feet		Feet	Feet	Feet	Feet
325	6 09 28.0	3,968.111	3.59497346 9.99749461 3.89748797 9.03069767 3.52810564	-3,935.26		378,552.84	2,262,958.10
326	6 09 28.0	*3,968	3.68369987 9.85718265 3.85690908 9.86112968 3.71798370	-4,943.99	-694.78	374,617.89 -0.03 1374,617.86	2,262,173.23 -0.08 12,262,173.24
327	46 34 43.0	7,191.377	3.80472790 9.89611172 3.61861618 9.90540135 3.42401728	-3,196.89	-5,233.15	369,674.60 -0.09 1969,674.51	2,246,950.17 -0.23 12,246,949.94
328	30 43 23.0	*15,305	3.72007856 9.93173006 3.76342300 9.64674908 3.41809188	-5,249.02	-3,654.71	368,477.71 -0.12 1968,477.59	2,244,293.46 -0.31 12,244,293.15
329	26 30 53.5	*21,171	3.49480356 9.99970057 3.46510399 8.85299915 3.04810314	-3,084.24	-2,618.74	361,223.09 -0.17 1961,223.92	2,241,676.72 -0.44 12,241,676.28
330	327 54 17.4	*94,327			+111.74	358,174.45 -0.19 1958,174.26	2,241,788.46 -0.80 12,241,787.96

330	60 41 48.8	6, 036, 883 *30, 264	3, 470,48009 9, 69858791 3, 73679118 9, 94033388 3, 72182356	-2, 954, 88	-5, 264, 18	335, 210.82 -0.24 1935, 210.86	2, 236, 634.33 -0.63 19, 236, 633.71
331	29 45 45.1	5, 332, 776 *35, 596	3, 68311304 9, 93564480 3, 72963354 9, 69858781 3, 42279055	-4, 629, 33	-2, 647, 23	350, 590.89 -0.28 1950, 590.31	2, 238, 877.11 -0.73 19, 238, 876.38
332	338 49 43.5	6, 308, 224 *41, 900	3, 70947199 9, 99649725 3, 79963274 3, 31048072 2, 11006946	-6, 301, 91	+128, 84	344, 288.68 -0.33 1944, 288.35	2, 224, 003.85 -0.86 19, 224, 003.09
333	336 07 33.0	6, 494, 733 *48, 394	3, 77371621 9, 94113837 3, 31252294 9, 60716485 3, 41972732	-5, 639, 04	+2, 628, 63	338, 349.64 -0.38 1938, 349.26	2, 236, 634.57 -1.00 19, 236, 633.57
Wanda	319 00 13.5	4, 091, 566 *32, 486	3, 43980414 9, 87786168 3, 31868986 3, 31691021 3, 42878977	-3, 088, 12	+2, 684, 11	335, 261.52 -0.41 1935, 261.11 335, 261.11 +0.41	2, 236, 318.68 -1.08 19, 236, 317.60 2, 236, 317.60 +1.08

*Distance from starting point of traverse.

†Adjusted coordinates.

‡Azimuths are clockwise from grid south.

z Factor = -2.08799×10^{-4} .

y Factor = -0.78116×10^{-4} .

D discrepancy =