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The Collimation Error Test— The U.S. Coast and Geodetic Survey Method

by Herbert W. Stoughton

Introduction

The U.S. Coast and Geodetic Survey's test for the collimation error in a geodetic level was developed, in its present form, between 1899 and 1901. The published instructions on performing the test appeared in 1903 (Hayford). The procedure outlined in Rappleye (1948) is unchanged from the earliest published procedure.

Derivation

Set two level rods in a level area no more than 100 m distance from each other (a distance of 50-60 m is preferred). Set the geodetic level near one level rod and then the second level rod as illustrated in Figures 1 and 2. At each instrument setup, the level is leveled, i.e., the bubble is centered, and the rod intercepts are observed. Also, the distance from the level to each level rod is measured. The ratio of the two distances at each setup is between 1:5 and 1:10. The latter ratio approaches the optimum.

The analytical expression for the condition equation resulting from the observations (Figs. 1 and 2) is:

$$O_1 + O_2 \equiv 0 \qquad (1)$$

Where O_i are the data collected/observed at setup i. Expanding equation (1):

$$r_{1,l} - r_{1,s} + r_{2,l} - r_{2,s} = 0$$

Rearranging:

$$(r_{1,s} + r_{2,s}) - (r_{1,l} + r_{2,l}) = 0 \qquad (2)$$

The "line of sight" is actually the "line of collimation." Before continuing with the derivation, it is appropriate to state rigorously the definition of certain technical terms. Failure to do so will result in misunderstanding.

Line of Collimation. The line through the second nodal point of the objective of a telescope and the center of the reticle.

Collimation Axis. The line through the second nodal point of the objective of a telescope perpendicular to the axis of rotation of the telescope.

Error of Collimation. The angle between the line of collimation of a telescope and its collimation axis. Error of collimation is a systematic error and in a series of observations is usually treated as being of the constant error type. When the error of collimation is zero, the line of collimation of the telescope and its collimation axis coincide. (Mitchell, 1948)

Figure 3 illustrates the collimation error, collimation axis, and line of collimation relationships for observations in Figures 1 and 2. The collimation error is the angle θ_C. The error in any level rod reading due to a collimation error is:

$$\epsilon_C = s \tan \theta_C$$

Where:

ϵ_C = error in the level rod reading,

s = the sight distance from the vertical axis of the level to the level rod,

θ_C = the collimation error.

Recall:

$$\tan \phi = \phi_r + \phi_r^3/3 + 2\phi_r^5/15 + 17\phi_r^7/315 + 62\phi_r^9/2835 + \ldots$$

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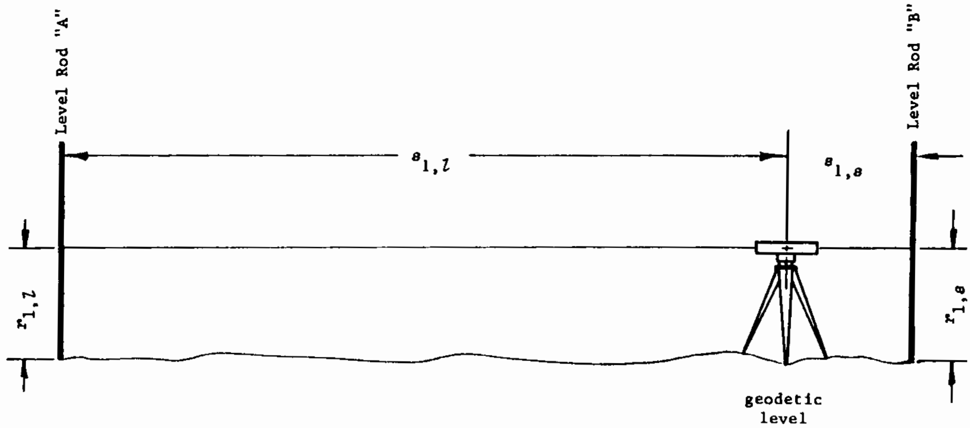

Figure 1. Setup No. 1.

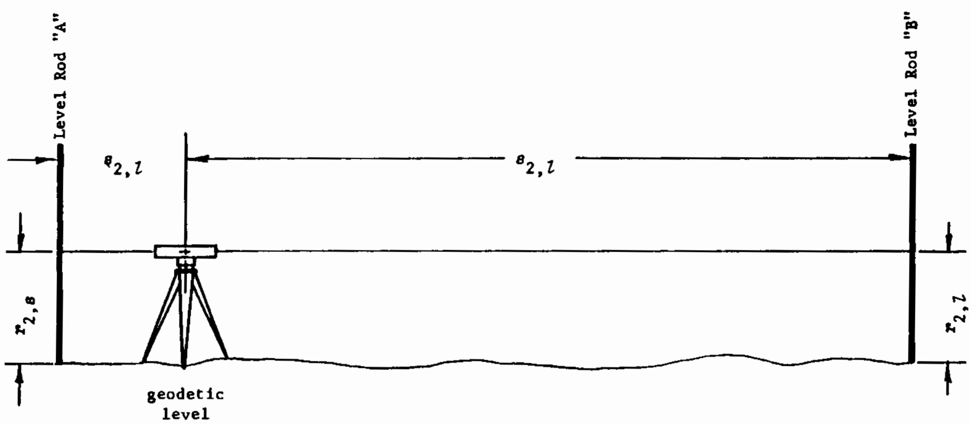

Figure 2. Setup No. 2.

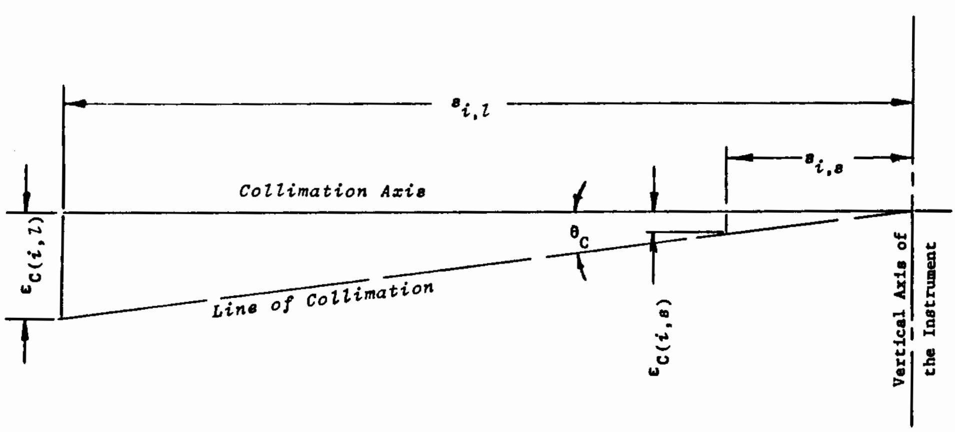

Figure 3.

Then:

$$\tan \phi = \phi_r(1. + \phi_r^2(1/3 + \phi_r^2(2/5 + \phi_r^2(17/315 + \ldots))))$$

$$\phi_r = \phi'' * 4.848136811*10^{-6}$$

$$\phi_r^2 = (\phi'')^2 * 2.35044305*10^{-11}$$

If ϕ equals 100 arc seconds, then:

$$\phi_r = 4.848136811*10^{-4}$$

$$\phi_r^2 = 2.35044305*10^{-7}$$

Then, because $\theta_C \lesssim 100''$:

$$\epsilon_C = s \tan \theta_C \quad (3.a)$$

or:

$$= s\,\theta_{C_r}(1. + (1/3)\,\theta_{C_r}^2) \quad (3.b)$$

Then, the maximum difference of employing either equation (3.a) or (3.b) is:

$$\delta\epsilon_C = (1.496*10^{-10})s$$

Recall, the maximum value of s is 10^6 level rod units, i.e., mm, mft, myd, etc. Therefore, the maximum value of $\delta\epsilon_C$ is $1.496*10^{-4}$ level rod units. Then:

$$\epsilon_C = s\,\theta_C''\text{ arc }1'' \quad (4)$$

Assume that a collimation error, θ_C, exists. Then, disregarding all other error sources, equation (2) becomes:

$$(r_{1,s} + s_{1,s}\,\theta_C''\text{ arc }1'' + r_{2,s} + s_{2,s}\,\theta_C''\text{ arc }1'') - (r_{1,l} + s_{1,l}\,\theta_C''\text{ arc }1'' + r_{2,l} + s_{2,l}\,\theta_C''\text{ arc }1'') = 0$$

Rearranging:

$$(r_{1,s} + r_{2,s}) - r_{1,l} + r_{2,l}) + [(s_{1,s} + s_{2,s}) - (s_{1,l} + s_{2,l})]\,\theta_C''\text{ arc }1'' = 0$$

Then:

$$\theta_C''\text{ arc }1'' = \frac{(r_{1,s} + r_{2,s}) - (r_{1,l} + r_{2,l})}{(s_{1,l} + s_{2,l}) - (s_{1,s} + s_{2,s})} \quad (5)$$

Where:

$(r_{1,s} + r_{2,s})$ = sum of the *near* rod readings.
$(r_{1,l} + r_{2,l})$ = sum of the *far* rod readings.
$(s_{1,l} + s_{2,l})$ = sum of the *far* sight distances.
$(s_{1,s} + s_{2,s})$ = sum of the *near* sight distances.
θ_C'' arc 1″ = collimation error (angle) in *radians.*

Note: The above discussion has addressed the *general case* of determining the collimation error.

Figures 4 and 5 illustrate the observing program for the U.S. Coast and Geodetic Survey procedure for three-wire differential leveling.

From Figures 4 and 5:

$$r_{1,s} = (1/3)(r_{1,s,T} + r_{1,s,M} + r_{1,s,B})$$

$$r_{2,s} = (1/3)(r_{2,s,T} + r_{2,s,M} + r_{2,s,B})$$

$$r_{1,l} = (1/3)(r_{1,l,T} + r_{1,l,M} + r_{1,l,B})$$

$$r_{2,l} = (1/3)(r_{2,l,T} + r_{2,l,M} + r_{2,l,B})$$

$$\delta_{1,s} = r_{1,s,T} - r_{1,s,B}$$

$$\delta_{2,s} = r_{2,s,T} - r_{2,s,B}$$

$$\delta_{1,l} = r_{1,l,T} - r_{1,l,B}$$

$$\delta_{2,l} = r_{2,l,T} - r_{2,l,B}$$

Recall from the laws of optics that:

$$s = K\,\delta\,\cos^2\theta + (C + f)\cos\theta$$

Where:

s = the horizontal distance from the center of the instrument to the level rod.
K = the instrument's stadia interval factor for a pair of wires.
δ = the difference between the two stadia wire readings.
θ = the vertical angle from the horizon for the line of sight.
(C + f) = the stadia constant – sum of the focal length and the distance between the objective lens and the instrument's vertical axis.

For leveling, θ equals 0°. Then:

$$s = K\,\delta + (C + f)$$

Then, the sight distances are:

$$s_{1,s} = K\,\delta_{1,s} + (C + f)$$

$$s_{2,s} = K\,\delta_{2,s} + (C + f)$$

$$s_{1,l} = K\,\delta_{1,l} + (C + f)$$

$$s_{2,l} = K\,\delta_{2,l} + (C + f)$$

And:

$$(s_{1,l} + s_{2,l}) = K\,\delta_{1,l} + K\,\delta_{2,l} + 2(C + f)$$

$$(s_{1,s} + s_{2,s}) = K\,\delta_{1,s} + K\,\delta_{2,s} + 2(C + f)$$

$$(s_{1,l} + s_{2,l}) - (s_{1,s} + s_{2,s}) = K[(\delta_{1,l} + \delta_{2,l}) - (\delta_{1,s} + \delta_{2,s})]$$

Then, equation (5) becomes:

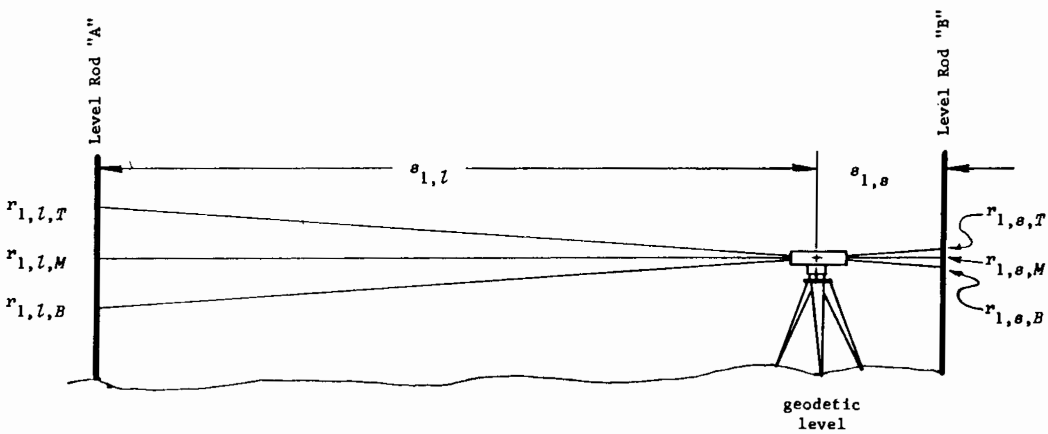

Figure 4. Setup No. 1.

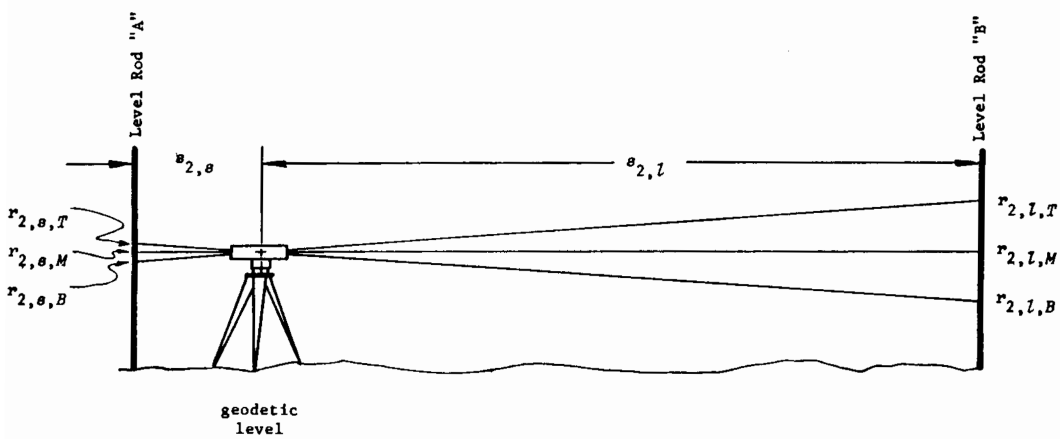

Figure 5. Setup No. 2.

θ''_C arc 1″ =

$$\frac{(r_{1,s} + r_{2,s}) - (r_{1,l} + 2_{2,l})}{K[(\delta_{1,l} + \delta_{2,l}) - (\delta_{1,s} + \delta_{2,s})]} \qquad (5.a)$$

Note: in the general usage of the stadia interval factor, the units are: m/mm, yd/myd, etc. To obtain θ''_C arc 1″ (a *unitless* number), K must be mm/mm. etc.

Finally, the collimation error equation can be written:

$C = K\ \theta''_C$ arc 1″ =

$$\frac{(r_{1,s} + r_{2,s}) - (r_{1,l} + r_{2,l})}{(\delta_{1,l} + \delta_{2,l}) - (\delta_{1,s} + \delta_{2,s})} \qquad (6)$$

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Hayford, John F. (1903), "Hypsometry: Precise Leveling in the United States 1900-1903 with a Readjustment of the Level Net and Resulting Elevations," *Annual Report for 1903*, Appendix No. 3, U.S. Coast and Geodetic Survey, p. 214.

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