UNITS AND SYSTEMS OF MEASURE

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From earliest time, primitive man needed rudimentary measures for a number of different tasks, from determining the size of his dwelling to noting the distance to food. As a basis of measure, man turned to easily identifiable objects such as body parts and the sun and moon as measuring tools.

The first unit of measure that is recorded is the "cubit". The cubit is normally taken to be the length of the forearm from the elbow to the tip of the middle finger. It was divided into the one-half cubit measured as the span of the hand, the palm which was the width of the hand or one-sixth of a cubit, and the digit which was one twenty-fourth of a cubit and measured by the width of a finger. The cubit is known to have been used by the Sumarians about 6,500 years ago. This cubit measured about 10.5 inches in length. One standard used in surveying and construction of buildings was the Royal Egyptian Cubit or Sacred Cubit. This equaled the forearm length the Pharaoh Khafu, the pyramid builder. A Royal Cubit Master was made in black granite and it was required that at each full moon the cubit sticks had to be returned for comparison with the Master. These cubit sticks were normally wood or ordinary granite. Failure to perform the comparison was punishable by death. This cubit was divided into 28 digits. The following relationships can be shown:

1 cubit = 28 digits
4 digits = 1 palm
5 digits = 1 hand
12 digits = small span
14 digits = large span
16 digits = remen
24 digits = small cubit

The early Greeks used the foot as linear measure which, according to legend, was based on the foot of Hercules. Modern measurements have shown that the Greek foot was equal to 12.1375 modern inches.

The Roman contribution to the present system of weights and measures include not only the words from which we derive our present names (for example, both the word "inch" and "ounce" can be derived from the Roman word "unciae") but also the use of 12 as a base. The Roman foot, or "pes" was divided into 12 unciae (it was also divided into 16 digits). The Romans are also credited with introducing the mile, which consisted of 1,000 paces or double steps. The word "mile" is derived from "mille passus" which translates into 1,000 paces. It appears that the Roman foot was only about 0.4" shorter than the foot used today. This 5,000 foot mile was introduced to England during the Roman occupation.

There is some disagreement as to the origin of the yard. Some believe that it evolved from the double cubit while others profess that it originated with the cubit. Nonetheless, it is known that the yard was divided into 2 (half-yard), 4 (span), 8 (finger), and 16 (nail) parts. Hence:

1 yard = 2 half-yards
= 4 spans
= 8 fingers
= 16 nails

The word "yard" is derived from the early Saxon word "gird" which means the circumference of a person's waist. The "standardized yard" was the sash or girdle worn around the waist of the Saxon kings. This could easily be removed and used as a convenient measuring tool. According to tradition, King Henry I, in the 12th century decreed that the length of the yard should be that distance from the tip of his nose to the end of his thumb.

While the Romans brought to England the 5,000 foot mile, Queen Elizabeth I declared, in the 16th century, that the mile was to be replaced by one of 5,280 feet. This was defined so that a convenient relationship between the mile and the furlong could be established (one mile is equal to 8 furlongs exactly). The length of the furlong, or furrow-long, was set by the early Tudor rulers as being 40 rods of five and one-half yards each (220 yards). This was the length of a furrow which could by plowed by an ox-driven plow. From this, the acre was defined by the amount of land that a farmer could plow in one day (40 rods by 4 rods). Rods are also referred to as perches and poles. One particular perch which was used in some parts of England for woodlands and in most parts of Ireland for all lands had a length of 18'. This value was used by the Scots and Irish in some of the New England towns.

While each ruler could determine the standard length of their linear measurements, chaos was especially prevalent during the Middle Ages since each feudal lord could set their own standard. Hence, in Europe, the length of the foot varied from 10" to 20" depending on the standard adopted by each lord. Some examples of standards developed during history are as follows:

- King David I of Scotland defined, in 1150, the inch to be the average length of the thumbs of a large man, medium-sized man and a small man.
- The inch was defined as the width of three barleycorns laid together. These barleycorns had to be round and dry.
- In 1305, Edward I defined the foot in England to be twelve three-barleycorn inches.
- The rod was originally established by the Saxons to be 15 feet long where the Saxon foot was 13.2 inches. When Edward I redefined the length of the foot, the

physical length of the rod remained the same but the number of feet in the rod became 16.5 feet, where it remains today. This is commonly referred to as the statute rod.

- In the 16th century the German author, Master Koebel wrote that the length of the rod was determined in the 16th century by the length of the left foot of the first 16 men, both big and small, who came out of church on a certain Sunday.

The 16.5' rod described by Koebel was taken to England by the Saxons. Even though Edward I standardized the rod to be 16.5' in length, there are numerous examples where rods of different length were used in surveying. These other lengths are given as [Toscano, 1991]:

Woodland lands18'
Fields16.5'
Arable lands12'
Meadows18'
Rocky land20' or 22'
Church land21'

A very common measurement tool was the chain. Its invention has been attributed to Aaron Rathborne. In his book, The Surveyor in Foure Bookes, published in 1616, the chain is described as being 2 statute rods (at most 3 rods) in length because Rathborne felt that a longer distance would make the chain susceptible to breaking. Each chain was subdivided into Units (1 unit = 1 rod) which were further divided into Primer (1 unit = 10 primer; a Prime measuring 19.8") and finally each Prime was separated into seconds (1 Prime = 10 seconds; each second measuring 1.98"). Rathborne suggested that a distance of 15 rods, 4 primes, and 52 seconds be recorded as 15µ 8' 52". [Toscano, 1991]

The decimal chain utilized in the U.S. was that developed by Edmund Gunter (an Oxford University professor and astronomer) in 1620. The Gunter chain is divided into 100 links, each link being 0.66' in length. Gunter's chain offered two advantages over Rathborne's which made the use of the chain easier to use thereby making it the preferred chain used by surveyors. The advantages are [Toscano, 1991]:

1. Unlike Rathborne's chain, Gunter did not have a zero-point. Thus, either end of the tape could be used for the zero-point.
2. The Gunter chain has only one division, the link, whereas Rathborne utilized three divisions.

The standard Gunter Chain was 4 poles long. Although the construction of the chain varied, many had tally marks located throughout its length [Toscano, 1991]. These marks were frequently brass tags or large rings that were used to break the tape into 10-link segments. Many had points on the mark that were used to located their place along the

tape. For example, at 20-links the brass tag would have two points; at 30-links, 3 points; etc. The mid-point of the chain (50-link) usually consisted of a tag that was larger than the others. It may also have a point. The length of the chain included the length of both handles.

There were a number of variations of the Gunter chain. For example, the Philadelphia chain was also 66' long (or sometimes 33') [Toscano, 1991]. This chain was divided into 80-links (40 links on the 33' chain). Therefore, one link was 1/20th of a rod which represents 9.9 inches. The Grumman chain was 66' in length but its construction was different. The chain itself was made of a light tempered wire. This chain was constructed to be used with a spring balance, thermometer and plumb bob. Thus, more precise measurements were possible.

In those areas that may have been under French control, an arpent can be found. The arpent is actually a unit of area. The length of the arpent varied in different localities. The following relationships have be defined for the arpent:

1 arpent = 0.8507 acres (Arkansas and Missouri)
1 arpent = 0.84625 acres (Mississippi, Alabama, and Florida)
1 arpent = 0.845 acres (Louisiana)
1 arpent = 30 toises
30 toises = 160 French feet
1065.75 feet(U.S.) = 1000 French feet (Louisiana)
arpent frontage = 192.50' (Arkansas and Missouri)
arpent frontage = 191.994' (Mississippi, Alabama, and Florida)

In the Southwest, a very common unit of measure is the vara which is around 33 inches in length. This value varies between states countries that were at one time under Spanish rule..

As one can see, there have been a number of different length units adopted around the world. This problem bothered scientists in the 17th and 18th century because they wanted one standard and uniform unit of length. In addition, the standards that were in existence, for example the Toise in France, could not be defined other than by a physical standard. This led the scientists to look for a better definition by which a standard could be measured and reproduced. On May 20, 1875, seventeen countries signed the International Metric Convention establishing the International Bureau of Weights and Measures. The physical standard was to be one ten-millionth of the earth's meridian, a distance from the North Pole to the Equator, running near Dunkirk in France and Barcelona in Spain. A platinum bar (called the metre des Archives) was created to be a tangible item of the length and it became the prototype meter. Subsequent surveying activities found that the prototype did not reflect the true distance, thus the use of the Earth as a standard was discontinued. In 1889, a new metric length called the International Prototype Meter, made of

platinum-iridium, was selected as the standard. Great care was employed in the selection of a metal which exhibited dimensional stability.

In the United States, the use of the metric system was legalized in 1866 by President Andrew Johnson even though it was used earlier by Ferdinand R. Hassler of the Coast Survey (now Coast and Geodetic Survey, NOS, NOAA) who brought an iron copy of the French meter to the U.S. One important aspect of the 1866 metric bills was the specification of the English equivalent. The foot/meter ratio was defined as 3937/1200 exact. After the International Prototype Meter was established, two copies were sent to the U.S. These were declared the fundamental standards in 1893 in what has been called the Mendenhall Order, after the Superintendent of the Weights and Measures. The same foot/meter relationship was maintained.

Because of inconsistencies in the yard and meter both in the United States and abroad, an agreement between the U.S. and the United Kingdom, in 1959, further refined this relationship. From that agreement, the yard was defined as being 0.9144 meter exact, or a foot being equal to 0.3048 meter exact. This had the effect of shortening the foot by two parts in a million. Thus, this refinement defines the international foot. Because of the problems in surveying and mapping, the old standard (3937/1200) was maintained for surveying purposes and this is called the U.S. Survey foot.

While the International Prototype Meter was an accurate standard when it was developed, there existed a need for higher accuracies in science. Therefore, in 1960 the length of the meter was redefined to be equal to 1,650,763.73 wavelengths in a vacuum of orange-red light of the krypton 86 atom. This became known as the National Prototype Meter. This modernized system was called Le Systeme International d'Unites (International System of Units) and is commonly referred to as the SI system. On December 23, 1975, President Ford signed the Metric Conversion Act of 1975 calling for gradual conversion to the metric system.

TABLE 1. LINEAR MEASURE (U.S. SURVEY FOOT)

12 inches	= 1 foot
3 feet	= 1 yard
16.5 feet	= 1 rod, pole, or perch
0.66 foot	= 1 link
100 links	= 1 chain
	= 4 rods
	= 66 feet
40 rods	= 1 furlong
	= 660 feet
1 mile	= 8 furlong
	= 80 chains
	= 320 rods
	= 5280 feet

TABLE 2. AREA MEASURE

144 squares inches	= 1 square foot
9 square feet	= 1 square yard
	= 1296 square inches
272.25 square feet	= 1 square rod
160 square rods	= 1 acre
	= 43560 square feet
640 acres	= 1 square mile
1 mile square	= 1 section of land
6 miles square	= 1 township
	= 36 sections
	= 36 square miles

TABLE 3. TABLE OF EQUIVALENTS USING U.S. SURVEY FOOT

1 centimeter	=	0.3937 inch
1 chain	=	66 feet (exact)
	=	20.1168 meters
1 fathom	=	6 feet (exact)
	=	1.8288 meters
1 foot	=	1200/3937 meters (exact)
	=	0.3048 meter
1 furlong	=	10 chains (exact)
	=	660 feet (exact)
	=	1/8 survey mile (exact)
	=	201.168 meters
1 inch	=	2.54 centimeters (exact, International ft)
1 kilometer	=	0.621 mile
1 league (land)	=	3 survey miles (exact)
	=	4.828 kilometers
1 link	=	0.66 foot (exact)
	=	0.201168 meter
1 meter	=	39.37 inches (exact)
	=	1.094 yards
1 mile	=	5280 feet (exact)
	=	1.609 kilometers
1 rod, pole, perch	=	16.5 feet (exact)
	=	5.0292 meters
1 yard	=	0.9144 meter
1 acre	=	43560 square feet
	=	0.405 hectare
1 hectare	=	2.471 acres
1 square mile	=	640 acres
	=	258.999 hectares

Like distance measurements, there are several different systems of angular measurement. From basic mathematics, one knows that the circumference of a circle can be written as

$$1 \text{ Circumference} = 2\pi r$$

where r is the radius of the circle. If one uses a unit circle (r = 1), then what remains is the radian measure where

$$1 \text{ Circumference} = 2\pi \text{ (radians)}$$

This is the system used in calculations in computers and calculators.

The most common angular measuring system for surveying and mapping in North America is the Sexagesimal System. In this system the angle is expressed in degrees, minutes, and seconds. The relationships are expressed as:

$$\begin{aligned} 1 \text{ Circumference} &= 360^{o} \text{ (Degrees of Arc)} \\ 1 \text{ Degree} &= 60' \text{ (Minutes of Arc)} \\ 1 \text{ Minute} &= 60'' \text{ (Seconds of Arc)} \end{aligned}$$

This system is very cumbersome for calculation purposes. Therefore, when working with calculators, the angle must usually be converted to a decimal form. This is easily done by the following formula:

Decimal Deg = Degree + (minutes/60) + (seconds/3600)

Going back to the degree-minute-second format is a little more time consuming. It can be done as follows:

Degree = Integer of decimal degree

Minutes = Integer of [(decimal degree - Degree)*60]

Seconds = [decimal minutes - Minutes]*60

For example, an angle expressed as 37^{o} 45' 17'' can be written in decimal form as:

$$\begin{aligned} \text{Decimal Degree} &= 37^{o} + (45'/60) + (17''/3600) \\ &= 37^{o} + 0.75^{o} + 0.00472222^{o} \\ &= 37.75472222^{o} \end{aligned}$$

Hence

$37^o\ 45'\ 17''\ = 37.75472222^o$

To obtain the degree-minute-second equivalent of this angle in decimal form the degrees can be written as

$$\text{Degree} = 37^o$$

Minutes are found by

$$\begin{aligned}\text{Minutes} &= (37.75472222^o - 37^o)*60 \\ &= 45.28333333'\end{aligned}$$

Therefore, taking the integer value results in

$$\text{Minutes} = 45'$$

and finally, seconds can be found by

$$\begin{aligned}\text{Seconds} &= (45.28333333' - 45')*60 \\ &= 17''\end{aligned}$$

It should be noted that in the degree-minute-second format, only seconds can contain a decimal portion. Most calculators have function keys that will perform his calculation automatically.

Another system commonly used in Europe and considered as the metric equivalent is the Centesimal System. The relationships present in this system are shown as:

$$\begin{aligned}1 \text{ Circumference} &= 400^g (\text{Gons - previously referred to as grads}) \\ 1 \text{ gon} &= 100^c (\text{Centigon or centesimal minutes}) \\ 1 \text{ Centigon} &= 100^{cc} (\text{Decimilligons or centesimal seconds})\end{aligned}$$

One can see from this system that decimal values are obtained directly.

A fourth system utilized very often in the military, especially in Artillery, is the Coast Artillery System where

$$1 \text{ circumference} = 6400 \text{ mils}$$

A final system, although not employed or accepted in surveying is the Metric Time-Angle System where [Clark, 1982]:

$$1 \text{ circumference} = 1.00\ \Theta \text{ (theta)}$$

Clark offered this system as the metric equivalent to the length standard. Most surveyors consider the centesimal system as being the metric angular system.

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