TRAVERSING AND TRAVERSE ADJUSTMENT

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COMPUTING TRUE VALUE

- True value is recorded value plus the correction:

$$T = R + C$$

  - Always added to the reading
- Error:

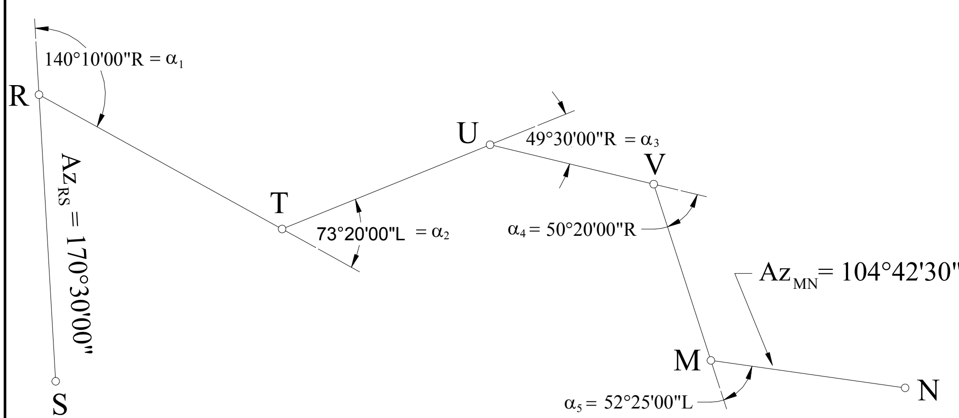
$$E = R - T$$

- Always subtracted from the reading

TRAVERSES

- Open v. closed traverses
- Open loop v. closed loop traverses
- Types of angles measured
  - Deflection angles
  - Interior angles of traverse
  - Exterior angles of traverse
  - Angles to the right

DEFLECTION ANGLE TRAVERSE



$$Az_1 + \sum_{i=1}^{n} \alpha_{R_i} - \sum_{i=1}^{n} \alpha_{L_i} - Az_2 - 360^{o} = 0$$

$$\sum \alpha_R = \alpha_1 + \alpha_3 + \alpha_4$$
$$= \left(140^o\ 10'00''\right) + \left(49^o\ 30'00''\right) + \left(50^o\ 20'00''\right)$$
$$= 240^o\ 00'00''$$

$$\sum \alpha_L = \alpha_2 + \alpha_5$$
$$= \left(73^o\ 20'00''\right) + \left(52^o\ 25'00''\right)$$
$$= 125^o\ 45'00''$$

$$Az_{S-R} + \sum \alpha_R - \sum \alpha_L - Az_{M-N} - 360^o = E_\angle$$

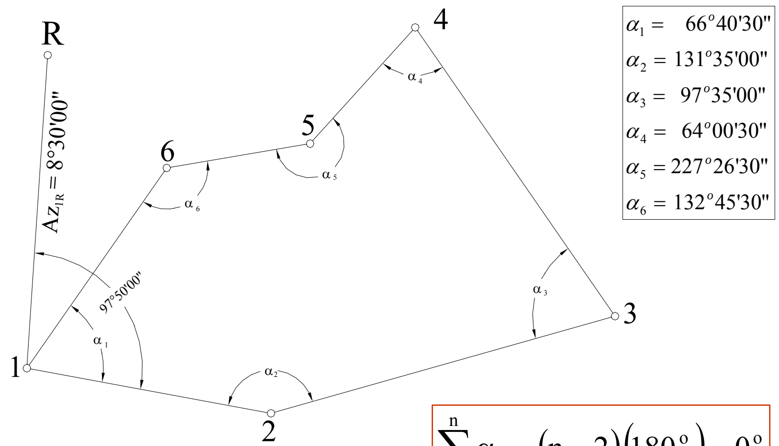
$$\left(350^o\ 30'00''\right) + \left(240^o\ 00'00''\right) - \left(125^o\ 45'00''\right) - \left(104^o\ 42'30''\right) - 360^o = +0^o\ 02'30''$$

$$cpa = -\frac{E_\angle}{n} = -\frac{0^o\ 02'30''}{5} = -30''$$

α1	=	+140° 10' 00"	- 30"	= +140° 09' 30"
α2	=	- 73° 20' 00"	- 30"	= - 73° 20' 30"
α3	=	+ 49° 30' 00"	- 30"	= + 49° 29' 30"
α4	=	+ 50° 20' 00"	- 30"	= + 50° 19' 30"
α5	=	- 52° 25' 00"	- 30"	= - 52° 25' 30"
				+114° 12' 30"

$$\left(350^o\ 30'00''\right) + \left(114^o\ 12'30''\right) - \left(104^o\ 42'30''\right) - 360^o = 0^o \quad \surd$$

INTERIOR ANGLE TRAVERSE



$$\sum_{i=1}^{n} \alpha_i - (n-2)(180^\circ) = 0^\circ$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 - (6-2)(180^\circ) = E_\angle$$
$$(720^\circ\ 03'\ 00'') - (720^\circ) = +3'\ 00''$$

The correction per angle (cpa) is next determined.

$$cpa = -\frac{30'\ 00''}{6} = -30''$$

The corrected angles are found by adding the cpa to each observation.

$$\alpha_1 = (66^\circ\ 40'\ 30'') + (-30'') = 66^\circ\ 40'\ 00''$$
$$\alpha_2 = (131^\circ\ 35'\ 00'') + (-30'') = 131^\circ\ 34'\ 30''$$
$$\alpha_3 = (97^\circ\ 35'\ 00'') + (-30'') = 97^\circ\ 34'\ 30''$$
$$\alpha_4 = (64^\circ\ 00'\ 30'') + (-30'') = 64^\circ\ 00'\ 00''$$
$$\alpha_5 = (227^\circ\ 26'\ 30'') + (-30'') = 227^\circ\ 26'\ 00''$$
$$\alpha_6 = (132^\circ\ 45'\ 30'') + (-30'') = 132^\circ\ 45'\ 00''$$
$$\sum \alpha_i = 720^\circ\ 00'\ 00''$$

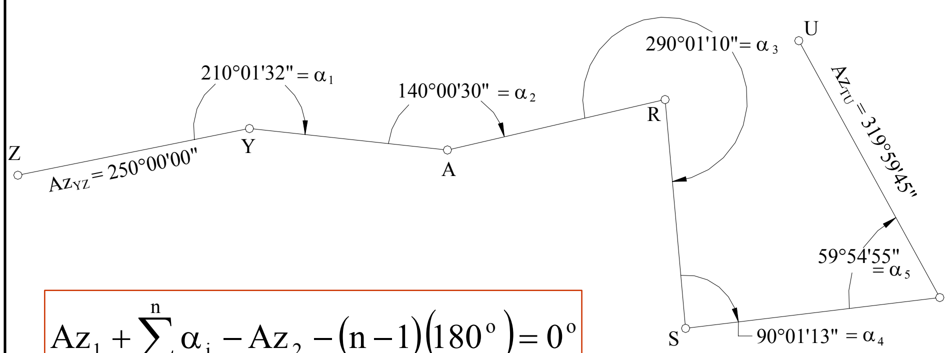
EXTERIOR ANGLE TRAVERSE

- Special case of interior angle traverse

$$\sum_{i=1}^{n} \alpha_{i_{Ext}} - (n+2)(180^{o}) = 0^{o}$$

- Note that interior and exterior angle traverses are closed loop traverses.

ANGLES TO THE RIGHT



$$Az_1 + \sum_{i=1}^{n} \alpha_i - Az_2 - (n-1)(180^{o}) = 0^{o}$$

From the example, the angular error of closure is found as

$$Az_{YZ} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 - Az_{TU} - (5-1)(180^\circ) = E_\angle$$

$$(250^\circ 00'00'') + (789^\circ 59'20'') - (319^\circ 59'45'') - 720^\circ = -00'\,25''$$

The correction per angle is

$$cpa = -\frac{25''}{5} = 5''$$

The corrected angles are found by adding the cpa to each measurement.

$$\alpha_1 = (210^\circ\ 01'\,32'') + 5'' = 210^\circ\ 01'\,37''$$
$$\alpha_2 = (140^\circ\ 00'\,30'') + 5'' = 140^\circ\ 00'\,35''$$
$$\alpha_3 = (290^\circ\ 01'\,10'') + 5'' = 290^\circ\ 01'\,15''$$
$$\alpha_4 = (90^\circ\ 01'\,13'') + 5'' = 90^\circ\ 01'\,18''$$
$$\alpha_5 = (59^\circ\ 54'\,55'') + 5'' = \underline{59^\circ\ 55'\,00''}$$
$$\sum \alpha_i = 789^\circ\ 59'\,45''$$

Check using the general relationship for the angles to the right.

$$(250^\circ 00'00'') + (789^\circ 59'45'') - (319^\circ 59'45'') - 720^\circ = 0^\circ \quad \surd$$

STEPS IN TRAVERSE ADJUSTMENT

1. Balance/adjust angles
2. Compute azimuths/bearings of lines
3. Compute latitudes and departures

$$\mathrm{Lat} = s \cdot \cos\alpha = \Delta Y$$

$$\mathrm{Dep} = s \cdot \sin\alpha = \Delta X$$

4. Determine closure error

$$e_D = dX = \sum_{i=1}^{n-1} \mathrm{Lat}_{i,i+1} - (X_n - X_1)$$

$$e_L = DY = \sum_{i=1}^{n-1} \mathrm{Dep}_{i,i+1} - (Y_n - Y_1)$$

STEPS IN TRAVERSE ADJUSTMENT

5. Compute linear error of closure, accuracy ratio, direction of closing line

$$E_c = \sqrt{e_D^2 + e_L^2}$$

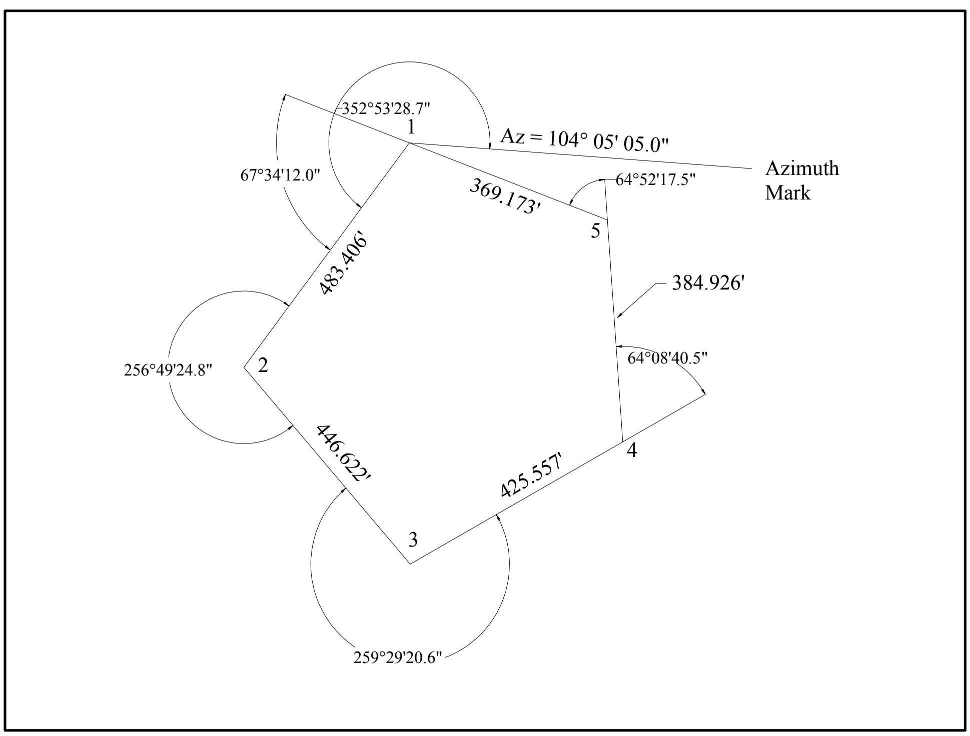
$$\mathrm{Accuracy} = E_c / P$$

$$\beta_e = \tan^{-1}\left(\frac{e_D}{e_L}\right)$$

6. Adjust traverse and compute adjusted distances, directions and coordinates
7. Compute area of traverse

EXAMPLE

Station	Angle	Distance
Az. Mark		
1	-67° 34’ 12.0” D	483.406’
2	256° 49’ 24.8” L	446.622’
3	259° 29’ 20.6” L	425.557’
4	-64° 08’ 40.5” D	384.926’
5	-64° 52’ 17.5” D	369.173’
1	352° 53’ 28.7” R	
Az. Mark		



Azimuth	1 ⇒ Az. Mark	104° 05’ 05.0”
	+180°	180° 00’ 00.0”
	Back Azimuth	284° 05’ 05.0”
	$-\angle_1$	-67° 34’ 12.0”
Azimuth	1 - 2	216° 30’ 53.0”
	$-\angle_2$	-256° 49’ 24.8”
		-40° 18’ 31.8”
	+ 180°	180° 00’ 00.0”
Azimuth	2 - 3	139° 41’ 28.2”
	+ 180°	180° 00’ 00.0”
	$-\angle_3$	-259° 29’ 20.6”
Azimuth	3 - 4	60° 12’ 07.6”
	$-\angle_4$	-64° 08’ 40.5”
	+ 360°	360° 00’ 00.0”
Azimuth	4 - 5	356° 03’ 27.1”
	$-\angle_5$	-64° 52’ 17.5”
Azimuth	5 - 1	291° 11’ 09.6”
	-180°	-180° 00’ 00.0”
	+ ∠ to Az. Mark	356° 53’ 28.7”
	-360°	-360° 00’ 00.0”
Azimuth	1 – Az. Mark	104° 04’ 38.2”

The angular error of closure is

$$E_{\angle} = \left(\text{Az. } 1 \rightarrow \text{Az Mark}_{\text{Calc}}\right) - \left(\text{Az. } 1 \rightarrow \text{Az Mark}_{\text{True}}\right)$$
$$= \left(104^{\circ}\ 04'38.3''\right) - \left(104^{\circ}\ 05'05.0''\right)$$
$$= -26.7''$$

The correction per angle is found using

$$\text{cpa} = -\frac{E_{\angle}}{n} = -\frac{-26.7''}{6} = 4.45''$$

ADJUSTED AZIMUTHS

Line	Preliminary Azimuth	Correction	Adjusted Azimuth
1-2	216° 30’ 53.0”	+04.5”	216° 30’ 57.5”
2-3	139° 41’ 28.2”	+08.9”	139° 41’ 37.1”
3-4	60° 12’ 07.6”	+13.4”	60° 12’ 21.0”
4-5	356° 03’ 27.1”	+17.8”	356° 03’ 44.9”
5-1	291° 11’ 09.6”	+22.3”	291° 11’ 31.9”
1-Az Mk	104° 04’ 38.3”	+26.7”	104° 05’ 05.0”

LATITUDES AND DEPARTURES

Traverse Adjustment Program						
		Azimuth				
Sta	Dist	Deg	Min	Sec	Departure	Latitude
1						
	483.406	216	30	57.5	-287.649	-388.509
2						
	446.622	139	41	37.1	288.908	-340.592
3						
	425.557	60	12	21	369.305	211.453
4						
	384.926	356	3	44.9	-26.432	384.017
5						
	369.173	291	11	31.9	-344.207	133.455
1						
	2109.684				-0.075	-0.176

Error in closure:

$$e_D = dX = \sum_{i=1}^{n-1} Lat_{i,i+1} = -0.075'$$

$$e_L = DY = \sum_{i=1}^{n-1} Dep_{i,i+1} = -0.176'$$

Linear error of closure

$$E_c = \sqrt{(-0.075)^2 + (-0.176)^2} = 0.19$$

Azimuth of the closing line

$$\beta_e = \tan^{-1}\left(\frac{-0.075'}{-0.176'}\right) = 203^o\ 04'50''$$

Accuracy ratio

$$\text{Accuracy} = \frac{0.191'}{2109.684} = \frac{1}{11{,}045}$$

$$\approx 1:11{,}000$$

COMPASS RULE ADJUSTMENT

$$c_L = \left(L/P\right) cl_L$$

$$c_D = \left(L/P\right) cl_D$$

where:
- c_L, c_D = corrections for latitudes and departures respectively,
- L = length of the line,
- P = perimeter distance around the traverse, and
- cl_L, cl_D = closure correction in latitude and departure, respectively, for the traverse.

From the example,

For line 1-2:

$$c_L = \left(\frac{0.176'}{2109.684'}\right) 483.406' = +0.040'$$

$$c_D = \left(\frac{0.075'}{2109.684'}\right) 483.406' = +0.017'$$

Line 2-3:

$$c_L = \left(\frac{0.176'}{2109.684'}\right) 446.622' = +0.037'$$

$$c_D = \left(\frac{0.075'}{2109.684'}\right) 446.622' = +0.016'$$

Traverse Adjustment Program

			Corrections to		Corrected	Corrected
Sta	Departure	Latitude	Departure	Latitude	Departure	Latitude
1						
	-287.649	-388.509	0.017	0.040	-287.632	-388.469
2						
	288.908	-340.592	0.016	0.037	288.924	-340.555
3						
	369.305	211.453	0.015	0.035	369.320	211.489
4						
	-26.432	384.017	0.014	0.032	-26.419	384.049
5						
	-344.207	133.455	0.013	0.031	-344.194	133.486
1						
	-0.075	-0.176	0.075	0.176	0.000	0.000

DETERMINE COORDINATES

$$X_{n+1} = X_n + Dep_{n-(n+1)}$$

$$Y_{n+1} = Y_n + Lat_{n-(n+1)}$$

Traverse Adjustment Program				
	Corrected	Corrected		
Sta	Departure	Latitude	X	Y
1			7885.572	7097.635
	-287.632	-388.469		
2			7597.940	6709.166
	288.924	-340.555		
3			7886.864	6368.611
	369.320	211.489		
4			8256.185	6580.100
	-26.419	384.049		
5			8229.766	6964.149
	-344.194	133.486		
1			7885.572	7097.635
	0.000	0.000		

COMPUTE ADJUSTED DISTANCES AND DIRECTIONS

$$D_{n-(n+1)} = \sqrt{(X_{n+1} - X_n)^2 + (Y_{n+1} - Y_n)^2}$$

$$= \sqrt{Dep^2_{n-(n+1)} + Lat^2_{n-(n+1)}}$$

$$\alpha_{n-(n+1)} = \tan^{-1}\left[\frac{X_{n+1} - X_n}{Y_{n+1} - Y_n}\right] = \tan^{-1}\left[\frac{Dep_{n-(n+1)}}{Lat_{n-(n+1)}}\right]$$

FROM EXAMPLE

Line 1-2:

$$D_{1-2} = \left[(5172.813' - 5460.445')^2 + (5849.543' - 6238.012')^2\right] = 483.364'$$

$$\alpha_{1-2} = \tan^{-1}\left[\frac{5172.813' - 5460.445'}{5849.543' - 6238.012'}\right] = 216° \, 31' \, 01.8''$$

Line 2-3:

$$D_{2-3} = \left[(5461.737' - 5172.813')^2 + (5508.988' - 5849.543')^2\right] = 446.604'$$

$$\alpha_{1-2} = \tan^{-1}\left[\frac{5461.737' - 5172.813'}{5508.988' - 5849.543'}\right] = 139° \, 41' \, 20.4''$$

etc….

Traverse Adjustment Program						
				Adjusted		
			Adjusted	Azimuth		
Sta	X	Y	Distance	Deg	Min	Sec
1	7885.572	7097.635				
			483.363	216	31	1.8
2	7597.940	6709.166				
			446.604	139	41	20.4
3	7886.864	6368.611				
			425.588	60	12	9.7
4	8256.185	6580.100				
			384.957	356	3	53.4
5	8229.766	6964.149				
			369.172	291	11	50.6
1	7885.572	7097.635				

TRANSIT RULE

$$c_L = \frac{cl_L}{\sum|Lat|} s_L$$

$$c_D = \frac{cl_D}{\sum|Dep|} s_D$$

where:

	c_L, c_D	=	corrections to the latitudes and departures of the line,
	cl_L, cl_D	=	closure corrections for the latitudes and departures for the traverse,
	$\Sigma\|Lat\|$, $\Sigma\|Dep\|$	=	sum of the absolute values of the latitudes and departures, and
	s_L, s_D	=	latitude and departure length of the line (absolute values).

FROM EXAMPLE

Line 1-2:

$$c_L = \left(\frac{0.176'}{1458.026'}\right) 388.509' = +0.047'$$

$$c_D = \left(\frac{0.075'}{1316.501'}\right) 287.649' = +0.016'$$

Line 2-3:

$$c_L = \left(\frac{0.176'}{1458.026'}\right) 340.592' = +0.041'$$

$$c_D = \left(\frac{0.075'}{1316.501'}\right) 288.908' = +0.016'$$

etc….

Traverse Adjustment Program						
	Transit Rule					
			Corrections to		Corrected	Corrected
Sta	Departure	Latitude	Departure	Latitude	Departure	Latitude
1						
	-287.649	-388.509	0.016	0.047	-287.633	-388.462
2						
	288.908	-340.592	0.016	0.041	288.925	-340.551
3						
	369.305	211.453	0.021	0.025	369.326	211.479
4						
	-26.432	384.017	0.002	0.046	-26.431	384.064
5						
	-344.207	133.455	0.020	0.016	-344.187	133.471
1						
	-0.075	-0.176	0.075	0.176	0.000	0.000

Traverse Adjustment Program								
						Adjusted		
	Corrected	Corrected			Adjusted	Azimuth		
Sta	Departure	Latitude	X	Y	Distance	Deg	Min	Sec
1			7885.572	7097.635				
	-421.071	-153.909			448.318	249	55	18.5
2			7464.501	6943.726				
	-159.696	-411.834			441.712	201	11	40.7
3			7304.805	6531.893				
	285.578	-380.184			475.494	143	5	15.7
4			7590.383	6151.708				
	430.139	49.312			432.957	83	27	36.2
5			8020.522	6201.020				
	187.862	339.346			387.877	28	58	8.2
6			8208.385	6540.367				
	-229.663	374.666			439.454	328	29	32.9
7			7978.722	6915.032				
	-93.150	182.603			204.989	332	58	22.2
1			7885.572	7097.635				

CRANDALL METHOD

$$A = \frac{e_D\left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right) - e_L\left(\sum_{i=1}^{n}\frac{Dep^2}{100s}\right)}{\left(\sum_{i=1}^{n}\frac{Dep^2}{100s}\right)\left(\sum_{i=1}^{n}\frac{Lat^2}{100s}\right) - \left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right)^2}$$

$$B = \frac{e_L\left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right) - e_D\left(\sum_{i=1}^{n}\frac{Lat^2}{100s}\right)}{\left(\sum_{i=1}^{n}\frac{Dep^2}{100s}\right)\left(\sum_{i=1}^{n}\frac{Lat^2}{100s}\right) - \left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right)^2}$$

CORRECTION APPLIED TO PARTICULAR LINE

$$c_{\ell_1} = Lat_1 A + Dep_1 B$$

$$c_{\ell_2} = Lat_2 A + Dep_2 B$$

$$c_{\ell_3} = Lat_3 A + Dep_3 B$$

$$\vdots$$

$$c_{\ell_n} = Lat_n A + Dep_n B$$

CORRECTIONS TO LATITUDES AND DEPARTURES

- General form of correction

$$c_L = c_\ell \frac{\text{Lat}}{100s}$$

$$c_D = c_\ell \frac{\text{Dep}}{100s}$$

- Substituting into previous equation:

$$c_L = A\frac{\text{Lat}^2}{100s} + B\frac{\text{Lat}\cdot\text{Dep}}{100s}$$

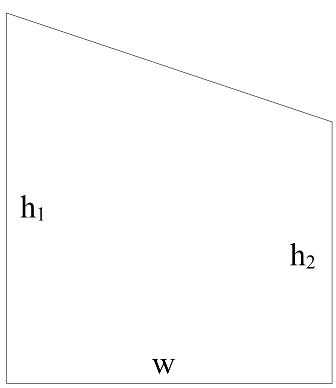
$$c_D = A\frac{\text{Lat}\cdot\text{Dep}}{100s} + B\frac{\text{Dep}^2}{100s}$$

FROM EXAMPLE

Traverse Adjustment Program										
							Correction to		Adjusted	
Point	Distance	Departure	Latitude	1	2	3	Departure	Latitude	Departure	Latitude
1										
	483.406	-287.6492	-388.5091	3.122412	1.711648	2.311811	0.048	0.064	-287.602	-388.445
2										
	446.622	288.9084	-340.5924	2.597346	1.868874	-2.203205	-0.022	0.025	288.887	-340.567
3										
	425.557	369.3052	211.4531	1.05068	3.20489	1.835024	0.050	0.029	369.356	211.482
4										
	384.926	-26.43238	384.0174	3.831109	0.018151	-0.2637	-0.004	0.058	-26.436	384.075
5										
	369.173	-344.207	133.4551	0.482437	3.209293	-1.2443	0.002	-0.001	-344.204	133.454
1										
Sum =	2109.684	-0.075009	-0.175764	11.08398	10.01286	0.435631	0.075	0.176	0.000	0.000

Traverse Adjustment Program								
					Adjusted			
	Adjusted					Azimuth		
Point	Departure	Latitude	X	Y	Distance	Deg	Min	Sec
1			5460.445	6238.012				
	-287.602	-388.445			483.326	216	30	57.5
2			5172.843	5849.567				
	288.887	-340.567			446.589	139	41	37.1
3			5461.730	5509.000				
	369.356	211.482			425.615	60	12	21
4			5831.086	5720.483				
	-26.436	384.075			384.984	356	3	44.9
5			5804.649	6104.558				
	-344.204	133.454			369.17	291	11	31.9
1			5460.445	6238.012				

AREA CALCULATIONS

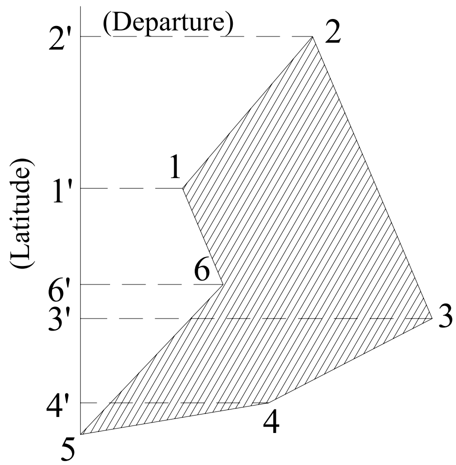


$$\text{Area} = \frac{h_1 + h_2}{2} \cdot w$$

Double area:

$$2A = (h_1 + h_2)w$$

DOUBLE MERIDIAN DISTANCE



$$2A = 2A_{2,2',3,3'} + 2A_{3,3',4,4'} + 2A_{4.4',5} - 2A_{5,6,6'} - 2A_{1,1',6,6'} - 2A_{1,1',2,2'}$$

DMD

1. DMD of first line equal departure of that line
2. DMD of each subsequent line equals the DMD of preceding line + departure of previous line + departure of line in question
3. DMD of last line should equal departure of last line, with opposite sign.

DMD

$$
\begin{aligned}
2A = {} & DMD_{5-6} * Lat_{5-6} + \left(DMD_{5-6} + Dep_{5-6} + Dep_{6-1}\right) * Lat_{6-1} \\
& + \left(DMD_{6-1} + Dep_{6-1} + Dep_{1-2}\right) * Lat_{1-2} \\
& + \left(DMD_{1-2} + Dep_{1-2} + Dep_{2-3}\right) * Lat_{2-3} \\
& + \left(DMD_{2-3} + Dep_{2-3} + Dep_{3-4}\right) * Lat_{3-4} \\
& + \left(DMD_{3-4} + Dep_{3-4} + Dep_{4-5}\right) * Lat_{4-5}
\end{aligned}
$$

FROM EXAMPLE

2A:	(-287.632) (-388.469)	=	111,736.1154
	(-287.632 – 287.632 + 288.924) (-340.555)	=	97,514.5187
	(-286.340 + 288.924 + 369.320) (211.489)	=	78,653.6051
	(371.904 + 369.320 – 26.418) (384.049)	=	274,520.5295
	(714.806 – 26.418 – 344.194) (133.486)	=	45,945.0803
	2A	=	608,369.8489
	A	=	304,184.82 sq. ft.
		=	6.98 ac.

AREA BY COORDINATES

$$2A = X_1(Y_2 - Y_n) + X_2(Y_3 - Y_1) + \cdots + X_{n-1}(Y_n - Y_{n-2}) + X_n(Y_1 - Y_{n-1})$$

2A:	5460.445 (5849.543 – 6104.526)	=	-1,392,320.6474
	+ 5172.813 (5508.988 – 6238.012)	=	-3,771,104.8245
	+ 5461.737 (5720.477 – 5849.543)	=	-704,924.5476
	+ 5831.057 (6104.526 – 5508.988)	=	3,472,616.0237
	+ 5804.639 (6238.012 – 5720.477)	=	3,004,103.8449
	2A	=	608,369.8489
	A	=	304,184.82 sq. ft.
		=	6.98 ac.

Point	X	Y
1	5460.445	6238.012
2	5172.813	5849.543
3	5461.737	5508.988
4	5831.057	5720.477
5	5804.639	6104.526
1	5460.445	6238.012

Sum 1 = 163,487,060.260
- Sum 2 = 162,878,690.411
2A = 608,369.849
A = 304,184.92 sq. ft. or 6.98 ac.