TAPE CORRECTIONS

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STANDARD CONDITIONS

- Foot system
  - Temperature - 68° F
  - Tape fully supported
  - Tension – 10 lbs
- Metric system
  - Temperature 20° C
  - Tape fully supported
  - Tension 50 N (Newtons)
    - 1 lb = 4.448 N

COLONIAL AMERICAN SURVEYOR'S SEMI-CIRCUMFERENTOR & CHAIN

ERRORS IN TAPING

- Systematic taping errors
  - Slope
  - Erroneous length
  - Temperature
  - Tension
  - Sag

- Random taping errors
  - Slope
  - Temperature
  - Tension
  - Sag
  - Alignment
  - Marking
  - Plumbing
  - Straightness of tape
  - Observational imperfections

APPLYING CORRECTIONS

$$T = R + C$$

or

$$T = R - E$$

- T = True Value
- R = Field Reading
- C = Correction
- E = Error

TAPING OPERATIONS

- Measuring between points
  - The value R is recorded in the field and the corrections computed
  - T is calculated
- Setting out a value
  - T is now known and the corrections are computed
  - R is calculated to the conditions in the field

SLOPE CORRECTION

- From right triangle geometry
  - H: horizontal distance
  - S: slope distance
  - V: vertical distance
  - θ: vertical angle
  - Z: zenith angle

$$H = S\cos\theta$$
$$= S\sin z$$

$$\mathbf{S^2 = H^2 + V^2}$$ **from which**

$$\mathbf{H = \sqrt{S^2 - V^2}}$$
$$\mathbf{V = \sqrt{H^2 - S^2}}$$

SLOPE CORRECTION

- Slope expressed as gradient or rate of grade
  - Ratio of vertical distance over horizontal distance
    - Rise over run
  - A +2% slope means 2 units rise in 100 units horizontal
  - A -3.5% slope means 2.5 units fall in 100 units horizontal

SLOPE CORRECTION EXAMPLE

- A road centerline gradient falls from station 0 + 00 (elevation = 564.22') to station 1 + 50 at a rate of -2.5%. What is the centerline elevation at station 1 + 50?
- Difference in elevation: $\text{Elev}_{\text{Diff}} = \text{Dist}\left(\frac{\text{rise}}{\text{run}}\right)$

$$= 150'\left(\frac{-2.5'}{100'}\right) = -3.75'$$

- Elevation at 1 + 50: $\text{Elev}_{1+50} = \text{Elev}_{0+00} + \text{Elev}_{\text{Diff}}$

$$= 564.22' - 3.75' = \underline{560.47'}$$

SLOPE CORRECTION EXAMPLE

- A road centerline runs from 1 + 00 (elevation = 471.37') to station 4 + 37.25 (elevation = 476.77'). What is the slope of the centerline grade line?
- Elevation difference: $$\text{Elev}_{\text{Diff}} = \text{Elev}_{4+37.25} - \text{Elev}_{1+00}$$ $$= 476.377' - 471.37' = +5.40'$$
- Distance: $$\text{Dist} = \text{STA}_{4+37.25} - \text{STA}_{1+00}$$ $$= 337.25'$$
- Gradient: $$\text{Slope} = \left(\frac{\text{Elev}_{\text{Diff}}}{\text{Dist}}\right) \cdot 100 = \left(\frac{+5.40'}{337.25'}\right) \cdot 100 = \underline{+1.60\%}$$

SLOPE CORRECTION EXAMPLE

- The slope distance between two points is 78.22' and the vertical angle is 1°20'. What is the corresponding horizontal distance?
- Horizontal distance:

$$\cos\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{H}}{\text{S}}$$

$$\text{H} = \text{S} \cdot \cos\theta = (78.22')\cos(1°20')$$
$$= \underline{78.20'}$$

SLOPE CORRECTION EXAMPLE

- The slope distance between two points is 78.22' and the zenith angle is 88°40'. What is the corresponding horizontal distance?

Note: sin z = cos (90°- θ)

Same answer as in previous example

- Horizontal distance:

$$\sin z = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{H}{S}$$

$$H = S \cdot \sin z = (78.22')\sin(88°40')$$
$$= \underline{78.20'}$$

SLOPE CORRECTION EXAMPLE

- A slope rises form one point, a distance of 156.777m, to another point at a rate of +1.5%. What is the corresponding horizontal distance between the points?

- Vertical angle:

$$\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{1.50'}{100'}$$
$$\theta = 0.85937°$$
$$= 0°51'34''$$

- Horizontal distance:

$$H = S\cos\theta$$
$$= 156.777\,\text{m}\cdot\cos(0°51'34'')$$
$$= \underline{156.759\,\text{m}}$$

SLOPE CORRECTION EXAMPLE

- The slope distance between two points is measured to be 199.908 m and the vertical distance between the points (i.e., the difference in elevation) is +2.435 m. What is the horizontal distance between the points?

- Horizontal distance:

$$H^2 = S^2 - V^2$$

$$H = \sqrt{(199.908\,\text{m})^2 - (2.435\,\text{m})^2}$$

$$= \underline{199.893\,\text{m}}$$

SLOPE CORRECTION

- So far – compute H and V directly
- Can compute correction
- Error due to slope: $E_h = S - H$
- Correction for slope: $C_h = H - S$
- Substitute $H = S \cdot \cos\theta$
- Correction for slope: $$\boxed{\begin{aligned} C_h &= S \cdot \cos\theta - S \\ &= S(\cos\theta - 1) \end{aligned}}$$

SLOPE CORRECTION

- If vertical distance given instead of vertical angle
- Correction:

$$C_h = H - S = \sqrt{S^2 - V^2} - S$$

$$= S\sqrt{1 - \frac{V^2}{S^2}} - S$$

- Use binomial theorem and expand radical $\sqrt{1 - \frac{V^2}{S^2}}$

$$C_h = S\left(1 - \frac{V^2}{2S^2} - \frac{V^4}{8S^4} - \frac{V^6}{16S^6} - \frac{5V^8}{128S^8} - \ldots\right) - S$$

- Reducing

$$C_h = -\frac{V^2}{2S} - \frac{V^4}{8S^3} - \frac{V^6}{16S^5} - \frac{5V^8}{128S^7} - \ldots$$

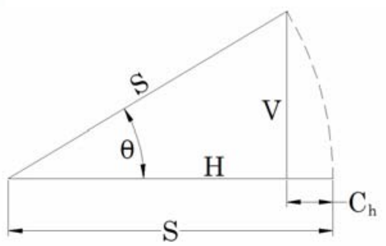
SLOPE CORRECTION

- Generally, only first term used
  - Valid for slope < 10-15%
  - Where required precision < 1:15,000

$$C_h = -\frac{V^2}{2S}$$

- If more precision necessary, additional terms required

SLOPE CORRECTION (ALTERNATIVE)



- Height from Pythagorean Theorem

$$V^2 = S^2 - H^2$$
$$= (S+H)(S-H)$$

- If slope not too large, slope and horizontal distances nearly the same

$$S + H = 2S \qquad \text{(approx.)}$$

SLOPE CORRECTION (ALTERNATIVE)

- Then,

$$V^2 = 2S(S-H) \Rightarrow \frac{V^2}{2S} = S - H \qquad \text{(approx)}$$

- Correction for slope: $C_h = H - S$
- Then,

$$\boxed{C_h = -\frac{V^2}{2S}}$$

SLOPE CORRECTION UNCERTAINTY

- Uncertainty in horizontal distance found by error propagation

$$e_h^{\ell} = \pm \ell (\sin\theta) e_\theta \qquad \text{or} \qquad e_h^{\ell} = \pm \frac{V}{\ell} e_V$$

  - e_h^{ℓ} = error in one tape length due to uncertainty in θ
  - ℓ = tape length (slope)
  - e_θ = error in vertical angle
  - e_V = uncertainty in elevation difference

SLOPE CORRECTION UNCERTAINTY

- If slope uniform over entire length of line, and θ or V is over entire length, then $L = \ell \cdot n$ and

$$e_h^{L} = \pm L (\sin\theta) e_\theta \qquad \text{or} \qquad e_h^{L} = \pm \frac{V}{L} e_V$$

- Otherwise use rule for addition of random error:

$$e_h^{L} = \pm \left[(g_1 e_{V_1})^2 + (g_2 e_{V_2})^2 + \cdots + (g_n e_{V_n})^2 \right]^{1/2}$$

  - $g_1, g_2, \ldots, g_n$, = slope $\left(V/\ell\right)$ for particular length $\ell_1, \ell_2, \ldots, \ell_n$

SLOPE CORRECTION EXAMPLE

- A measurement was made along a line inclined by 3° 22'. The slope distance is 3,236.86'. What is the horizontal distance?

- Correction for slope

$$C_h = S(\cos\theta - 1)$$
$$= 3{,}236.86'[(\cos 3°22') - 1]$$
$$= -5.586'$$

- Horizontal distance

$$T = R + C$$
$$= 3{,}236.86' + (-5.59')$$
$$= \underline{3{,}231.27'}$$

SLOPE CORRECTION EXAMPLE

- If the uncertainty in θ was 1', what is the uncertainty in the entire length?

- Uncertainty: $e_h^L = \pm L(\sin\theta)e_\theta$

$$= \pm(3{,}236.86')\sin 3°22'\left(1'\frac{2\pi^{rad}}{180°}\right)$$
$$= \underline{\pm 0.06'}$$

- Corrected distance:

$$\mathrm{T} = R + C = \underline{3{,}231.27' \pm 0.06'}$$

STANDARD LENGTH CORRECTION

- Tapes assumed to be correct as manufactured
  - Exception is for precise taping
  - Wear – tapes become kinked and stretched
- Correction:

$$C_\ell = \ell_S - 100' \quad \text{or} \quad C_\ell = \ell_S - 30\,m$$

  - ℓ_S = calibrated value of tape
- Discrepancy found through tape comparison to a standard tape

STANDARD LENGTH CORRECTION

- Estimate of error in calibration: e_ℓ
- Uncertainty in length of line due to error in calibration:

$$e_\ell^L = \pm e_\ell \cdot n$$

  - n = number of tape lengths
- Error tends ***not*** to compensate

STANDARD LENGTH EXAMPLE

- **A measurement was recorded as 171.278 m with a 30-m tape that was only 29.996 m under standard conditions. What is the corrected measurement?**
- **Correction per tape length:** $C_\ell = \ell_S - 30\,\text{m} = 29.996\,\text{m} - 30\,\text{m}$
  $= -0.004\,\text{m}$
- **Number of tape lengths:** $\text{No. Tape Lengths} = \dfrac{\text{Dist}}{30\,\text{m}} = \dfrac{171.278\,\text{m}}{30\,\text{m}}$
  $= 5.71$
- **Total correction:** $C_\ell^L = C_\ell \cdot \text{No. Tape Lengths}$
  $= (-0.004\,\text{m}) \cdot (5.71\text{ tape lengths}) = -0.023\,m$
- **Corrected distance:** $T = R + C_\ell^L = 171.278\,\text{m} + (-0.023\,\text{m})$
  $= \underline{171.255\,m}$

STANDARD LENGTH EXAMPLE

- If the uncertainty in the calibrated length of the line is ±0.0012 m, what is the uncertainty in the entire length of the line?
- Total uncertainty is:

$$e_\ell^L = \pm e_\ell \cdot \text{n} = \pm(0.0012\,\text{m}) \cdot (5.71\text{ tape lengths})$$
$$= \pm 0.007\,m$$

- The length of the line is then

$$\text{Dist} = \underline{171.255\,\text{m} \pm 0.007\,\text{m}}$$

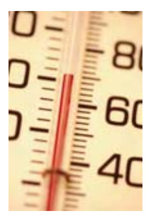
STANDARD LENGTH EXAMPLE

- A surveyor is using a tape whose standard length is 100.02'. Two pins need to be set out 600.00' apart. What is the field measurement needed to set out the correct length?
- Correction per tape length $$C_{\ell} = \ell_S - 100' = 100.02' - 100'$$ $$= 0.02' \text{ per tape length}$$
- Total correction $$C_{\ell}^{L} = C_{\ell} \cdot \text{n} = (0.02')(6 \text{ tape lengths})$$ $$= 0.12'$$
- Field reading $$R = T - C = 600.00' - 0.12'$$ $$= \underline{599.88'}$$

TEMPERATURE

- Tape susceptible to dimensional change due to variation in temperature
- Correction:

$$C_T = \ell \alpha (T - T_S)$$



- C_T = correction due to temperature
  ℓ = length of tape
  α = coefficient of thermal expansion
  α = 0.00000645 per 1°F
  α = 0.0000116 per 1°C
  T = field temperature
  T_S = standard temperature (Normally 68° F or 20°C)

TEMPERATURE

- Uncertainty – differentiate w.r.t. T $\frac{\partial C_T}{\partial T} = \ell\,\alpha$
- From error propagation, error in one tape length due to error in temp is:

$$e_T^{\ell} = \pm\sqrt{(\ell\,\alpha)^2 e_T^2} = \pm \ell\,\alpha\,e_T$$

  - e_T^{ℓ} = error in one tape length
    e_T = error in determining temp.
- Total error of length of line:

$$e_T^{L} = \pm \ell\,\alpha\,e_T\,\sqrt{n}$$

TEMPERATURE UNCERTAINTY

- Last formula – assumes uncertainty same for each length
  - Generally average temp. used over line
  - Thus, error would not tend to compensate
- Total error estimate: $e_T^{L} = \pm \ell\,\alpha\,e_T\,n$
- Recognizing L = ℓ n,

$$e_T^{L} = \pm L\,\alpha\,e_T$$

TEMPERATURE CORRECTION EXAMPLE

- Line is measured as 876.42 m. The field temperature is 24° C. A 30-m tape with correct length at 20° C was used. Find the corrected length of the line.
- The correction per tape length:

$$C_T = \ell\,\alpha(T - T_S) = (30\text{ m})(0.0000116\text{ per }1°\text{C})(24° - 20°)$$
$$= 0.001\text{ m/tape length}$$

- The correction for the line:

$$n = \frac{L}{30\text{ m}} = \frac{876.42\text{ m}}{30\text{ m}} = 29.214\text{ tape lengths}$$
$$C_T^L = n\,C_T = (29.214\text{ tape lengths})\cdot(0.001\text{ m/tape length}) = 0.04\text{ m}$$

- The corrected length of line:

$$T = R + C_T^L = 876.42\text{ m} + 0.04\text{m}$$
$$= \underline{876.46\text{ m}}$$

TEMPERATURE CORRECTION EXAMPLE

- If e_T = 1° C, the total error estimate is

$$e_T^L = \pm L\,\alpha\,e_T = (876.42\text{ m})\cdot(0.0000116)\cdot(0.0000116)\cdot 1°\text{C}$$
$$= \underline{\pm 0.01}$$

- The distance is: 876.46m ± 0.01m

TEMPERATURE CORRECTION EXAMPLE

- You must lay out two points in the field that will be exactly 100.000 m apart. Field conditions indicate that the temperature of the tape is 27° C. What distance will be laid out?

- Correction for temperature:

$$C_T = \ell\,\alpha(T - T_S) = (100.000\,\text{m})(0.0000116\text{ per }1°\text{C})(27° - 20°)$$
$$= +0.008\,m$$

- Distance used to lay out:

$$R = T - C = 100.000\,\text{m} - 0.008\,\text{m}$$
$$= \underline{99.992\,\text{m}}$$

TENSION CORRECTION

- Applied stress of wire – force per unit area
- Resultant strain – elongation per unit length

$$\text{Stress} = P/A \qquad \text{Strain} = eL/\ell$$

- where: P = tension (force)
  A = cross-sectional area of tape
  eL = elongation produced by tension
  ℓ = length of tape

TENSION CORRECTION

- Hooke's Law – P proportional to eL and stress proportion to strain $$\frac{P}{A} \propto \frac{eL}{\ell}$$
- Insert constant: $\frac{P}{A} = E\frac{eL}{\ell}$ or $E = \frac{P\ell}{A(eL)}$

Proportionality constant (E) called Young's modulus of elasticity

TENSION CORRECTION

- Error found by comparing elongation produced from standardized tension (P_S) and field tension (P) $$eL = \frac{P\ell}{AE} \quad \text{and} \quad eL = \frac{P_S\ell}{AE}$$
- Correction: $$C_P^\ell = \frac{P\ell}{AE} - \frac{P_S\ell}{AE} = \frac{(P - P_S)\ell}{AE}$$ $$\text{or} \quad C_P^L = \frac{(P - P_S)L}{AE}$$
- Young's modulus of elasticity normally between 28,000,000 psi to 30,000,000 psi

TENSION CORRECTION

- Uncertainty – only variable: field tension

$$e_P^{\ell} = \pm\left(\frac{\ell}{AE}\right)e_P$$

where: $e_P{}^{\ell}$ = error in one tape length
e_P = error in determining tension

- Error for total length of line (error propagation)

$$e_P^{L} = \pm\left(\frac{\ell}{AE}\right)e_P\sqrt{n}$$

TENSION CORRECTION EXAMPLE

- Given: A = 0.0040 sq. in P = 25 lbs
  E = 29,000,000 psi P_S = 15 lbs
  L = 1,000.00'
- Correction for tension:

$$C_P^L = \frac{(P - P_S)L}{AE} = \frac{(25 - 15\,\text{lbs})(1{,}000.00')}{(0.0040\,\text{sq in})(29{,}000{,}000\,\text{psi})}$$

$$= 0.086'$$

- Corrected distance:

$$T = R + C = 1{,}000.00' + 0.09'$$

$$= \underline{1{,}000.09'}$$

TENSION CORRECTION EXAMPLE

- If the uncertainty in P is 1-lb, what is the uncertainty due to tension?
- Uncertainty per tape length:

$$e_P^{\ell} = \pm\left(\frac{\ell}{AE}\right)e_P = \pm\left[\frac{100.00'}{(0.0040\,\text{sq in})(29{,}000{,}000\,\text{psi})}\right]$$
$$= \pm 0.0009'$$

- Total uncertainty:

$$e_P^{L} = \pm e_P^{\ell}\sqrt{n} = \pm(0.0009')\sqrt{10}$$
$$= \underline{\pm 0.003'}$$

- Distance = 1,000.09’ ± 0.003’

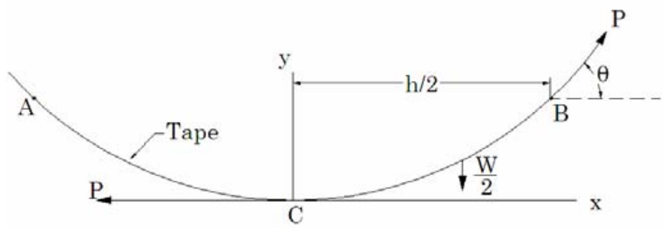
TENSION CORRECTION

- Cross-sectional area
  - Measured with micrometer
  - Taken from manufacturer’s specifications
  - Computed from:

$$\text{Tape area} = \frac{\text{weight}}{(\text{length})(\text{specific weight of tape steel})}$$

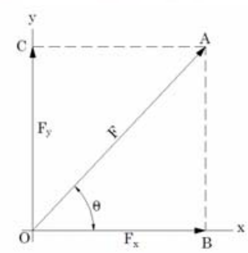
SAG CORRECTION

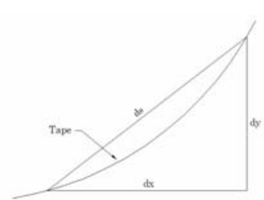
- Tape supported at ends will sag in center
- Amount of sag depends on
  - Weight of tape per unit length
  - Applied tension
- Arc forms catenary curve or approx. parabola



SAG CORRECTION

- F is force and the components are shown here – can distinguish between forces in x and y directions
- Differentiation of equation of parabola gives slope at support B
- Tape forms differentially short segments of curve – "ds" found by differentiation
- Integrate to find total length of curve
- Horizontal force will approach tension and horizontal distance approaches the curve





SAG CORRECTION

- Correction for sag given as:

$$C_S = -\frac{W^2 \ell}{24 P^2} \qquad \text{or} \qquad C_S^L = -n\left(\frac{W^2 \ell}{24 P^2}\right)$$

- Also expressed in terms of weight per foot

$$C_S = -\frac{w^2 \ell^3}{24 P^2} \qquad \text{or} \qquad C_S^L = -n\left(\frac{w^2 \ell^3}{24 P^2}\right)$$

- Sag correction always negative
- Sag varies with tension

SAG CORRECTION

- Uncertainty – take derivative of correction equation w.r.t. P and use general propagation formula

$$e_S^\ell = \pm\frac{W^2 \ell}{12 P^3} e_P \qquad \text{or} \qquad e_S^\ell = \pm\frac{w^2 \ell^3}{12 P^3} e_P$$

- Assuming came conditions for all tape lengths, error due to sag for total length:

$$e_S^L = \pm\frac{W^2 \ell}{12 P^3} e_P \sqrt{n} \qquad \text{or} \qquad e_S^L = \pm\frac{w^2 \ell^3}{12 P^3} e_P \sqrt{n}$$

SAG CORRECTION EXAMPLE

- A 100' steel tape weights 0.02 lbs/ft and supported at the ends only with a tension of 12 lbs. A distance of 350.00' was measured. What is the correction for sag?
- Correction per tape length (100') is:

$$C_S = -\frac{w^2\ell^3}{24P^2} = -\frac{(0.02\text{ lbs/ft})^2(100.00')^3}{24(12\text{ lbs})^2}$$
$$= -0.116'$$

- Correction for 50' section is

$$C_S = -\frac{w^2\ell^3}{24P^2} = -\frac{(0.02\text{ lbs/ft})^2(50.00')^3}{24(12\text{ lbs})^2}$$
$$= -0.014'$$

- Total correction:

$$C_S^L = (-0.116')\cdot 3 + (-0.014') = -0.36'$$

SAG CORRECTION EXAMPLE

- If the uncertainty in tension was 1 lb., what is the uncertainty in the total length?
- For 3 full tape lengths

$$e_S^{300'} = \pm\frac{w^2\ell^3}{12P^2}e_P\sqrt{n} = \pm\left[\frac{(0.02\text{ lbs/ft})^2(100.00')^3}{12(12\text{ lbs})^3}\right](1\text{ lb})\sqrt{3} = \pm 0.033'$$

- For 50' tape length

$$e_S^{50'} = \pm\frac{w^2\ell^3}{12P^2}e_P = \pm\left[\frac{(0.02\text{ lbs/ft})^2(50.00')^3}{12(12\text{ lbs})^3}\right](1\text{ lb}) = \pm 0.002'$$

- For full length:

$$e_S^L = e_S^{300'} + e_S^{50'} = \pm 0.04'$$

- Distance:

$$T = R + C = 350.00' + (-0.36')$$
$$= \underline{349.64' \pm 0.04'}$$

NORMAL TENSION

- Use tape correction to negate effects of sag in tape
- Make error in sag = correction for tension

$$\frac{w^2\ell^3}{24P^2} - \frac{(P - P_S)\ell}{AE} = 0$$

- Define normal tension P → P_n

$$\frac{w^2\ell^3}{24P_n^2} - \frac{(P_n - P_S)\ell}{AE} = 0$$

$$\vdots$$

$$P_n = \frac{0.204\, w\, \ell \sqrt{AE}}{\sqrt{P_n - P_S}}$$

NORMAL TENSION

- Note that normal tension (P_n) on both sides of equation
- Use as a first approximation

$$P_n = \sqrt[3]{\frac{AEW^2}{15}}$$

- Then take this value for P_n into the previous equation and solve for new normal tension
- Continue until difference below criteria

NORMAL TENSION EXAMPLE

- Find the normal tension given:
  A = 0.0040 sq. in. W = 1.3 lbs
  E = 29,000,000 psi P_S = 15 lbs
- Initial estimate of normal tension

$$P_n = \sqrt[3]{\frac{AEW^2}{15}} = \sqrt{\frac{(0.0040\,\text{sq in})(29{,}000{,}000\,\text{psi})(1.3\,\text{lbs})^2}{15}} = 24\,\text{lbs}$$

- Adjusted value for normal tension

$$P_n = \frac{0.204\,W\sqrt{AE}}{\sqrt{P_n - P_S}} = \frac{(0.204)(1.3\,\text{lbs})\sqrt{(0.004\,\text{sq in})(29{,}000{,}000\,\text{psi})}}{\sqrt{24\,\text{lbs} - 15\,\text{lbs}}} = 30\,\text{lbs}$$

- Use mean value of the last 2 P_n values: 27 lbs

$$P_n = \frac{0.204\,W\sqrt{AE}}{\sqrt{P_n - P_S}} = \frac{(0.204)(1.3\,\text{lbs})\sqrt{(0.004\,\text{sq in})(29{,}000{,}000\,\text{psi})}}{\sqrt{27\,\text{lbs} - 15\,\text{lbs}}} = \underline{26\,\text{lbs}}$$

INCORRECT ALIGNMENT

- May occur when more than one tape length measured in field
- Error: random in nature
  systematic in effect
- Lateral displacement from true line causes systematic error

$$E_a^{\ell} = \frac{d_a}{2\ell}$$

- Correction for alignment for entire length found by

$$C_a^L = -n\,E_a^{\ell}$$

E_a^{ℓ} = alignment error
d_a = lateral displacement

TAPE NOT STRAIGHT

- Taping in brush and when wind blowing
  - Impossible to have all parts in perfect alignment
- Error systematic & variable
  - Same as measuring with tape that is too short
- Amount of error
  - Less if bend in in center
  - Increases as it gets closer to ends
- Reduced by careful field procedures

IMPERFECTIONS IN OBSERVATIONS

- Personal errors or blunders
  - Plumbing
  - Marking tape ends with tape fully supported
  - Adding or dropping full tape length
  - Adding a foot or decimeter
  - Other points incorrectly taken as end mark on tape
  - Reading numbers incorrectly
  - Calling numbers incorrectly or not clearly

SPECIFICATION FOR 1/5,000 TAPING

Source of Error	Max Effect on 1 Tape Length: 100 ft	Max Effect on 1 Tape Length: 30 m
Temp estimated to closest 7° F	±0.005’	±0.0014m
Tension known within 5 lbs	±0.006’	±0.0018m
Slope errors no larger than 1’/100’	±0.005’	±0.0015m
Alignment errors no larger than 0.5’/100’	±0.001’	±0.0004m
Plumbing & marking errors max 0.015’/100’	±0.015’	±0.0046m
Length of tape known within ±0.005’	±0.005’	±0.0015m

SPECIFICATION FOR 1/5,000 TAPING

- Total random error in 1 tape length = $\sqrt{\sum \text{errors}^2}$

$$\sum E = \sqrt{0.000337} = 0.018' \qquad \sum E = \sqrt{0.0000031} = 0.0056\text{m}$$

$$\text{accuracy} = \frac{0.018'}{100'} = \frac{1}{5{,}400} \quad \text{or} \quad \text{accuracy} = \frac{0.0056\text{m}}{30\text{m}} = \frac{1}{5{,}400}$$

- This assumes systematic errors already corrected

<table>
<tr><td>Source</td><td>Error of 0.01’/100’ caused by</td><td>Makes tape too:</td><td>Importance in 1:5,000 taping</td><td colspan="2">Procedure to reduce or eliminate</td></tr>
<tr><td>Tape not of standard length</td><td>-</td><td>Long or short</td><td>Usually small – check</td><td colspan="2">Standardize and apply correction</td></tr>
<tr><td>Temperature</td><td>15° F</td><td>Long or short</td><td>Only in hot or cold weather</td><td colspan="2">Measure temp & apply correction
Use invar tape</td></tr>
<tr><td>Change in tension</td><td>15 lbs</td><td>Long or short</td><td>Negligible</td><td>Apply correction;
Use spring balance</td><td rowspan="2">Use normal tension
Use standard tension</td></tr>
<tr><td>Sag</td><td>$\Delta P = 0.6$ lb too small</td><td>Short</td><td>Large esp. for heave tape</td><td>Apply correction;
Use only fully supported</td></tr>
<tr><td>Slope</td><td>1.4’ in h
0°48’ in θ</td><td>Short</td><td></td><td colspan="2">At breaks in slope, determine differences in height; apply correction</td></tr>
<tr><td>Imperfect horizontal alignment</td><td>1.4’</td><td>short</td><td>Not serious</td><td colspan="2">Use reasonable care in alignment; keep tape taut and reasonably straight</td></tr>
</table>

Source	Governing condition causes and manner of cumulation	Estimated value per tape length
Plumbing to mark tape ends	Rugged terrain, breaking tape often; cumulates randomly $\propto \sqrt{n}$	0.05’ – 0.10’
Marking tape ends with tape fully supported	Tape graduated to hundredths of ft; cumulates randomly $\propto \sqrt{n}$	0.01’
Applying tension	Change in sag correction due to variation in tension of ±2 lb from standard tension; cumulates $\propto \sqrt{n}$	0.01’
Determining elevation diff or slope angle	In h = ±0.8’ in θ = 0°28; Cumulates $\propto$ n	0.050’
standardization	Field tapes compared to standardized tape kept in office; cumulates $\propto$ n	0.005’