1 TRAVERSING AND TRAVERSE ADJUSTMENTS

Surveying Engineering Department
Ferris State University

Introduction

Control surveying is that part of surveying in which high precision instruments and techniques are employed to locate points for subsequent surveying operations. Because it is used as a base for further work, it needs to be performed with more care and greater accuracy. Control is used for many purposes. Photogrammetry and topographic mapping require control, as does construction layout. Control provides the basic framework of coordinates required for many other surveying applications.

The basis for evaluation of errors and corrections is based on a very simple relationship.

$$T = R + C$$

Where: T = the true value
R = the recorded field value
C = the correction to be applied to the field measurement

What we see is that the correction term is <u>always</u> added to the recorded value. The correction term could be either positive or negative. It should be recognized that the error (E) has the same magnitude as the correction but with opposite sign. Thus, E = -C. The error can be expressed as:

$$E = R - T$$

This formula states that the error is equal to the difference between the recorded value and the true value. The true value is always subtracted from the measured field recorded value. Since the true value is never really known, it is usually approximated by some value, such as the mean.

Kinds of Traverse

There are a number of different kinds of traverses employed in surveying. They include: deflection-angle traverse, interior angle traverse, exterior angle traverse, and a traverse where all the angles are measured to the right, as one would experience with a directional theodolite[1]. Each has its own geometry from which the angular error of closure can be computed.

---

[1] Conceptually, one could also have all angles measured to the left, although there are no instruments I know that measures angles <u>just</u> to the left.

Deflection-Angle Traverse

The deflection-angle traverse (figure 1) is primarily used in route surveys such a highways. One can see that the measured angle is that formed from the prolonged backsight line from the last station to the line in the direction of next point. The direction, left or right, from the prolonged line is a necessary part of the angle. The general form from the geometry states that,

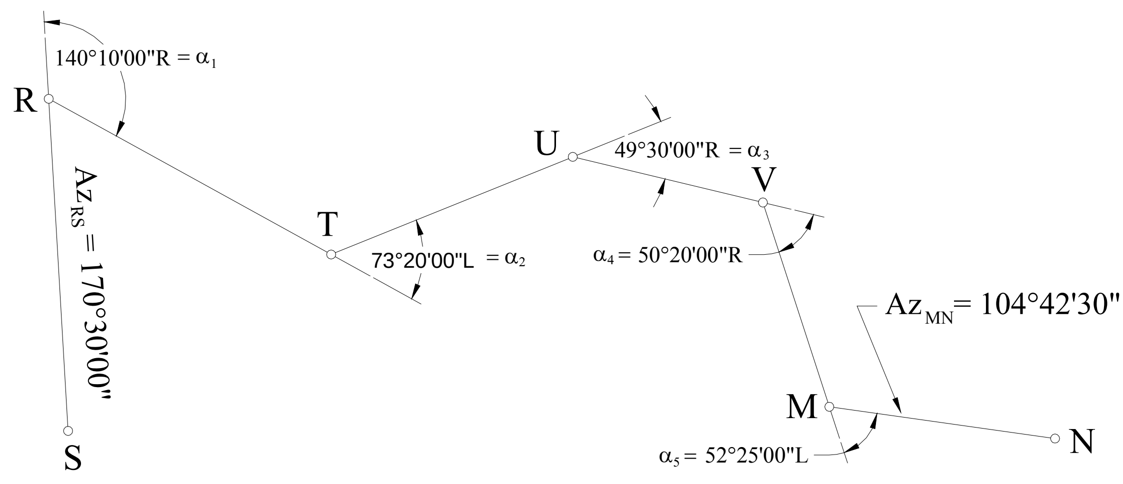

Figure 1. Deflection angle traverse [from Anderson and Mikhail, 1998].

$$Az_1 + \sum_{i=1}^{n} \alpha_{R_i} - \sum_{i=1}^{n} \alpha_{L_i} - Az_2 - 360^\circ = 0$$

where Az_1 = the forward azimuth at the origin,
Az_2 = the azimuth at the closing, and
$\alpha_{R,L}$ = the deflection angles to the right and left respectively

The equation expresses the geometric relationship that should, theoretically, exist. Thus, this represents the true value (0). In the example in figure 1 [Anderson and Mikhail, 1998], the angular error of closure is found using the following relationships.

$$\begin{aligned}\sum \alpha_R &= \alpha_1 + \alpha_3 + \alpha_4 \\ &= (140^\circ\ 10'00'') + (49^\circ\ 30'00'') + (50^\circ\ 20'00'') \\ &= 240^\circ\ 00'\ 00''\end{aligned}$$

$$\begin{aligned}\sum \alpha_L &= \alpha_2 + \alpha_5 \\ &= \left(73^\circ\ 20'00''\right) + \left(52^\circ\ 25'00''\right) \\ &= 125^\circ\ 45'00''\end{aligned}$$

The angular error is:

$$Az_{S-R} + \sum \alpha_R - \sum \alpha_L - Az_{M-N} - 360^\circ = E_\angle$$

$$\left(350^\circ\ 30'00''\right) + \left(240^\circ\ 00'00''\right) - \left(125^\circ\ 45'00''\right) - \left(104^\circ\ 42'30''\right) - 360^\circ = +0^\circ\ 02'30'$$

Note that in a deflection-angle traverse that the angles to the right are considered as positive angles while those to the left are treated as negative angles. The correction to each angle (cpa) is found by dividing the total angular error by the number of angles (recognizing that C = -E).

$$cpa = -\frac{E_\angle}{n} = -\frac{0^\circ\ 02'30''}{5} = -30''$$

In this formula, n is the total number of angles measured. The corrected angles are computed by adding the correction per angle (cpa) to each of the measured angles.

$$\begin{aligned}\alpha_1 &= +140^\circ\ 10'\ 00'' - 30'' = +140^\circ\ 09'\ 30'' \\ \alpha_2 &= -\ \ 73^\circ\ 20'\ 00'' - 30'' = -\ \ 73^\circ\ 20'\ 30'' \\ \alpha_3 &= +\ \ 49^\circ\ 30'\ 00'' - 30'' = +\ \ 49^\circ\ 29'\ 30'' \\ \alpha_4 &= +\ \ 50^\circ\ 20'\ 00'' - 30'' = +\ \ 50^\circ\ 19'\ 30'' \\ \alpha_5 &= -\ \ 52^\circ\ 25'\ 00'' - 30'' = \underline{-\ \ 52^\circ\ 25'\ 30''} \\ & \qquad\qquad\qquad\qquad\qquad\quad +114^\circ\ 12'\ 30''\end{aligned}$$

Check to make sure that the corrections have been applied correctly.

$$\left(350^\circ\ 30'00''\right) + \left(114^\circ\ 12'30''\right) - \left(104^\circ\ 42'30''\right) - 360^\circ = 0^\circ \quad \surd$$

The corrections have been applied correctly. Note that in this example the angles to the right decreased numerically while the angles to the left increased.

Interior Angle Traverse

The interior angle traverse (figure 2) is probably the most common kind of traverse encountered in surveying. Here, all of the angles within the polygon are observed. The general form of the geometric relationship of the angles is

$$\sum_{i=1}^{n} \alpha_i - (n-2)(180^\circ) = 0^\circ$$

From the example in figure 2 [Anderson and Mikhail, 1998], the angular error of closure is any difference found in the previous equation that deviates from 0°.

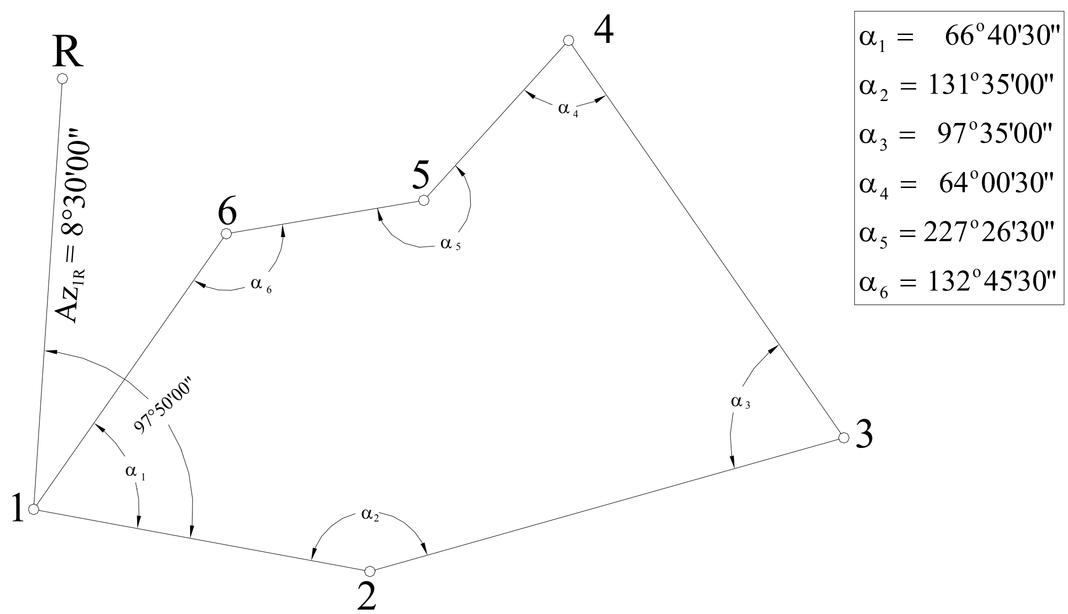

Figure 2. Interior angle traverse [from Anderson and Mikhail, 1998].

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 - (6-2)(180^\circ) = E_\angle$$
$$(720^\circ\ 03'\ 00'') - (720^\circ) = +3'\ 00''$$

The correction per angle (cpa) is determined next.

$$\text{cpa} = -\frac{30'\ 00''}{6} = -30''$$

The corrected angles are found by adding the cpa to each observation.

$$\alpha_1 = \left(66^\circ\ 40'\ 30''\right) + \left(-30''\right) = \quad 66^\circ\ 40'\ 00''$$
$$\alpha_2 = \left(131^\circ\ 35'\ 00''\right) + \left(-30''\right) = 131^\circ\ 34'\ 30''$$
$$\alpha_3 = \left(97^\circ\ 35'\ 00''\right) + \left(-30''\right) = \quad 97^\circ\ 34'\ 30''$$
$$\alpha_4 = \left(64^\circ\ 00'\ 30''\right) + \left(-30''\right) = \quad 64^\circ\ 00'\ 00''$$
$$\alpha_5 = \left(227^\circ\ 26'\ 30''\right) + \left(-30''\right) = 227^\circ\ 26'\ 00''$$
$$\alpha_6 = \left(132^\circ\ 45'\ 30''\right) + \left(-30''\right) = \underline{132^\circ\ 45'\ 00''}$$
$$\sum \alpha_i = 720^\circ\ 00'\ 00''$$

A special case of figure 2 is the case where the exterior angles have been measured. Here geometric angular relationship is shown as:

$$\sum_{i=1}^{n} \alpha_{i_{Ext}} - (n+2)\left(180^\circ\right) = 0^\circ$$

Note that the interior and exterior angle traverses are all closed-loop traverses.

Angles to the Right

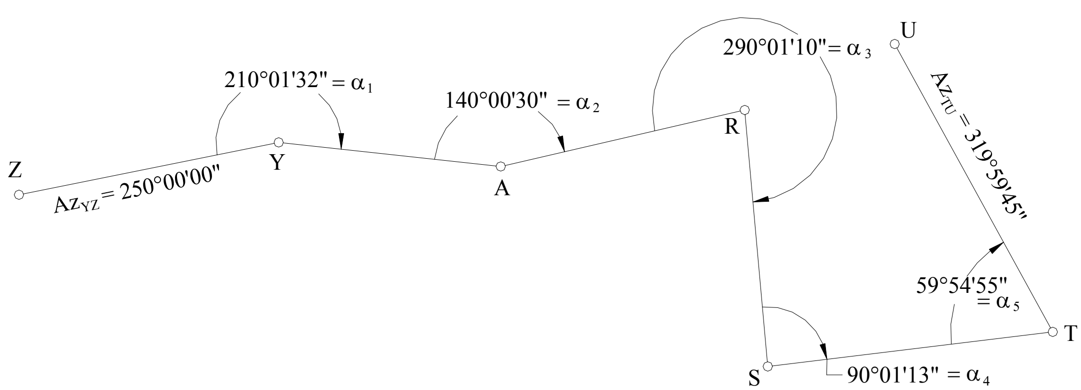

Figure 3. Traverse where angles were measured to the right [from Anderson and Mikhail, 1998].

With directional instruments, angles are measured to the right. An example is shown in figure 3 [Anderson and Mikhail, 1998]. The general form for the angular geometry is shown as:

$$Az_1 + \sum_{i=1}^{n} \alpha_i - Az_2 - (n-1)(180^\circ) = 0^\circ$$

From the example, the angular error of closure is found as

$$Az_{YZ} + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 - Az_{TU} - (5-1)(180^\circ) = E_{\angle}$$
$$(250^\circ 00'00'') + (789^\circ 59'20'') - (319^\circ 59'45'') - 720^\circ = -00'\,25''$$

The correction per angle is

$$cpa = -\frac{25''}{5} = 5''$$

The corrected angles are found by adding the cpa to each measurement.

$$\begin{aligned}
\alpha_1 &= (210^\circ\ 01'\,32'') + 5'' = 210^\circ\ 01'\,37'' \\
\alpha_2 &= (140^\circ\ 00'\,30'') + 5'' = 140^\circ\ 00'\,35'' \\
\alpha_3 &= (290^\circ\ 01'\,10'') + 5'' = 290^\circ\ 01'\,15'' \\
\alpha_4 &= (90^\circ\ 01'\,13'') + 5'' = 90^\circ\ 01'\,18'' \\
\alpha_5 &= (59^\circ\ 54'\,55'') + 5'' = \underline{59^\circ\ 55'\,00''} \\
\sum \alpha_i &= 789^\circ\ 59'\,45''
\end{aligned}$$

Check using the general relationship for the angles to the right.

$$(250^\circ 00'00'') + (789^\circ 59'45'') - (319^\circ 59'45'') - 720^\circ = 0^\circ \qquad \surd$$

Steps in Traverse Computations

The process of computing and adjusting a traverse can be broken down into 6 basic steps.

1. Balance/adjust the angles.
2. Compute the azimuths/bearings of the survey lines. Note that the directions of the lines can be computed using the raw field angles and these preliminary directions can then be adjusted for the angular misclosure.
3. Compute the latitudes and departures

$$\text{Lat} = s \cdot \cos\alpha = \Delta Y$$

$$\text{Dep} = s \cdot \sin\alpha = \Delta X$$

where: s = distance between two points
α = azimuth/bearing of the line
ΔX = difference in the X-coordinate between the two ends of the line
ΔY = difference in the Y-coordinate between the two ends of the line.

The latitudes and departures are also referred to by the northing or easting value.

4. Determine the closure error in departure and latitude, e_D and e_L respectively. Expressed in this form the closures are error terms (recall that E = R –T).

$$e_D = DX = \sum_{i=1}^{n-1} \text{Lat}_{i,i+1} - (X_n - X_1)$$

$$e_L = DY = \sum_{i=1}^{n-1} \text{Dep}_{i,i+1} - (Y_n - Y_1)$$

In other words, the error in departure/latitude is equal to the sums of the departures/latitudes minus the difference in the coordinates between the last and first point. If the traverse is a closed loop traverse where the beginning and ending points are the same, in other words $X_n = X_1$ and $Y_n = Y_1$, then the closure errors are simply the sums of the departures and latitudes.

$$e_D = DX = \sum_{i=1}^{n-1} \text{Lat}_{i,i+1}$$

$$e_L = DY = \sum_{i=1}^{n-1} \text{Dep}_{i,i+1}$$

5. Compute the linear error of closure, accuracy ratio and the direction of the closing line. The linear error of closure (E_c) is the square root of the sum of the latitude and departure closure errors squared.

$$E_c = \sqrt{e_D^2 + e_L^2}$$

The accuracy ratio is defined as the ratio of the linear error of closure to the perimeter distance (P)

$$\text{Accuracy} = E_c / P$$

Finally, the direction of the closing line is found by the arctangent of the error in departure to the error in latitude.

$$\beta_e = \tan^{-1}\left(\frac{e_D}{e_L}\right)$$

6. Adjust the traverse and compute the adjusted distances, directions and coordinates. The methods of adjusting the traverse will be discussed after the example is introduced.
7. The last step is to normally compute the area of the traverse, assuming that the traverse is a closed-loop traverse.

Example: A traverse was run as shown in figure 4. The data are also given in the following table:

Station	Angle	Distance
Az. Mark		
1	-67° 34' 12.0" D	483.406'
2	256° 49' 24.8" L	446.622'
3	259° 29' 20.6" L	425.557'
4	-64° 08' 40.5" D	384.926'
5	-64° 52' 17.5" D	369.173'
1	352° 53' 28.7" R	
Az. Mark		

Note that the angle codes are as follows: R = angle to the right, L = angle to the left, D = deflection angle with a positive being to the right and a negative to the left.

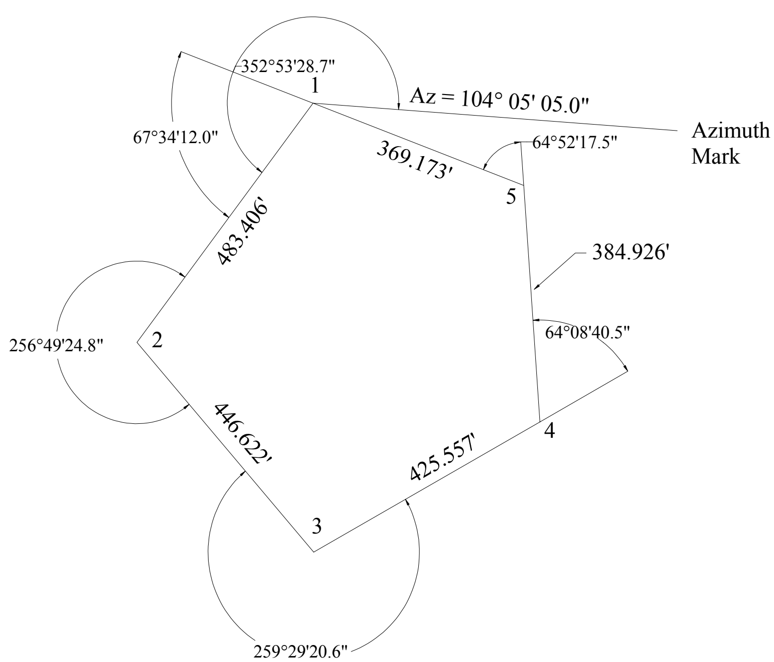

Figure 4. Example for traverse adjustment.

Because this problem uses mixed angle types, the first part of the problem is to compute the azimuth of the lines based on the measured quantities and then find the angular error by comparing the known azimuth from point 1 to the azimuth mark to the computed valued based on the measured angles.

The azimuths of the lines are then found as follows:

Azimuth	1 ⇒ Az. Mark	104° 05' 05.0"
	+180°	180° 00' 00.0"
	Back Azimuth	284° 05' 05.0"
	$-\angle_1$	-67° 34' 12.0"
Azimuth	1 - 2	216° 30' 53.0"
	$-\angle_2$	-256° 49' 24.8"
		-40° 18' 31.8"
	+ 180°	180° 00' 00.0"
Azimuth	2 - 3	139° 41' 28.2"
	+ 180°	180° 00' 00.0"
	$-\angle_3$	-259° 29' 20.6"
Azimuth	3 - 4	60° 12' 07.6"
	$-\angle_4$	-64° 08' 40.5"
	+ 360°	360° 00' 00.0"
Azimuth	4 - 5	356° 03' 27.1"
	$-\angle_5$	-64° 52' 17.5"
Azimuth	5 - 1	291° 11' 09.6"
	-180°	-180° 00' 00.0"
	+ ∠ to Az. Mark	356° 53' 28.7"
	-360°	-360° 00' 00.0"
Azimuth	1 – Az. Mark	104° 04' 38.2"

The angular error of closure is the difference between the calculated azimuth of the closing line to the known or "true" azimuth

$$\begin{aligned} E_\angle &= \left(\text{Az. } 1 \rightarrow \text{Az Mark}_{\text{Calc}}\right) - \left(\text{Az. } 1 \rightarrow \text{Az Mark}_{\text{True}}\right) \\ &= \left(104^\circ\ 04'38.3''\right) - \left(104^\circ\ 05'05.0''\right) \\ &= -26.7'' \end{aligned}$$

The correction per angle is found using

$$\text{cpa} = -\frac{E_\angle}{n} = -\frac{-26.7''}{6} = 4.45''$$

The adjusted azimuths become

Line	Preliminary Azimuth	Correction	Adjusted Azimuth
1-2	216° 30’ 53.0”	+04.5”	216° 30’ 57.5”
2-3	139° 41’ 28.2”	+08.9”	139° 41’ 37.1”
3-4	60° 12’ 07.6”	+13.4”	60° 12’ 21.0”
4-5	356° 03’ 27.1”	+17.8”	356° 03’ 44.9”
5-1	291° 11’ 09.6”	+22.3”	291° 11’ 31.9”
1-Az Mk	104° 04’ 38.3”	+26.7”	104° 05’ 05.0”

In this particular example, the correction accumulates when being added to the preliminary azimuth. The first line receives the computed cpa. The second preliminary azimuth must be adjusted not only by the cpa but must also accommodate the adjustment that was placed on the first line. In other words, the correction is 2*cpa. The next line receives a 3*cpa correction. This continues along the entire traverse, each new azimuth receiving he same correction as the previous line plus the cpa for the angle measured at the new station.

The latitudes and departures are shown as follows in the Excel program:

Traverse Adjustment Program						
		Azimuth				
Sta	Dist	Deg	Min	Sec	Departure	Latitude
1						
	483.406	216	30	57.5	-287.649	-388.509
2						
	446.622	139	41	37.1	288.908	-340.592
3						
	425.557	60	12	21	369.305	211.453
4						
	384.926	356	3	44.9	-26.432	384.017
5						
	369.173	291	11	31.9	-344.207	133.455
1						
	2109.684				-0.075	-0.176

The error in closure for the latitude and departure are:

$$e_D = DX = \sum_{i=1}^{n-1} Lat_{i,i+1} = -0.075'$$

$$e_L = DY = \sum_{i=1}^{n-1} Dep_{i,i+1} = -0.176'$$

If the error in closure is expressed as e_D or e_L, the corresponding corrections can be expressed as cl_D and cl_L respectively, where the value of cl_D and cl_L are equal to the corresponding error in closure with opposite sign. In this example, cl_D = +0.075' while cl_L = +0.176'. The linear error of closure is computed as:

$$E_c = \sqrt{(-0.075)^2 + (-0.176)^2} = 0.191'$$

The azimuth of the closing line becomes[2]

$$\beta_e = \tan^{-1}\left(\frac{-0.075'}{-0.176'}\right) = 203^\circ\ 04'50''$$

To compute the accuracy ratio, or relative error of closure, of the traverse, divide the linear error of closure by the perimeter distance.

$$\text{Accuracy} = \frac{0.191'}{2109.684} = \frac{1}{11{,}045}$$
$$\approx 1 : 11{,}000$$

Traverse Adjustment

Compass Rule/Bowditch Method

There are several different types of traverse adjustment, only a couple of which will be discussed here. The first method is called the Compass Rule or Bowditch Method (named after Nathaniel Bowditch[3], see figure 5, who devised this method of adjustment

[2] The arctangent function requires some thought in its application. In order to determine the proper quadrant, one must evaluate the sign of the numerator and denominator. If both the numerator and denominator are positive then the azimuth is the in the northeast quadrant and β_e is the value returned from the calculator. If the signs are both negative, β_e will still be positive and the proper quadrant is in the southwest. Therefore, add the angle returned from the calculator to 180˚. In this example, the calculator will show an angular value of 23˚ 04' 50". Add 180˚ to place it in the proper quadrant and the correct azimuth becomes 203˚ 04' 50". If the numerator is positive and the denominator is negative, β_e will be a negative angle. In this particular case, the proper quadrant is the southeast. Therefore, add the angle to 180˚ and the result should fall between 90˚ and 180˚. Finally, when the numerator is negative and the denominator is positive then the azimuth of the line is in the northwest quadrant. The angle displayed in the calculator should be a negative value. Add this angle to 360˚ to obtain the proper azimuth. The value should fall between 270˚ and 360˚.

[3] Nathaniel Bowditch (March 26, 1773 - March 16, 1838) was a self-taught mathematician whose formal education ended at the age of 10. During his teens he studied at night and taught himself Latin and Greek as well as mathematics, science and astronomy. Taking the navigation tables of John Hamilton Moore, Bowditch made several thousand corrections to the calculations, eventually publishing ***The New American Practical Navigator*** that is still being published today.

for sea navigation purposes). This method of adjustment[4] makes certain assumptions about the data. First, it assumes that the errors in distance are proportional to the square root of the lengths of the traverse sides. Second, it assumes that the probable errors in these distances are equal to the probable error of the direction multiplied by the distance. The corrections to the latitudes and departures can be written as:

Figure 5. Nathaniel Bowditch.

$$c_L = \left(L/_P\right) cl_L$$

$$c_D = \left(L/_P\right) cl_D$$

where: c_L, c_D = corrections for latitudes and departures respectively,
L = length of the line,
P = perimeter distance around the traverse, and
cl_L, cl_D = closure correction in latitude and departure, respectively, for the traverse.

From the example,

For line 1-2:

[4] From Dr. David Gibson of the University of Florida: "Bowditch was an early American navigator and author of a famous book on navigation: American Practical Navigator. He developed a method for correcting a series of courses (vectors) at sea. Each course would be plotted on a chart from the known starting map point by plotting azimuths measured by compass and distances by "dead reckoning" (time x velocity). When "closing" onto a reliable map feature, the end of the last plotted course would miss the known map point forming an error of closure gap. Bowditch adjusted all intermediate vertex points parallel with the error of closure a distance proportional to the waypoint's distance into the route. A waypoint one quarter of the way along the route should be moved a quarter of the error of closure. This method was later applied to compass surveys and became known in surveying as the Compass Rule. To correct a traverse, adjust each latitude and departure separately:

$$\frac{\text{Corr. (lat, dep)}}{\text{Error (lat, dep)}} \equiv \frac{\text{Length of Line}}{\text{Trav Perimeter}}$$

Calculate the corrections and always reverse the sign of the error for the sign of the correction. Check that the sum of corrections equals the total error of latitude or departure with opposite sign." http://www.surv.ufl.edu/sur2101/LectOutlines/Lecture%2010%20Outline.doc. Last accessed 8//25/04.

$$c_L = \left(\frac{0.176'}{2109.684'}\right) 483.406' = +0.040'$$

$$c_D = \left(\frac{0.075'}{2109.684'}\right) 483.406' = +0.017'$$

Line 2-3:

$$c_L = \left(\frac{0.176'}{2109.684'}\right) 446.622' = +0.037'$$

$$c_D = \left(\frac{0.075'}{2109.684'}\right) 446.622' = +0.016'$$

etc ….

Using the spreadsheet:

Traverse Adjustment Program						
			Corrections to		Corrected	Corrected
Sta	Departure	Latitude	Departure	Latitude	Departure	Latitude
1						
	-287.649	-388.509	0.017	0.040	-287.632	-388.469
2						
	288.908	-340.592	0.016	0.037	288.924	-340.555
3						
	369.305	211.453	0.015	0.035	369.320	211.489
4						
	-26.432	384.017	0.014	0.032	-26.419	384.049
5						
	-344.207	133.455	0.013	0.031	-344.194	133.486
1						
	-0.075	-0.176	0.075	0.176	0.000	0.000

As a check, sum the latitudes and departures and their sum should add up to zero for a closed loop traverse.

Once the latitudes and departures have been adjusted, the surveyor may want to determine the coordinates of each point along the traverse. Given the X-coordinate of a point at one end of the line, the X-coordinate of the point of the other end is found by adding the departure from the first to second point to the X-coordinate of the first point. In a similar fashion, the latitude is applied to the Y-coordinate. This is shown as:

$$X_{n+1} = X_n + Dep_{n-(n+1)}$$

$$Y_{n+1} = Y_n + Lat_{n-(n+1)}$$

Assuming that the coordinates for point 1 are known to be X_1 = 5460.445' and Y_1 = 6238.012', the coordinates of the remaining traverse points are:

Traverse Adjustment Program				
	Corrected	Corrected		
Sta	Departure	Latitude	X	Y
1			7885.572	7097.635
	-287.632	-388.469		
2			7597.940	6709.166
	288.924	-340.555		
3			7886.864	6368.611
	369.320	211.489		
4			8256.185	6580.100
	-26.419	384.049		
5			8229.766	6964.149
	-344.194	133.486		
1			7885.572	7097.635
	0.000	0.000		

Make sure to compute the coordinates of point 1 again from the last traverse station, point 5 in this case. For a closed loop traverse, this coordinate must agree with the coordinates of the first point. This provides a check on the calculations. If the traverse is a closed traverse, but not a closed loop traverse, the final coordinates must agree with the known coordinates of that point.

Since the latitudes and departures are adjusted, the corresponding distances and directions do not coincide with these adjusted values. Therefore, the adjusted distances and directions need to be determined. The distance is found using the Pythagorean theorem:

$$D_{n-(n+1)} = \sqrt{(X_{n+1} - X_n)^2 + (Y_{n+1} - Y_n)^2}$$
$$= \sqrt{Dep^2_{n-(n+1)} + Lat^2_{n-(n+1)}}$$

The adjusted directions are computed using the arctangent function.

$$\alpha_{n-(n+1)} = \tan^{-1}\left[\frac{X_{n+1} - X_n}{Y_{n+1} - Y_n}\right] = \tan^{-1}\left[\frac{Dep_{n-(n+1)}}{Lat_{n-(n+1)}}\right]$$

For the example, the adjusted distances and azimuths for the lines are found as follows

Line 1-2:

$$D_{1-2} = \left[(5172.813' - 5460.445')^2 + (5849.543' - 6238.012')^2\right] = 483.364'$$

$$\alpha_{1-2} = \tan^{-1}\left[\frac{5172.813' - 5460.445'}{5849.543' - 6238.012'}\right] = 216° \, 31' \, 01.8''$$

Line 2-3:

$$D_{2-3} = \left[(5461.737' - 5172.813')^2 + (5508.988' - 5849.543')^2\right] = 446.604'$$

$$\alpha_{1-2} = \tan^{-1}\left[\frac{5461.737' - 5172.813'}{5508.988' - 5849.543'}\right] = 139° \, 41' \, 20.4''$$

etc….

The following table shows the results from the distance and azimuth calculations for all points within the traverse.

Traverse Adjustment Program						
				Adjusted		
			Adjusted	Azimuth		
Sta	X	Y	Distance	Deg	Min	Sec
1	7885.572	7097.635				
			483.363	216	31	1.8
2	7597.940	6709.166				
			446.604	139	41	20.4
3	7886.864	6368.611				
			425.588	60	12	9.7
4	8256.185	6580.100				
			384.957	356	3	53.4
5	8229.766	6964.149				
			369.172	291	11	50.6
1	7885.572	7097.635				

Transit Rule

A second traverse adjustment method is called the Transit Rule. This method is preferred when the angular measurements are more precise than the distance measurements. There is also some evidence to suggest that this method may be better than the Compass Rule, even when the distance and angle measurements have comparable precisions, when the traverse legs run parallel to the axes used in the traverse coordinate system. The corrections to the latitudes and departures for the traverse are computed as:

$$c_L = \frac{cl_L}{\sum|Lat|} s_L$$

$$c_D = \frac{cl_D}{\sum|Dep|} s_D$$

where: c_L, c_D = corrections to the latitudes and departures of the line,
cl_L, cl_D = closure corrections for the latitudes and departures for the traverse,
$\Sigma|Lat|$, $\Sigma|Dep|$ = sum of the absolute values of the latitudes and departures, and
s_L, s_D = latitude and departure length of the line (absolute values).

Using the example, the adjusted latitudes and departures are shown for the Compass Rule as follows. First, the sums of the absolute values of the latitudes and departures are found and then the corrections to the latitudes and departures.

$$\sum|Lat| = |-388.509| + |-340.592| + 211.453 + 384.017 + 133.455 = 1{,}458.026'$$

$$\sum|Dep| = |-287.632| + 288.924 + 369.320 + |-26.419| + |-344.194| = 1{,}316.501'$$

Line 1-2:

$$c_L = \left(\frac{0.176'}{1458.026'}\right) 388.509' = +0.047'$$

$$c_D = \left(\frac{0.075'}{1316.501'}\right) 287.649' = +0.016'$$

Line 2-3:

$$c_L = \left(\frac{0.176'}{1458.026'}\right) 340.592' = +0.041'$$

$$c_D = \left(\frac{0.075'}{1316.501'}\right) 288.908' = +0.016'$$

etc….

The adjusted values for all lines are presented in the following spreadsheet:

Traverse Adjustment Program						
	Transit Rule					
			Corrections to		Corrected	Corrected
Sta	Departure	Latitude	Departure	Latitude	Departure	Latitude
1						
	-287.649	-388.509	0.016	0.047	-287.633	-388.462
2						
	288.908	-340.592	0.016	0.041	288.925	-340.551
3						
	369.305	211.453	0.021	0.025	369.326	211.479
4						
	-26.432	384.017	0.002	0.046	-26.431	384.064
5						
	-344.207	133.455	0.020	0.016	-344.187	133.471
1						
	-0.075	-0.176	0.075	0.176	0.000	0.000

As with the Compass Rule, the next step is to compute the coordinates and adjusted distances and directions of the traverse lines. This is shown in the following table.

Traverse Adjustment Program								
						Adjusted		
	Corrected	Corrected			Adjusted	Azimuth		
Sta	Departure	Latitude	X	Y	Distance	Deg	Min	Sec
1			7885.572	7097.635				
	-421.071	-153.909			448.318	249	55	18.5
2			7464.501	6943.726				
	-159.696	-411.834			441.712	201	11	40.7
3			7304.805	6531.893				
	285.578	-380.184			475.494	143	5	15.7
4			7590.383	6151.708				
	430.139	49.312			432.957	83	27	36.2
5			8020.522	6201.020				
	187.862	339.346			387.877	28	58	8.2
6			8208.385	6540.367				
	-229.663	374.666			439.454	328	29	32.9
7			7978.722	6915.032				
	-93.150	182.603			204.989	332	58	22.2
1			7885.572	7097.635				

Again, make sure that the coordinates of point 1 are recalculated instead of just putting the known value into the solution.

Crandall Method

The third method of traverse adjustment is based on the method developed by Charles L. Crandall and referred to as the Crandall Method. Crandall based his development on least squares and assumed that after the angular error is accounted for that the random errors remaining were attributed to the distances. This approach implies that the angle measurements are much more accurate than the distance measurements.

To compute the corrections to the latitudes and departures, Crandall identified two intermediate calculations: A and B, shown as follows.

$$A = \frac{e_D\left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right) - e_L\left(\sum_{i=1}^{n}\frac{Dep^2}{100s}\right)}{\left(\sum_{i=1}^{n}\frac{Dep^2}{100s}\right)\left(\sum_{i=1}^{n}\frac{Lat^2}{100s}\right) - \left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right)^2}$$

$$B = \frac{e_L\left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right) - e_D\left(\sum_{i=1}^{n}\frac{Lat^2}{100s}\right)}{\left(\sum_{i=1}^{n}\frac{Dep^2}{100s}\right)\left(\sum_{i=1}^{n}\frac{Lat^2}{100s}\right) - \left(\sum_{i=1}^{n}\frac{Lat_i \cdot Dep_i}{100s_i}\right)^2}$$

where: e_L, e_D = total errors in latitude and departure,
Lat, Dep = latitude and departure length for a given line, and
s = distance between the two points.

The corrections applied to a particular line, c_ℓ, is found from the following relationships:

$$\begin{aligned} c_{\ell_1} &= Lat_1 A + Dep_1 B \\ c_{\ell_2} &= Lat_2 A + Dep_2 B \\ c_{\ell_3} &= Lat_3 A + Dep_3 B \\ &\vdots \\ c_{\ell_n} &= Lat_n A + Dep_n B \end{aligned}$$

With these corrections to the lines, the corrections to the latitudes and departures are given in this general form:

$$c_L = c_\ell \frac{Lat}{100s}$$

$$c_D = c_\ell \frac{Dep}{100s}$$

Substituting the value for c_ℓ into these equations yields the final form of the correction to the latitudes and departures.

$$c_L = A\frac{Lat^2}{100s} + B\frac{Lat \cdot Dep}{100s}$$

$$c_D = A\frac{Lat \cdot Dep}{100s} + B\frac{Dep^2}{100s}$$

Using the same example as before, the adjusted latitudes and departures using the Crandall Method are presented in the next table.

Point	Distance	Departure	Latitude	1	2	3	Departure	Latitude	Departure	Latitude
1										
	483.406	-287.649	-388.509	3.122412	1.711648	2.311811	0.048	0.064	-287.602	-388.445
2										
	446.622	288.9084	-340.592	2.597346	1.868874	-2.20321	-0.022	0.025	288.887	-340.567
3										
	425.557	369.3052	211.4531	1.05068	3.20489	1.835024	0.050	0.029	369.356	211.482
4										
	384.926	-26.4324	384.0174	3.831109	0.018151	-0.2637	-0.004	0.058	-26.436	384.075
5										
	369.173	-344.207	133.4551	0.482437	3.209293	-1.2443	0.002	-0.001	-344.204	133.454
1										
Sum =	2109.684	-0.07501	-0.17576	11.08398	10.01286	0.435631	0.075	0.176	0.000	0.000
A =	0.0156									
B =	0.0068									

The coordinates of the traverse points and the adjusted distances and azimuths are computed as before and are shown in the next table.

Traverse Adjustment Program								
					Adjusted			
	Adjusted					Azimuth		
Point	Departure	Latitude	X	Y	Distance	Deg	Min	Sec
1			5460.445	6238.012				
	-287.602	-388.445			483.326	216	30	57.5
2			5172.843	5849.567				
	288.887	-340.567			446.589	139	41	37.1
3			5461.730	5509.000				
	369.356	211.482			425.615	60	12	21
4			5831.086	5720.483				
	-26.436	384.075			384.984	356	3	44.9
5			5804.649	6104.558				
	-344.204	133.454			369.17	291	11	31.9
1			5460.445	6238.012				

A comparison of the results of the three different adjustments is given in the next Excel spreadsheet. While there are some differences, they are all less than 0.04'. It is evident that any of the adjustment techniques will have similar results, although one cannot state that this level of agreement will occur with all traverses.

Comparison of Adjustment Results						
	Compass Rule		Transit Rule		Crandall Method	
Point	Departure	Latitude	Departure	Latitude	Departure	Latitude
1						
	-287.632	-388.469	-287.633	-388.462	-287.602	-388.445
2						
	288.924	-340.555	288.925	-340.551	288.887	-340.567
3						
	369.32	211.489	369.326	211.479	369.356	211.482
4						
	-26.419	384.049	-26.431	384.064	-26.436	384.075
5						
	-344.194	133.486	-344.187	133.471	-344.204	133.454
1						

Area Calculations

Once the traverse has been adjusted, the area is normally calculated, assuming that the traverse is a closed polygon. The area of a polygon can be computed using the basic geometry for computing the area of a trapezoid (figure 6). The area is simply the average height times the width.

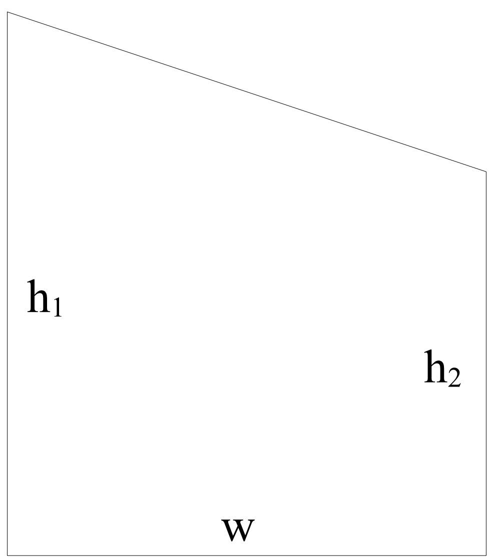

Figure 6. Area of a trapezoid.

$$\text{Area} = \frac{h_1 + h_2}{2} \cdot w$$

The double area is then written as $2A = (h_1 + h_2)w$.

There are two basic methods of computing the area of a polygon: Double Meridian Distance (DMD) or by coordinates. Both methods will give identical results.

Referring to figure 5, the area by Double Meridian Distance is based on the area of a series of trapezoids. Draw a meridian through the left most point in the traverse and then draw lines perpendicular to that meridian line to each of the other points in the traverse[5]. The width of the trapezoid is designated as the distance along the meridian (the latitude of the line) while the heights are the perpendicular distances from the meridian to the traverse point (departures of the line). Thus, from figure 5, the double area of traverse 1-2-3-4-5 is found as follows:

$$2A = 2A_{2,2',3,3'} + 2A_{3,3',4,4'} + 2A_{4.4',5} - 2A_{5,6,6'} - 2A_{1,1',6,6'} - 2A_{1,1',2,2'}$$

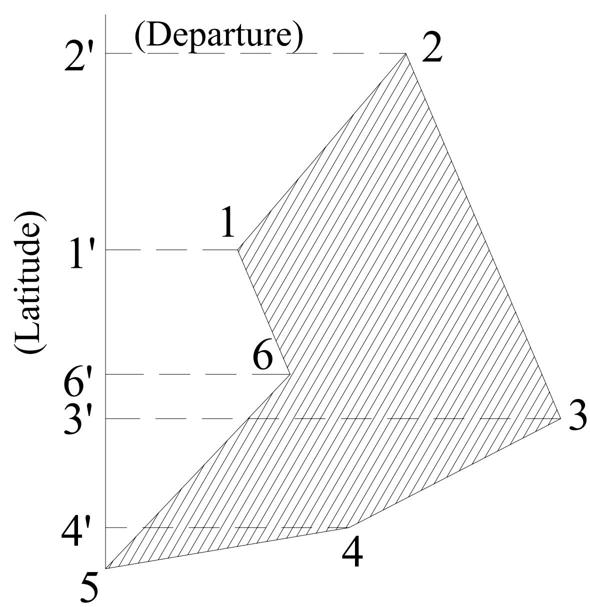

Figure 7. Area by Double Meridian Distance [from Moffitt and Bouchard, 1992].

In other words, the area of the traverse, shown as shaded in the figure, is the total area of the polygon 5-2'-2-3-4 minus the area of polygon 5-2'-2-1-6.

The computation of the DMD is given in 3 basic rules:

[5] Drawing the line through the western most point is done for convenience so that areas come out as positive quantities. In theory, the meridian line can be placed anywhere.

1. The DMD of the first line is equal to the departure of that line.
2. The DMD of each subsequent line is equal to the DMD of the preceding line plus the departure of the preceding line plus the departure of the line in question.
3. The DMD of the last line should equal the departure of the last line, with opposite sign.

In other words, the double area is found for the traverse in figure 5 using:

$$\begin{aligned}2A = {} & DMD_{5-6} * Lat_{5-6} + \left(DMD_{5-6} + Dep_{5-6} + Dep_{6-1}\right) * Lat_{6-1} \\ & + \left(DMD_{6-1} + Dep_{6-1} + Dep_{1-2}\right) * Lat_{1-2} \\ & + \left(DMD_{1-2} + Dep_{1-2} + Dep_{2-3}\right) * Lat_{2-3} \\ & + \left(DMD_{2-3} + Dep_{2-3} + Dep_{3-4}\right) * Lat_{3-4} \\ & + \left(DMD_{3-4} + Dep_{3-4} + Dep_{4-5}\right) * Lat_{4-5}\end{aligned}$$

Using the results from the Compass Rule adjustment, the area by DMD of the example traverse is:

2A:	(-287.632) (-388.469)	=	111,736.1154
	(-287.632 – 287.632 + 288.924) (-340.555)	=	97,514.5187
	(-286.340 + 288.924 + 369.320) (211.489)	=	78,653.6051
	(371.904 + 369.320 – 26.418) (384.049)	=	274,520.5295
	(714.806 – 26.418 – 344.194) (133.486)	=	45,945.0803
	2A	=	608,369.8489
	A	=	304,184.82 sq. ft.
		=	6.98 ac.[6]

The area of a polygon can also be written in terms of coordinates. The area can be written as

$$2A = X_1\left(Y_2 - Y_n\right) + X_2\left(Y_3 - Y_1\right) + \cdots + X_{n-1}\left(Y_n - Y_{n-2}\right) + X_n\left(Y_1 - Y_{n-1}\right)$$

or

$$2A = Y_1\left(X_2 - X_n\right) + Y_2\left(X_3 - X_1\right) + \cdots + Y_{n-1}\left(X_n - X_{n-2}\right) + Y_n\left(X_1 - X_{n-1}\right)$$

Using the values from the Compass Rule adjustment, the area is calculated in the following manner:

[6] 1 acre = 43,564 square feet

2A:	5460.445 (5849.543 – 6104.526)	=	-1,392,320.6474
	+ 5172.813 (5508.988 – 6238.012)	=	-3,771,104.8245
	+ 5461.737 (5720.477 – 5849.543)	=	-704,924.5476
	+ 5831.057 (6104.526 – 5508.988)	=	3,472,616.0237
	+ 5804.639 (6238.012 – 5720.477)	=	3,004,103.8449
	2A	=	608,369.8489
	A	=	304,184.82 sq. ft.
		=	6.98 ac.

This calculation is often presented in the following form:

Point	X	Y
1	5460.445	6238.012
2	5172.813	5849.543
3	5461.737	5508.988
4	5831.057	5720.477
5	5804.639	6104.526
1	5460.445	6238.012

Multiply the i^{th} value of X by the $(i+1)^{th}$ value of Y and sum those values. Lets assume this is Sum 1. Similarly, multiply the i^{th} value of Y by the $(i+1)^{th}$ value of X and sum those values. Lets call this Sum 2. Then the difference between the two sums is the double area. Hence,

Sum 1 = 163,487,060.260
- Sum 2 = 162,878,690.411
2A = 608,369.849
A = 304,184.92 sq. ft. or 6.98 ac.

References

Anderson, J. and E. Mikhail, 1998. **Surveying: Theory and Practice**, 7th edition, WCB McGraw Hill, Boston, 1167p.

Moffitt, F. and H. Bouchard, 1992. **Surveying**, 9th edition, Harper Collins, New York, 848p.