TAPING CORRECTIONS

SURVEYING ENGINEERING
FERRIS STATE UNIVERSITY

1. STANDARD LENGTH CORRECTION

Because a tape is manufactured, it is inherently in error due to imperfections present in the manufacturing process. While these imperfections could be reduced, it would be expensive. Thus, by comparison with a standard tape whose dimensions between the ends is known precisely, the true reading between the ends can be determined. The correction for standardization is found by comparing the nominal length to the calibrated length. For a 100' tape, this is shown as

$$C_\ell = \ell_s - 100' \qquad \textbf{(1)}$$

where: C_ℓ = correction due to the nominal length variation
ℓ_s = the actual length of the tape by standardization.

For a normal metric tape, the formula can be written as

$$C_\ell = \ell_s - 30m \qquad \textbf{(2)}$$

Like all measurements, one can not assume that the calibrated length (ℓ_s) is the absolute true value since errors exist in the calibration process and in the calibration line. While this error is small, to obtain a reasonable estimate of the expected error in a distance, it should be considered. This is done by estimating the uncertainty in the calibrated length since that is the only term in the equation that can vary. If this estimate is e_ℓ, then one finds the total uncertainty due to errors in calibration as

$$e_\ell^L + \pm e_\ell \cdot n \qquad \textbf{(3)}$$

where n is the number of tape lengths. This error would not tend to compensate.

EXAMPLE: A 100' tape is used to measure a distance at 705.76'. The true length of the tape is 100.02'. What is the correct true length of the line?

SOLUTION: ℓ_s = 100.02'
n = 7.06 tape length

the correction per tape length is

$$c_\ell = \ell_s - 100'$$
$$= 100.02' - 100'$$
$$= 0.02 \text{ ft/tape length}$$

the total tape correction for the line becomes

$$c_\ell^L = c_\ell \cdot n$$
$$= (0.02')(7.06)$$
$$= 0.14'$$

the correct length is

$$T = R + C$$
$$= 705.76' + 0.14'$$
$$= 705.90'$$

If the uncertainty of ℓ_s is e_ℓ = ± 0.004', the total uncertainty of the length of the line due to the uncertainty in ℓ_s is:

$$e_e^L = + e_\ell \cdot n$$
$$= + (0.004')(7.04)$$
$$= + 0.0028'$$

the length is 705.90' + 0.03'

<u>EXAMPLE</u>: The surveyor needs to set out two iron pins 600.00' apart. The tape used is known to be 100.02'. What length should the surveyor measure in the field?

SOLUTION: The correction per tape length is:

$$C_\ell = \ell_s - 100'$$
$$= 100.02 - 100$$
$$= 0.02 \text{ ft/ tape length}$$

The total tape corrections becomes:

$$C = C_\ell \cdot n$$
$$= (0.02')(6)$$
$$= 0.12'$$

The desired field reading is:

$$T = R + C \Rightarrow R = T - C$$
$$= 600.00' - 0.12'$$
$$= 599.88'$$

2. TEMPERATURE CORRECTION

All things in nature are prone to dimensional changes as temperature fluctuates. In man-made structures, this is accounted for in the construction process by putting in expansion joints, as an example. The steel tape is very susceptible to dimensional change due to variation in temperature. Normally, the temperature used in standardizing the tape is 68° F (20° C). Thus, when the tape temperature is less than 68° F, the length of the tape will be less than its standard length and vice versa. The correction for temperature is found by the formula:

$$C_t = \ell\alpha(t - t_s) \qquad \textbf{(4)}$$

where:
- C_t = correction due to temperature
- ℓ = length of the tape
- α = the coefficient of thermal expansion given as
  - α = 0.00000645 per 1° F
  - α = 0.0000116 per 1° C
- t = field temperature
- t_s = standard temperature (normally 68° F or 20° C).

To determine the uncertainty in length due to the error in temperature, differentiate equation (4) with the variable t

$$\frac{\partial C_t}{\partial t} = \ell\alpha \qquad \textbf{(5)}$$

Then from the general formula for propagation of random errors, the uncertainty in one tape length due to the error in temperature is

$$e_t^{\ell} = \sqrt{(\ell\alpha)^2 e_t^2} \qquad \textbf{(6)}$$

Yielding

$$e_t^{\ell} = \pm \ell \alpha e_t \quad (7)$$

where: e_t^{ℓ} = error in one tape length
e_t = error in determining temperature for that tape length.

The error in the total length of the line is then found by

$$e_t^{L} = \pm \ell \alpha e_t \sqrt{n} \quad (8)$$

This assumes that the temperature was read for each tape length and that the uncertainty in the temperature, e_t, is the same. Generally, though, only an average temperature is used in determining the tape correction over the entire line. This means that the error would not tend to compensate and thus the total error estimate would be

$$e_t^{L} = \pm \ell \alpha e_t n \quad (9)$$

Designating L as the total length of the line (i.e., L = ℓn), equation (9) becomes

$$e_t^{L} = \pm L \alpha e_t \quad (10)$$

3. SAG CORRECTION

If the tape is only supported at the ends, it will sag in the center and the amount of this sag is dependent upon the weight of the tape per unit length and the applied tension. The tape forms an arc approximating either a catenary curve or parabola. From figure 1 one can see the effect of this curve. Assuming the figure is a parabola, the equation of the parabola is

$$y = Ax^2 \quad (11)$$

and the slope becomes upon differentiation

$$\frac{dy}{dx} = 2Ax \quad (12)$$

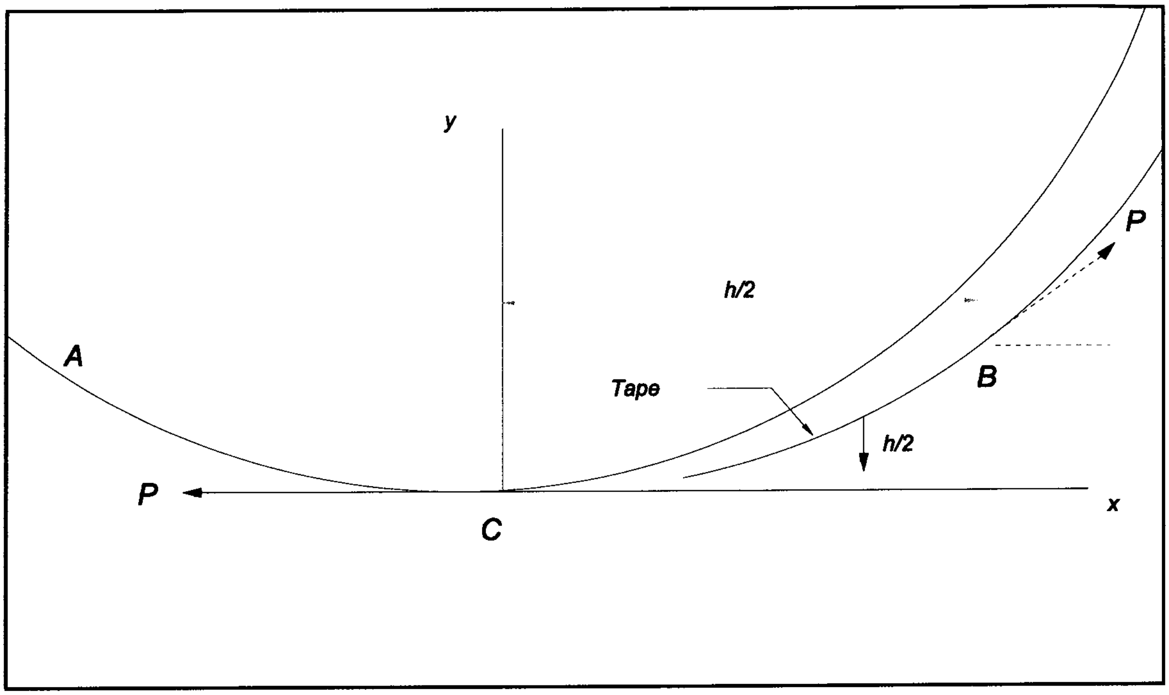

Figure 1.

From figure 2 the component horizontal and vertical forces can be found by the basic trigonometric relations as

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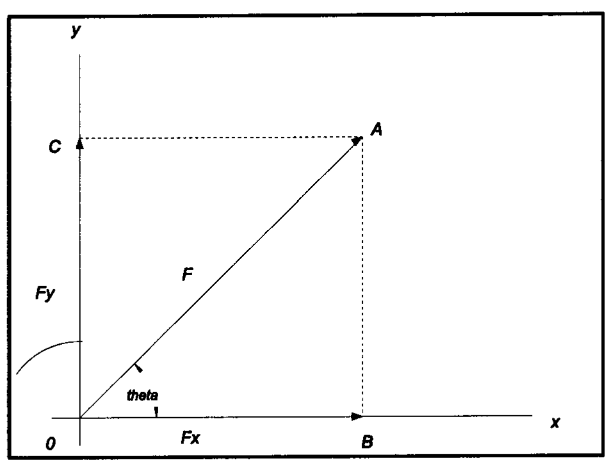

Figure 2 Resolution of a force into components.

Designating the force (F) as the applied tension (p), i.e., F=p, one then finds that

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(14)

$$\text{in } \alpha = \frac{w}{2}$$

Again from figure 2 one can see the following relationship

$$\tan\theta = \frac{Fy}{Fx} = \frac{\sin\theta}{\cos\theta} = \frac{W}{2Fx} \tag{15}$$

At support B, we have

$$\tan\theta = \frac{dy}{dx} = \frac{2Ah}{2} = Ah \tag{16}$$

Rearranging (16)

$$A = \frac{\tan\theta}{h} = \frac{W}{2hFx} \tag{17}$$

and

$$\frac{dy}{dx} = \frac{2Wx}{2hFx} = \frac{Wx}{hFx} \tag{18}$$

The length of a curved surface can be found through calculus by taking very small segments and adding them together (figure 3). Thus, one can write

$$ds^2 = dx^2 + dy^2 \tag{19}$$

Differentiating yields

$$\left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2 \tag{20}$$

or

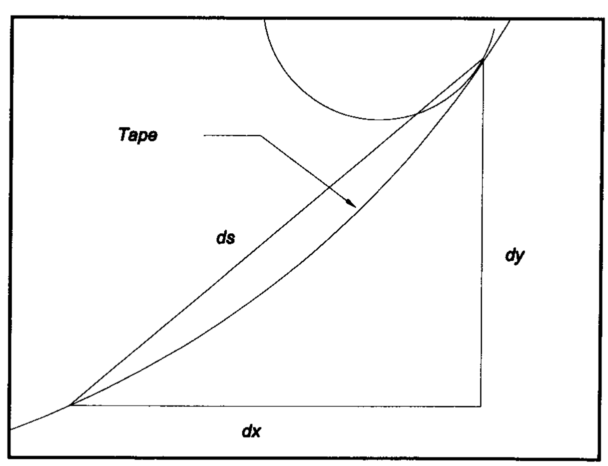


Figure 3

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \tag{21}$$

Integrating, the total length of the curve is

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \tag{22}$$

Using the notation developed for the tape, equation (22) becomes

$$\frac{\ell}{2} = \int_0^{\frac{h}{2}} \sqrt{1 + \left(\frac{Wx}{hFx}\right)^2}\, dx \tag{23}$$

Using the binomial theorem, which is generally given as

$$= x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)\ (n-2)}{3!}. \tag{24}$$

and expanding the term under the radical results in

$$\ell = 2\int_0^{\frac{h}{2}} \left[1 + \frac{W^2x^2}{2h^2 2Fx^2} - \frac{W^4x^4}{8h^4 8Fx^4} + \cdots\right] dx \tag{25}$$

Integrating and keeping only the first two terms yield

$$\ell = 2\left[\frac{h}{2} + \frac{W^2h}{48Fx^2}\right] \tag{26}$$

Thus

$$\ell = h + \frac{W^2h}{24Fx^2} \tag{27}$$

As the horizontal force approaches the tension and as the horizontal distance approaches the curve, the error due to sag is

$$\ell - h = \frac{W^2 \ell}{24 p^2} = E_s \tag{28}$$

and the correction is

$$C_s = -\frac{W^2 \ell}{24 p^2} \tag{29}$$

where the weight, W, and length, ℓ, is that between the portion of the tape suspended. Equation (29) can be expressed in a slightly different form if the weight of the tape per foot is employed. In that case, designating w as the weight per foot, the correction for sag is

$$C_s = -\frac{w^2 \ell^3}{24 p^2} \tag{30}$$

The sag correction is always negative. From equations (29) and (30) one can see that the sag varies with tension. Thus, applying the general propagation formula by taking the derivative of equation (29) with respect to the tension, p, results in the uncertainty error shown as

$$e_s^{\ell} = \pm \frac{W^2 \ell}{12 p^3} e_p \tag{31}$$

where: e_s^{ℓ} = error in one tap length
e_p = error in tension for one tape length

If we assume the same care in applying the tension and the amount of tension applied to all tape lengths, the error due to sag for

$$e_s^L = \pm \frac{W^2 \ell}{12 p^3} e_p \sqrt{n} \tag{32}$$

4. TENSION CORRECTION

From physics, the applied stress of a wire is defined as the force per unit area and the resultant strain is the elongation per unit length. Thus

$$STRESS = \frac{p}{A} \qquad (a)$$

:nd **(33)**

$$STRAIN = \frac{e}{\ell} \qquad (b)$$

where: p = tension (force)
A = cross-sectional area of wire (tape)
e = elongation of the tape produced by the applied tension
ℓ = length of the tape

According to Hooke's Law, force (p) is proportional to e and stress is proportional to strain. Therefore,

$$\frac{p}{A} \propto \frac{e}{\ell} \qquad \textbf{(34)}$$

Inserting a constant (E) results in

$$\frac{p}{A} = E\frac{e}{\ell} \qquad \textbf{(35)}$$

or

$$E = \frac{p\ell}{Ae} \qquad \textbf{(36)}$$

The proportionality constant is called Young's modulus of elasticity. The error in tension is found by comparing the elongation produced by the standardized tension with that of the field tension. Designating p as the field tension and p_s as the standardized tension, we have

$$e = \frac{p\ell}{AE} \qquad and \qquad e = \frac{p_s\ell}{AE} \qquad \textbf{(37)}$$

from which the correction is found by the relationship

$$C_p = \frac{p\ell}{AE} - \frac{p_s\ell}{AE} \qquad \textbf{(38)}$$

yielding the normal form of the tension correction

$$C_p = \frac{(p - p_s)\,\ell}{AE} \tag{39}$$

Young's modules of elasticity is normally between 28,000,000 psi to 30,000,000 psi (pounds per square inch). Looking for the uncertainty error in the tension correction formula, the only variable considered is the field tension. From error propagation theory

$$e_p^{\ell} = \pm\left(\frac{\ell}{AE}\right) e_p \tag{40}$$

where: e_p^{ℓ} = error of one tape length due to e_p
e_p = error in determining tension for one tape length.

Since tension must be read or estimated for each tape length, errors propagate according to the law of compensation. The error estimate for the total length of the line is

$$e_p^{L} + \pm\left(\frac{\ell}{AE}\right) e_p\sqrt{n} \tag{41}$$

It is possible to use the tension correction as a means of negating the effects of sag in the tape such that the tape will have its correct length as if it was supported throughout. This is called normal tension. Recall that the correction for sag is shown as:

$$C_s = -\frac{w^2\ell^3}{24p^2} \tag{42}$$

The error due to sag is

$$E_s = -C_s = \frac{w^2\ell^3}{24p^2} \tag{43}$$

The error for sag must equal the correction for tension. Thus

$$\frac{w^2\ell^3}{24p^2} + \frac{(p - p_s)\ell}{AE}$$

$$\frac{w^2\ell^3}{24p^2} - \frac{(p - p_s)\ell}{AE} = 0 \tag{44}$$

Let's define p_n as the normal tension, i.e. the tension applied to the tape to eliminate the sag. Then the above equation becomes.

$$\frac{w^2\ell^3}{24p_n} - \frac{(p_n - p_s)\ell}{AE} = 0$$

$$p_n^3 - p_n^2 p_s = \frac{w^2\ell^2\sqrt{AE}}{24} \tag{45}$$

$$p_n\sqrt{p_n - p_s} = w\ell\sqrt{\frac{AE}{24}} \tag{46}$$

The normal tension is then

$$p_n = \frac{0.204\, w\ell\sqrt{AE}}{\sqrt{p_n - p_o}} = \frac{0.204\, W\sqrt{AE}}{\sqrt{p_n - p_o}} \tag{47}$$

Notice that the normal tension is on both sides of the equation. One can use equation (46) and solve for the cubic equation or the normal equation can be approximated by

$$p_n = \sqrt[3]{\frac{AEW}{15}} \tag{48}$$

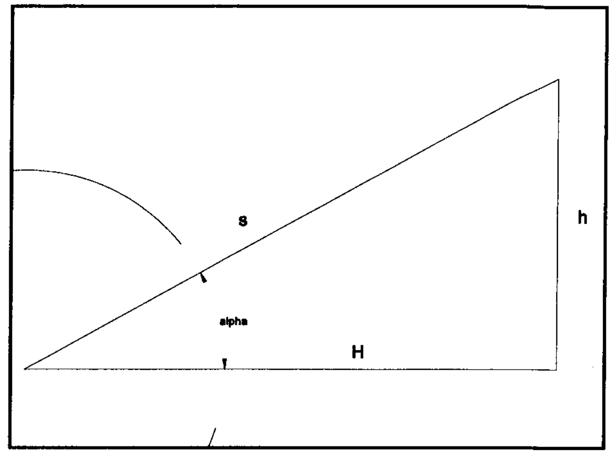
Substitute the value computed in equation (48) and substitute into equation (47). If the value computed in (47) is within 1 lb of the value inserted into the right hand side of the equation, then the normal tension is found. Otherwise, take the newly computed value of p and substitute it into the right hand side of the equation and compute a new normal tension.

5. SLOPE CORRECTION

In slope measurement, the distance measured in the taping process is s. The angle (α) is the elevation (or depression) angle measured by some means, usually a transit/theodolite. The horizontal distance is H. See figure 4. The difference in elevation between the two points is h.

The horizontal distance can be easily computed using the trigonometric formula

$$H = s\cos\alpha \qquad \textbf{(49)}$$



Another approach would be to compute a correction. The error in slope is found by

$$E_h = s - H \qquad \textbf{(50)}$$

The corresponding correction becomes

$$C_h = H - s \qquad \textbf{(51)}$$

Substitute the value for H from equation (49) results in

$$C_h = s\cos\alpha - s$$

$$C_h = s(\cos\alpha - 1) \qquad \textbf{(52)}$$

If one has a difference in height (h) between the two points, the horizontal distance can be found without measuring the angle. From the Pythagorean Theorem the horizontal distance is computed as

$$H = \sqrt{s^2 - h^2} \qquad \textbf{(53)}$$

As before, a correction can also be computed and then applied to the measured slope distance. The correction is found by

$$C_h = H - s \qquad \textbf{(54)}$$

$$C_h = \sqrt{s^2 - h^2} - s$$

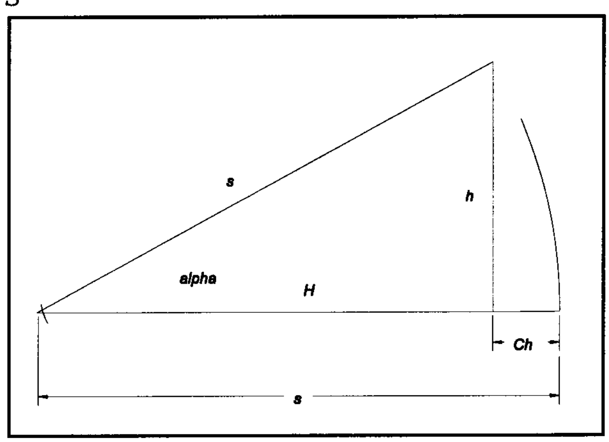
$$C_h = s\sqrt{1 - \frac{h^2}{s^2}} - s \qquad \textbf{(55)}$$

Recall the binomial theorem (equation 24) and expand the radical. This results in the formula

$$C_h = s\left[1 - \frac{h^2}{2s^2} - \frac{h^4}{8s^4} - \frac{h^6}{16s^6} - \frac{5h^8}{128s^8} - \ldots\right] - \qquad \textbf{(56)}$$

Reducing yields

$$C_h = -\frac{h^2}{2s} - \frac{h^4}{8s^3} - \frac{h^6}{16s^5} - \frac{5h^8}{128s^7} - \ldots \qquad \textbf{(57)}$$



For most cases only the first term in equation (57) is used to compute the correction. It is valid for slopes not exceeding 10-15% or when the required precision is less than 1:5,000. If either of these conditions are present then additional terms, as shown in equation (57), would be required. Thus, the correction for slope is normally expresses as

$$C_h = -\frac{h^2}{2s} \qquad \textbf{(58)}$$

An alternative derivation of the slope correction can be shown from the the following figure as:

$$h^2 = s^2 - H^2$$
$$= (s + H)(s - H) \quad \textbf{(59)}$$

The height (h) can be found by the Pythagorean Theorem as

If the slope is not too large, then the slope distance and the horizontal distance are almost the same. Therefore,

$$s + H = 2s \qquad (approximately) \quad \textbf{(60)}$$

Hence

$$2s(s - H) \quad \Rightarrow \quad \frac{h^2}{2s} = s - H \qquad (approxima \quad \textbf{(61)}$$

The correction for slope is found by

$$C_h = H - s \quad \textbf{(62)}$$

From equation (61) the correction becomes

$$C_h = -\frac{h^2}{2s} \quad \textbf{(58)}$$

The uncertainty in the horizontal distance is again found through error propagation. Equation (49) expresses the horizontal distance in terms of the variable vertical angle (α). Thus, the error in the horizontal distance is

$$e_h^{\ell} = \pm \ell (\sin \alpha)\, e_{\alpha} \quad \textbf{(63)}$$

where: e_h^{ℓ} = error in one tape length due to the uncertainty in α
ℓ = one tape length (slope)
e_{α} = error in the vertical angle α

If the vertical distance is used, then

$$e_h^{\ell} = \pm \frac{h}{\ell} \, e_H \tag{64}$$

where e is the error in the elevation difference. If the slope is uniform over the entire length of the line (L) and the vertical angle or elevation difference is over the entire line, then (63) becomes

$$e_h^{L} = \pm L (\sin\alpha) \, e_{\alpha} \tag{65}$$

and equation (64) becomes

$$e_h^{L} = \pm \frac{h}{L} \, e_V \tag{66}$$

If the slopes are different then the errors propagate according to the rule for addition of random errors

$$e_h^{L} = \pm \sqrt{(g_1 e_{h_1})^2 + (g_2 e_{h_2})^2 + \cdots + (g_n e_{h_n})^2} \tag{67}$$

where g_1, g_2, ⋯ g_n is the slope (h/ℓ) for a particular length ℓ_1, ℓ_2, etc.

6. <u>INCORRECT ALIGNMENT</u>

If there is more than one tape length measured in the field then errors due to incorrect alignment can result. Normally, it is the rear tapeperson's responsibility to ensure that the taping is done in a straight line. This is easily accomplished by placing a range pole at the terminus. This error is peculiar in that it is random in nature but systematic in effect. The reading will always be too high whether the pin is right or left of the line. For a 100-ft tape the error due to alignment amounts to 0.005' when one end of the tape is off line with respect to the other end by 1 foot and to only 0.001' when the error in alignment is 0.5' off. A lateral displacement from the true line would cause a systematic error

$$e_a^{\ell} = \frac{d_a}{2\ell} \tag{68}$$

where: e_a^{ℓ} = alignment error, one tape length
d_a = lateral displacement
ℓ = length of the tape

As one can see, this has the same form as the tape correction. A correction can be found for the entire length as

$$C_a = -n e_a^{\ell} \quad \textbf{(69)}$$

where: c_a = alignment correction
n = number of tape lengths

But this error is generally considered to be negligible since if higher accuracy is needed the ends of the tape can be aligned by using a transit/theodolite.

7. TAPE NOT STRAIGHT

It is not only important that the taping be performed in a straight line, it is equally important that the tape is straight. In taping through grass and brush or when the wind is blowing, it is impossible to have all parts of the tape in perfect alignment. The error arising from this is systematic and variable and is the same as measuring with a tape that is too short. The amount of error is less if the bend is in the center of the tape and increases as it gets closer to the ends. Only careful field procedures can reduce this error.

8. IMPERFECTIONS IN OBSERVATIONS

There are a number of sources of errors associated with taping that can be termed personal errors and/or blunders. Plumbing to mark the ends of the tape in the measurement can be a major contributor to the error in taping. As the plumbing process gets farther away from the points, the uncertainty increases. To reduce this problem, the tapeperson must stop the plumb bob from swaying by using the ground to steady the motion. They must also constantly check to see that the string has not shifted in its position and that the plumb bob is directly over the point. In all instances, the tape should be as close as possible to the points during the measurement. Other sources of errors associated with observations include:

- marking the tape ends with the tape fully supported
- adding or dropping a full tape length
- adding a foot or a decimeter
- other points incorrectly taken as 0- or 100-ft (or 0- and 30-m) marks on the tape
- reading numbers incorrectly
- calling numbers incorrectly or not clearly.

A summary of the random and systematic errors in taping are presented in the following two tables.

SOURCE	GOVERNING CONDITIONS CAUSES AND MANNER OF CUMULATION	ESTIMATED VALUE PER TAPE LENGTH
Plumbing to mark tape ends	Rugged terrain, breaking tape frequently; cumulative randomly $\propto \sqrt{n}$	0.05 - 0.10 ft. (15 - 30 mm)
Marking tape ends with tape fully supported	Tape graduated to hundredths of ft. or mm; cumulates randomly $\propto \sqrt{n}$	0.01 ft (3 mm)
Applying tension	Change in sag correction due to variations in tension of ±2 lbs or 0.9 kg from standard tension; cumulates $\propto \sqrt{n}$	0.01 ft (3 mm)
Determining elevation difference or slope angle (assume a max 6% slope)	in h = ±0.8 ft ±0.25 m in α = ±0°28'; cumulates $\propto$ n	0.050 ft (15 mm)
Standardization	Field tapes compared to standardized tape kept in office; cumulates $\propto$ n	0.005 ft (1.5 mm)

Table 1. Random errors in taping.

SOURCE	AMOUNT	ERROR OF 0.01 FT/100 FT OR 3 MM/30 M TAPE LENGTH CAUSED BY	MAKES TAPE TOO:
Tape not of standard length	$C_\ell = \ell_s - 100'$	-	Long or Short
Temperature	$C_t = \ell\alpha(t - t_s)$	15° F or 9° C	Long or Short
Change in pull or tension	$C_p = \dfrac{(p - p_s)}{AE}$	15 lbs or 4.2 kg	Long or Short
Sag	$C_s = -\dfrac{W^2\ell}{24P^2} = -\dfrac{w^2\ell^3}{24p^2}$	Δp = 0.6 lb or 1 kg too small	Short
Slope	$H = s\cos\alpha = \sqrt{s^2 - h^2}$ $C_h = s(\cos\alpha - 1) = -\dfrac{h^2}{2s}$ (approx)	1.4 ft or 0.42 m in h, 0°48' in α (Both from horizontal)	Short
Imperfect Horizontal Alignment	Same as slope	1.4 ft or 0.42 m	Short

Table 2. Systematic errors in taping.