Transformation Between Cartesian to Geodetic Coordinates - Borkowski's Iterative Method

Some useful angle functions

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d\cdot\dfrac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{array} \right.$$

$$r2d := \frac{180}{\pi}$$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

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Constants for GRS 80:

$a := 6378137$ $b := 6356752.3141$ $e_2 := 0.00669438002290$

$ep_2 := 0.00673949677548$ $f := 0.00335281068118$

a and b are the semi-major and semi-minor axes of the ellipsoid respectively, e_2 and ep_2 are the first and second eccentricities squared respectively, f is the flattening

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Given Cartesian Coordinates of the Point:

$X := 472239.0061$ $Y := -4493054.0133$ $Z := 4487560.5408$

Note that the notation used here generally follows the notation used in Borkowski [1989].

Solution:

The longitude, λ, is shown in ddd.mmsss format in highlighted region:

$\lambda := atan2(X, Y)$

$$fac := \left| \begin{array}{l} (-1) \text{ if } \lambda < 0 \\ 1 \text{ if } \lambda > 0 \end{array} \right.$$

fac is a factor to keep track of the sign of the angle for output.

$long := |\lambda|\cdot r2d + 0.000000001$

$fac\cdot dms(long) = -84.000000000$

long is the longitude

Designating r as the distance from the polar axis to the point

$r := \sqrt{X^2 + Y^2}$ $r = 4517803.010902$

$$c := \frac{a^2 - b^2}{\sqrt{(a \cdot r)^2 + (b \cdot Z)^2}}$$ $c = 0.006716$

$\Omega := \mathrm{atan2}(a \cdot r, b \cdot Z)$ $\mathrm{dms}(\Omega \cdot r2d) = 44.424096$

The initial estimate of the reduced or parametric latitude, Ψ

$\Psi_0 := \mathrm{atan2}(b \cdot r, a \cdot Z)$ $\mathrm{dms}(\Psi_0 \cdot r2d) = 44.541367$

Beginning the Newton-Raphson iteration where $f\Psi_1$ is the original function and $f\Psi_2$ is the first derivative.

$$f\Psi_1 := 2 \cdot \sin(\Psi_0 - \Omega) - c \cdot \sin((2 \cdot \Psi_0))$$

$$f\Psi_2 := 2 \cdot (\cos(\Psi_0 - \Omega) - c \cdot \cos(2 \cdot \Psi_0))$$

$\delta\Psi$ is the ratio of the change in the function and the first derivative to determine whether or not to stop the iterations. Here the minimum criteria is 0.00005" of arc.

$$\delta\Psi := \frac{f\Psi_1}{f\Psi_2}$$ $\mathrm{dms}(\delta\Psi) = 0.000000057$

$\Psi := \Psi_0 - \delta\Psi$ $\mathrm{dms}(\Psi \cdot r2d) = 44.541364$

$\Psi_0 := \Psi$ Make the current value of the reduced latitude equal to the initial estimate and calculate a new value for the reduced latitude.

$$f\Psi_1 := 2 \cdot \sin(\Psi_0 - \Omega) - c \cdot \sin((2 \cdot \Psi_0))$$

$$f\Psi_2 := 2 \cdot (\cos(\Psi_0 - \Omega) - c \cdot \cos(2 \cdot \Psi_0))$$

$$\delta\Psi := \frac{f\Psi_1}{f\Psi_2}$$ $\mathrm{dms}(\delta\Psi) = 0.000000000$

The critia has been met therefore stop the iterations.

$\Psi := \Psi_0 - \delta\Psi$ $\mathrm{dms}(\Psi \cdot r2d) = 44.541364$

The latitude, ϕ, of the point in ddd.mmsss format is shown in the highlighted region:

$$\phi := \mathrm{atan}\left(\frac{a}{b} \cdot \tan(\Psi)\right)$$ $\mathrm{dms}(\phi \cdot r2d) = 45.000000000$

The height above the ellipsoid is:

$H := (r - a \cdot \cos(\Psi)) \cdot \cos(\phi) + (Z - b \cdot \sin(\Psi)) \cdot \sin(\phi)$ $H = 300.0000$

Transformation Between Cartesian to Geodetic Coordinates - Borkowski's Non-Iterative Method

Some useful angle functions

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d\cdot \dfrac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{array} \right.$$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

$$r2d := \frac{180}{\pi}$$

Constants for GRS 80:

$a := 6378137$ $b := 6356752.3141$ $e_2 := 0.00669438002290$

$ep_2 := 0.00673949677548$ $f := 0.00335281068118$

a and b are the semi-major and semi-minor axes of the ellipsoid respectively, e_2 and ep_2 are the first and second eccentricities squared respectively, f is the flattening

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Given Quantities:

$X := 472239.0061$ $Y := -4493054.0133$ $Z := 4487560.5408$

Note that the notation used here generally follows the notation used in Borkowski [1989]

Solution:

The longitude, λ, shown in DDD.MMSSs format is shown in the highlighted region:

$\lambda := atan2(X, Y)$

$$fac := \left| \begin{array}{l} (-1) \text{ if } \lambda < 0 \\ 1 \text{ if } \lambda > 0 \end{array} \right.$$

fac is a factor to keep track of the sign of the angle for output.

$long := |\lambda|\cdot r2d + 0.000000001$

$fac\cdot dms(long) = -84.000000000$

Designating r as the distance from the polar axis to the point

long is the longitude

$$r := \frac{X}{\cos(\lambda)}$$

$r = 4517803.010902$

$z := Z$ $\quad z = 4487560.5408$

$$E := \frac{b \cdot z - \left(a^2 - b^2\right)}{a \cdot r} \qquad E = 0.980525$$

$$F := \frac{b \cdot z + \left(a^2 - b^2\right)}{a \cdot r} \qquad F = 0.999427$$

$$P := \frac{4}{3} \cdot (E \cdot F + 1) \qquad P = 2.63995$$

$$Q := 2 \cdot \left(E^2 - F^2\right) \qquad Q = -0.07485$$

$$D := P^3 + Q^2 \qquad D = 18.404296$$

$$v := \sqrt[3]{\sqrt{D} - Q} - \sqrt[3]{\sqrt{D} + Q} \qquad v = 0.018901$$

$$G := \frac{\sqrt{\left(E^2 + v\right)} + E}{2} \qquad G = 0.98532$$

$$t := \sqrt{G^2 + \frac{F - v \cdot G}{2 \cdot G - E}} - G \qquad t = 0.415198$$

The latitude, ϕ, of the point in DDD.MMSSs format is shown in the highlighted area:

$$\phi := \mathrm{atan}\left[a \cdot \frac{\left(1 - t^2\right)}{2 \cdot b \cdot t}\right] \qquad \mathrm{dms}(\phi \cdot r2d) = 45.00000000$$

The height above the ellipsoid is shown in the highlighted region

$$H := (r - a \cdot t) \cdot \cos(\phi) + (z - b) \cdot \sin(\phi) \qquad H = 300.000015$$

Transformation Between Cartesian to Geodetic Coordinates
Bowring Method

Some useful angle functions

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \frac{minutes}{60.0} + \frac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d\cdot \frac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \frac{mins}{100} + \frac{secs}{10000} \end{array} \right.$$

$$r2d := \frac{180}{\pi}$$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

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Constants for GRS 80 (semi-major axis (a), semi-minor axis (b), first eccentricity squared (e_2):

$a := 6378137$ $\quad b := 6356752.3141$ $\quad e_2 := 0.00669438002290$

Given Quantities:

$X := 472239.0061$ $\quad Y := -4493054.0133$ $\quad Z := 4487560.5408$

Note that the notation used here generally follows the notation used in Bowring [1976].

Solution:

e_{p2} is the second eccentricity squred (often depicted as e'^2)

$$e_{p2} := \frac{a^2 - b^2}{b^2}$$

$$e_{p2} = 0.0067394968$$

The longitude is shown in highlighted region in DD.MMSSss format

$\lambda := atan2(X, Y)$

$$fac := \left| \begin{array}{l} (-1) \ \text{ if } \ \lambda < 0 \\ 1 \ \text{ if } \ \lambda > 0 \end{array} \right.$$

fac is a factor to keep track of the sign of the angle for output.

$$long := |\lambda|\cdot r2d + 0.000000001$$

$$fac\cdot dms(long) = -84.000000000$$

long is the longitude

W is the distance from the polar axis to the point

$W := \sqrt{X^2 + Y^2}$ $W = 4517803.010902$

The initial estimate of the parameter β_o (referred to as the parametric or reduced latitude) is

$\beta_o := \operatorname{atan}\left(\frac{a}{b} \cdot \frac{Z}{W}\right)$ $\beta_o = 0.7837191028$

$\phi := \operatorname{atan}\left[\frac{Z + e_{p2} \cdot b \cdot \left(\sin\left(\beta_o\right)\right)^3}{W - a \cdot e_2 \cdot \left(\cos\left(\beta_o\right)\right)^3}\right]$ $\operatorname{dms}(\phi \cdot r2d) = 45.00000000$

$\beta := \operatorname{atan}\left(\frac{b}{a} \cdot \tan(\phi)\right)$ $\beta = 0.7837189446$

$\operatorname{dms}\left(\left|\beta - \beta_o\right| \cdot r2d\right) = 0.0000032637$

The second iteration. Since the difference between the two values for the two versions of the parametric latitude are above the criteria (criteria for β - β_o is 0.00005" and current value is 0.0326") make $\beta_o := \beta$ and recalculate β.

$\beta_o := \beta$ $\beta_o = 0.7837189446$

$\phi := \operatorname{atan}\left[\frac{Z + e_{p2} \cdot b \cdot \left(\sin\left(\beta_o\right)\right)^3}{W - a \cdot e_2 \cdot \left(\cos\left(\beta_o\right)\right)^3}\right]$

$\beta := \operatorname{atan}\left(\frac{b}{a} \cdot \tan(\phi)\right)$ $\beta = 0.7837189446$

$\operatorname{dms}\left(\left|\beta - \beta_o\right| \cdot r2d\right) = 0$

The criteria between the new estimate of β and the current estimate are met. Stop iteration.

The latitude is shown in the highlighted region in DD.MMSSss format

$\operatorname{dms}(\phi \cdot r2d) = 45.000000000$

N is the radius of curvature in the prime vertical

$$N := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi))^2}}$$ $N = 6388838.290174$

The geodetic height is given in the highlighted region

$$h := \frac{W}{\cos(\phi)} - N$$ $h = 300.0000$

Transformation Between Cartesian to Geodetic Coordinates
Czarnecki's Iterative Method

Some useful angle functions

$$dd(ang) := \begin{vmatrix} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree) \cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes) \cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{vmatrix}$$

$$radians(ang) := \begin{vmatrix} d \leftarrow dd(ang) \\ d \cdot \dfrac{\pi}{180.0} \end{vmatrix}$$

$$dms(ang) := \begin{vmatrix} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree) \cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1 \cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{vmatrix}$$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

$$r2d := \frac{180}{\pi}$$

Constants for GRS 80:

$a := 6378137$ $\qquad$ $ep_2 := 0.00673949677548$

a is the semi-major axis of the ellipse and ep2 is the second eccentricity squared

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Given Quantities:

$X := 472239.0061$ $\qquad$ $Y := -4493054.0133$ $\qquad$ $Z := 4487560.5408$

Note that the notation used here generally follows the notation used in Czarnecki [1988]

Solution:

The longitude, λ, is shown in ddd.mmsss format in hightlighted region:

$long := atan2(X, Y)$

$$fac := \begin{vmatrix} (-1) & \text{if } long < 0 \\ 1 & \text{if } long > 0 \end{vmatrix}$$

fac is a factor to keep track of the sign of the angle for output.

$\lambda := |long| \cdot r2d + 0.000000001$

$fac \cdot dms(\lambda) = -84.000000000$

long is the longitude

$$e_2 := \frac{ep_2}{1 + ep_2}$$

variable e_2 is the first eccentricity squared

$$GradE := 2\cdot\sqrt{X^2 + Y^2 + \frac{Z^2}{(1 - e_2)^2}}$$ $$GradE = 12778278.602186$$

$$\Delta E := X^2 + Y^2 + \frac{Z^2}{1 - e_2} - a^2$$ $$\Delta E = 3833393207.40625$$

The initial approximation of the height is h_0

$$h_0 := \frac{\Delta E}{GradE}$$ $$h_0 = 299.992928$$

Begin the first iteration for the height, h

$$h := \frac{\left(\Delta E + {h_0}^2\right)}{GradE}$$ $$h = 300$$

$$dh := \left|h - h_0\right|$$ $$dh = 0.00704$$

$$h_0 := h$$

Since dh > criteria, make h_0 = h and calculate a new height value

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Second iteration:

$$h := \frac{\left(\Delta E + {h_0}^2\right)}{GradE}$$ $$h = 300.0000$$

$$dh := \left|h - h_0\right|$$ $$dh = 0.00000$$

$$h_0 := h$$

Since h < criteria, stop the iteration and the height is displayed in the highlighted region above.

$$t := 2\cdot h\cdot\frac{ep_2}{GradE}$$ $$t = 316.4508927564 \times 10^{-9}$$

The latitude is identified in the highlighted area and is displayed in DDD.MMSSs format

$$\phi := \operatorname{atan}\left(\frac{1 + ep_2}{1 + t}\cdot\frac{Z}{\sqrt{X^2 + Y^2}}\right)$$ $$dms(\phi\cdot r2d) = 45.000000000$$

Transformation Between Cartesian to Geodetic Coordinates - Hirvonen and Moritz's Iterative Method

Some useful angle functions

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree) \cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes) \cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d \cdot \dfrac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree) \cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1 \cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{array} \right.$$

$r2d := \cdot$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

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Constants for GRS 80:

$a := 6378137$ $b := 6356752.3141$ $e_2 := 0.00669438002290$

$ep_2 := 0.00673949677548$ $f := 0.00335281068118$

a and b are the semi-major and semi-minor axes of the ellipsoid respectively, e_2 and ep_2 are the first and second eccentricities squared respectively, f is the flattening

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Given Cartesian Coordinates of the Point:

$X := 472239.0061$ $Y := -4493054.0133$ $Z := 4487560.5408$

Solution:

The longitude, λ, is shown in ddd.mmsss format in highlighted region:

$long := atan2(X, Y)$

$$fac := \left| \begin{array}{l} (-1) \text{ if } long < 0 \\ 1 \text{ if } long > 0 \end{array} \right.$$

fac is a factor to keep track of the sign of the angle for output.

$\lambda := |long| \cdot r2d + 0.000000001$

$fac \cdot dms(\lambda) = -84.000000000$

The distance from the polar axis to the point is designated as p.

$p := \sqrt{X^2 + Y^2}$ $p = 4517803.01090182$

The initial estimate of the latitude is desingate as ϕ_1 and the radius of curvature in the prime vertical is shown as N.

$$\phi_1 := \operatorname{atan}\left[\left(\frac{Z}{p}\right)\cdot\left(\frac{1}{1-e_2}\right)\right]$$

$$\operatorname{dms}(\phi_1\cdot r2d) = 45.00000326$$

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_1))^2}}$$

$$N = 6388838.29356868$$

Compute a new estimate of the latitude

$$\phi := \operatorname{atan}\left[\frac{Z}{p}\cdot\left(1 + \frac{e_2\cdot N\cdot\sin(\phi_1)}{Z}\right)\right]$$

$$\operatorname{dms}(\phi\cdot r2d) = 45.000000011$$

$$d\phi := |\phi - \phi_1|$$

$$\operatorname{dms}(d\phi\cdot r2d) = 0.000003253$$

$$\phi_1 := \phi$$

Since the difference in the current and new estimate of the latitude is > 0.03", make the new estimate of ϕ_0 be equal to ϕ and compute a new estimate of the latitude.

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_1))^2}}$$

$$N = 6388838.29018512$$

$$\phi := \operatorname{atan}\left[\frac{Z}{p}\cdot\left(1 + \frac{e_2\cdot N\cdot\sin(\phi_1)}{Z}\right)\right]$$

$$\operatorname{dms}(\phi\cdot r2d) = 45.000000000$$

$$d\phi := |\phi - \phi_1|$$

$$\operatorname{dms}(d\phi\cdot r2d) = 0.000000011$$

$$\phi_1 := \phi$$

Iterate one more time.

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_1))^2}}$$

$$N = 6388838.29017376$$

$$\phi := \operatorname{atan}\left[\frac{Z}{p}\cdot\left(1 + \frac{e_2\cdot N\cdot\sin(\phi_1)}{Z}\right)\right]$$

$$\operatorname{dms}(\phi\cdot r2d) = 45.000000000$$

$$d\phi := |\phi - \phi_1|$$

$$\operatorname{dms}(d\phi\cdot r2d) = 0.000000000$$

The latitude, ϕ, of the point in ddd.mmsss format is shown in the highlighted region above.

The height above the ellipsoid is shown in the highlighted area below.

$$h := \begin{cases} \dfrac{Z}{\sin(\phi)} - N + e_2 \cdot N & \text{if } (\phi \cdot r2d) > 45 \\ \dfrac{p}{\cos(\phi)} - N & \text{otherwise} \end{cases}$$

$h = 300.0000$

Transformation Between Cartesian to Geodetic Coordinates
Lin and Wang's Iterative Method

Some useful angle functions

$$\mathrm{dd}(ang) := \left| \begin{array}{l} degree \leftarrow \mathrm{floor}(ang) \\ mins \leftarrow (ang - degree) \cdot 100.0 \\ minutes \leftarrow \mathrm{floor}(mins) \\ seconds \leftarrow (mins - minutes) \cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{array} \right.$$

$$\mathrm{radians}(ang) := \left| \begin{array}{l} d \leftarrow \mathrm{dd}(ang) \\ d \cdot \dfrac{\pi}{180.0} \end{array} \right.$$

$$\mathrm{dms}(ang) := \left| \begin{array}{l} degree \leftarrow \mathrm{floor}(ang) \\ rem \leftarrow (ang - degree) \cdot 60 \\ mins \leftarrow \mathrm{floor}(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1 \cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{array} \right.$$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

$$r2d := \frac{1}{\;}$$

Constants for GRS 80:

$a := 6378137 \qquad b := 6356752.3141 \qquad e_2 := 0.00669438002290$

a and b are the semi-major and semi-minor axes of the earth, respectively, and e_2 is the second eccentricity squared.

Given Quantities:

$X := 472239.0061 \qquad Y := -4493054.0133 \qquad Z := 4487560.5408$

Note that the notation here generally follows the notation used in Lin and Wang [1995].

Solution:

The longitude is shown ind DDD.MMSSs format in the highlighted region.

$\lambda := \mathrm{atan2}(X, Y)$

$$fac := \left| \begin{array}{l} (-1) \ \text{ if } \ \lambda < 0 \\ 1 \ \text{ if } \ \lambda > 0 \end{array} \right.$$

fac is a factor to keep track of the sign of the angle for output.

$long := |\lambda| \cdot r2d + 0.000000001$

$fac \cdot \mathrm{dms}(long) = -84.000000000$

long is the longitude

W is the distance from the polar axis to the point.

$W := \sqrt{X^2 + Y^2}$

$W = 4517803.010902$

The initial estimate of the parameter m is

$$m := \frac{a \cdot b \cdot \left(a^2 \cdot Z^2 + b^2 \cdot W^2\right)^{\frac{3}{2}} - a^2 \cdot b^2 \cdot \left(a^2 \cdot Z^2 + b^2 \cdot W^2\right)}{2 \cdot \left(a^4 \cdot Z^2 + b^4 \cdot W^2\right)} \qquad m = 955118091.105655$$

The next 3 lines are solved for iteratively using the Newton-Raphson method until f_m converges to zero

$$f_m := \frac{W^2}{\left(a + \frac{2 \cdot m}{a}\right)^2} + \frac{Z^2}{\left(b + \frac{2 \cdot m}{b}\right)^2} - 1 \qquad f_m = 0.000000000$$

$$f_{pm} := (-4) \cdot \left[\frac{W^2}{a \cdot \left(a + \frac{2 \cdot m}{a}\right)^3} + \frac{Z^2}{b \cdot \left(b + \frac{2 \cdot m}{b}\right)^3}\right] \qquad f_{pm} = -0.000000000$$

$$m_1 := m - \frac{f_m}{f_{pm}} \qquad m_1 = 955118091.859665$$

End of iterative steps of the algorithm

W_e is the polar distance to the point P projected along the normal to the ellipsoid.

$$W_e := \left|\frac{W}{\left(1 + \frac{2 \cdot m}{a^2}\right)}\right| \qquad W_e = 4517590.878857$$

$$Z_e := b \cdot \sqrt{1 - \frac{W_e^{\,2}}{a^2}} \qquad Z_e = 4487348.408755$$

The geodetic latitude, ϕ, given in DDD.MMSSs format is shown in the highlighted area.

$$\phi := \operatorname{atan}\left(\frac{a^2 \cdot Z_e}{b^2 \cdot W_e}\right) \qquad \operatorname{dms}(\phi \cdot r2d) = 45.000000000$$

The height is shown in the next highlighted region.

$$H := \sqrt{\left(W - W_e\right)^2 + \left(Z - Z_e\right)^2} \qquad H = 300.0000$$

Transformation Between Cartesian to Geodetic Coordinates - Seemkooei's Iterative Method

Some useful angle functions

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \dfrac{minutes}{60.0} + \dfrac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d\cdot \dfrac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \dfrac{mins}{100} + \dfrac{secs}{10000} \end{array} \right.$$

$$r2d := \frac{180}{\pi}$$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

Constants for GRS 80:

$a := 6378137 \quad b := 6356752.3141 \quad e_2 := 0.00669438002290$

$ep_2 := 0.00673949677548 \quad f := 0.00335281068118$

a and b are the semi-major and semi-minor axes of the ellipsoid respectively, e_2 and ep_2 are the first and second eccentricities squared respectively, f is the flattening

Given Cartesian Coordinates of the Point:

$X := 472239.0061 \quad Y := -4493054.0133 \quad Z := 4487560.5408$

Note that the notation used here generally follows the notation used in Seemkooei [2002]

Solution:

The longitude, λ, is shown in ddd.mmsss format in hightlighted region:

$long := atan2(X, Y)$

$$fac := \left| \begin{array}{l} (-1) \text{ if } long < 0 \\ 1 \text{ if } long > 0 \end{array} \right.$$

fac is a factor to keep track of the sign of the angle for output.

$\lambda := |long|\cdot r2d + 0.000000001$

$fac\cdot dms(\lambda) = -84.000000000$

The distance from the minor axis to the point:

$p := \sqrt{X^2 + Y^2}$ $\quad p = 4517803.010902$

The initial estimate of the latitude

$\phi_0 := \text{atan2}\left[p\cdot(1 - e_2), Z\right]$ $\qquad$ $\text{dms}(\phi_0\cdot r2d) = 45.000003$

The radius of curvature in the prime vertical based on the initial estimate of ϕ

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_0))^2}} \qquad N = 6388838.293569$$

$$num := Z + N\cdot e_2\cdot(\sin(\phi_0))^3$$

$$den := p - N\cdot e_2\cdot(\cos(\phi_0))^3$$

Compute a new estimate of the latitude, ϕ

$\phi := \text{atan2}(den, num)$ $\qquad$ $\text{dms}(\phi\cdot r2d) = 45.000000000$

$\delta\phi := |\phi - \phi_0|$ $\qquad$ $\text{dms}(\delta\phi\cdot r2d) = 0.000003264$

$\phi_0 := \phi$

Difference in estimates of latitude. If $\delta\phi$ > 0.0001" then make ϕ_0 = 0 and callculate a new estimate for ϕ

Start 2nd iteration:

$$N := \frac{a}{\sqrt{1 - e_2\cdot(\sin(\phi_0))^2}} \qquad N = 6388838.290174$$

$$num := Z + N\cdot e_2\cdot(\sin(\phi_0))^3$$

$$den := p - N\cdot e_2\cdot(\cos(\phi_0))^3$$

$\phi := \text{atan2}(den, num)$

$\delta\phi := |\phi - \phi_0|$ $\qquad$ $\text{dms}(\delta\phi\cdot r2d) = 0.000000000$

Difference smaller than criteria therefore stop iterations

$\text{dms}(\phi\cdot r2d) = 45.000000000$

The latitude is shown in DDD.MMSSs format in the highlighted region above.

$$N := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi))^2}}$$

N is the radius of curvature based on the final value for the latitude.

$N = 6388838.290174$

The goedetic height is shown in the next highlighted area.

$$h := \frac{p}{\cos(\phi)} - N$$

$h = 300.0000$

Transformation Between Cartesian to Geodetic Coordinates - Sjoberg's Iterative Method

Some useful angle functions

$$dd(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ mins \leftarrow (ang - degree)\cdot 100.0 \\ minutes \leftarrow floor(mins) \\ seconds \leftarrow (mins - minutes)\cdot 100.0 \\ degree + \frac{minutes}{60.0} + \frac{seconds}{3600.0} \end{array} \right.$$

$$radians(ang) := \left| \begin{array}{l} d \leftarrow dd(ang) \\ d\cdot\frac{\pi}{180.0} \end{array} \right.$$

$$dms(ang) := \left| \begin{array}{l} degree \leftarrow floor(ang) \\ rem \leftarrow (ang - degree)\cdot 60 \\ mins \leftarrow floor(rem) \\ rem1 \leftarrow (rem - mins) \\ secs \leftarrow rem1\cdot 60.0 \\ degree + \frac{mins}{100} + \frac{secs}{10000} \end{array} \right.$$

dd takes degrees, minutes and seconds and places it in decimal degrees. radians converts decimal degrees to radians. dms convers decimal degree to degrees, minutes, seconds. r2d converts radians into decimal degrees

$$r2d := \frac{180}{\pi}$$

Constants for GRS 80:

$a := 6378137$ $b := 6356752.3141$ $e_2 := 0.00669438002290$

$ep_2 := 0.00673949677548$ $f := 0.00335281068118$

a and b are the semi-major and semi-minor axes of the ellipsoid respectively, e_2 and ep_2 are the first and second eccentricities squared respectively, f is the flattening

Given Cartesian Coordinates of the Point:

$X := 472239.0061$ $Y := -4493054.0133$ $Z := 4487560.5408$

Note that the notation used h ere generally follows that used in Sjoberg [1999].

Solution:

The longitude, λ, is shown in ddd.mmsss format in hightlighted region:

$\lambda := atan2(X, Y)$

$$fac := \left| \begin{array}{l} (-1) \text{ if } \lambda < 0 \\ 1 \text{ if } \lambda > 0 \end{array} \right.$$

fac is a factor to keep track of the sign of the angle for output.

$long := |\lambda|\cdot r2d + 0.000000001$

$fac\cdot dms(long) = -84.000000000$

The variable p is the distance from the polar axis to the point,

$p := \sqrt{X^2 + Y^2}$ $p = 4517803.010902$

The initail estimate of the tan ϕ, designated as α_0. Begin the iteration on α.

$$\alpha_0 := \frac{Z}{p} \cdot \frac{1}{1 - e_2} \qquad \alpha_0 = 1.000000$$

$$\alpha_{00} := \frac{Z}{p} \qquad \alpha_{00} = 0.993306$$

$$\delta := \frac{e_2 \cdot a}{p} \qquad \delta = 0.009451$$

$$A := 1 - e_2 \qquad A = 0.993306$$

Updating the estimate of the tan ϕ, designated as α_1.

$$\alpha_1 := \alpha_{00} + \delta \cdot \frac{\alpha_0}{\sqrt{1 + A \cdot \alpha_0^2}}$$

$$\delta\alpha := \operatorname{atan}\left(\left|\alpha_1 - \alpha_0\right|\right) \qquad \operatorname{dms}(\delta\alpha \cdot r2d) = 0.0000065053$$

$\alpha_0 := \alpha_1$ Since the difference in estimates of the latitude are greater than 0.06", make α_0 equal to the current estimate of the tan ϕ and calculate α_1 again.

$$\alpha_1 := \alpha_{00} + \delta \cdot \frac{\alpha_0}{\sqrt{1 + A \cdot \alpha_0^2}}$$

$$\delta\alpha := \operatorname{atan}\left(\left|\alpha_1 - \alpha_0\right|\right) \qquad \operatorname{dms}(\delta\alpha \cdot r2d) = 0.0000000218$$

$\alpha_0 := \alpha_1$ Update the calculation for α again.

$$\alpha_1 := \alpha_{00} + \delta \cdot \frac{\alpha_0}{\sqrt{1 + A \cdot \alpha_0^2}}$$

$$\delta\alpha := \operatorname{atan}\left(\left|\alpha_1 - \alpha_0\right|\right) \qquad \operatorname{dms}(\delta\alpha \cdot r2d) = 0.0000000001$$

Since the difference in α estimates is less than the criteria, stop the iteration and the latitude, ϕ , is given in the highlighted region in DDD.MMSSss format

$$\phi := \operatorname{atan}(\alpha_1) \qquad \operatorname{dms}(\phi \cdot r2d) = 45.000000000$$

N is the radius of curvature in the prime vertical. The value for the height is given in the highlighted area below.

$$N := \frac{a}{\sqrt{1 - e_2 \cdot (\sin(\phi))^2}}$$

$N = 6388838.290174$

$$h := \frac{p}{\cos(\phi)} - N$$

$h = 300.0000$